HUMBOLDT-UNIVERSITÄT ZU BERLIN

Berlin PUM Workshop 2012

Analysis and Application of the GFEM, XFEM, MM

22 - 24 August 2012
Berlin, Germany

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Preface

The intrinsic concept of the Partition of Unity Method (PUM) as devised by Babuška and Melenk may be found in many approaches such as Generalized Finite Element Methods (GFEM), eXtended Finite Element Methods (XFEM) and Meshless Methods (MM). Given this common concept, the aim of the Berlin PUM Workshop 2012 is to provide an opportunity for researchers and practitioners to discuss recent research results that may support a wide applicability in PUM related approaches. To build a foundation for these discussions, a number of experts has been invited to talk about their research. The covered topics will range from theoretical analysis of PUM based methods to applications and aspects of implementation.

We – the organizors – would like to take the chance to thank all participants for their contributions. Furthermore, we would like to express our gratitude to Ramona Klaass-Thiele, Friederike Hellwig and Paul Boeck for their support with the preparation of this workshop.

Andreas Byfut, **Martin Eigel** and **Andreas Schröder**

Berlin, August 2012

Contents

Internet Access

If you are from an institution that supports `eduroam` and have your device configured appropriately, you can access the `eduroam` wifi network without any further configuration. If that does not work for you, you may use the following procedure for guest internet access.

First, connect to the open wifi network `HU-VPN`. Second, start your favorite (state of the art) web browser and open any website. You will be redirected to `https://ssl.cms.hu-berlin.de/`. Third, enter the username and password that you can find on the back of your name badge. This will allow you to visit the websites of Humboldt University only. At last, to obtain full internet access, you have to push the `Start` button next to the `Network Connect` as can be seen below. For this procedure to work, you need to have administrator rights and to have a current Java software installed. If the `Network Connect` was successful, you have now full internet access.

For legal reasons, we have to associate your name and institution with your username and password. Please do not share your username and password with anybody else.

Lunch and Dinner

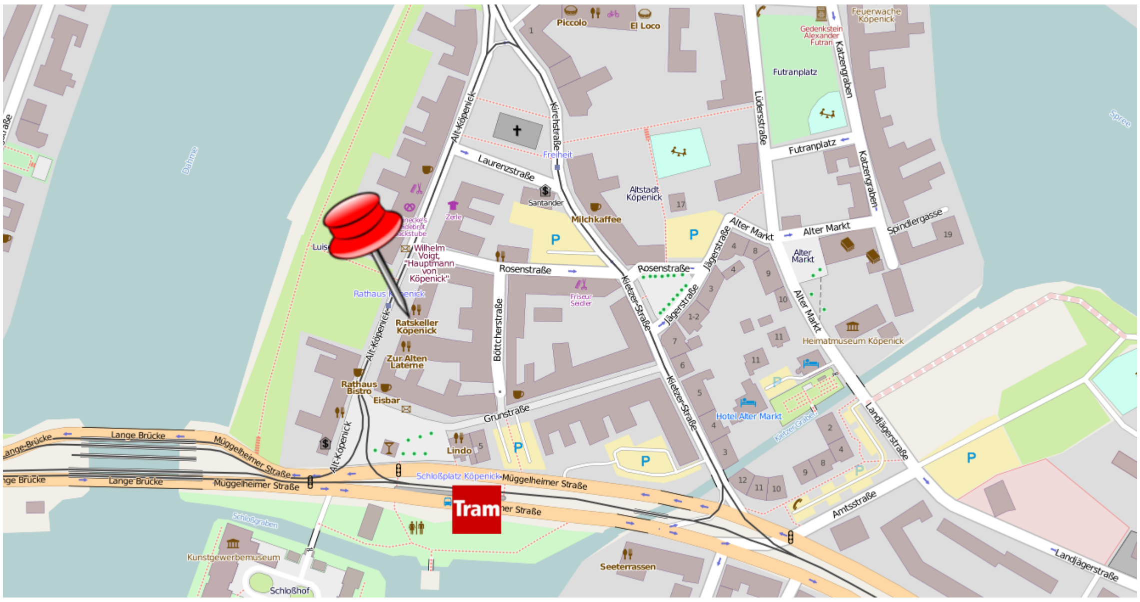
For the lunch breaks, we have reserved a table at a nearby cafeteria that provides five different dishes every day (including a vegetarian dish and a soup) as well as a salad bar. The joint dinner at a traditional German and a Turkish restaurant are optional opportunities to also discuss non-PUM related stuff.

Please note, that the lunch at the cafeteria and the dinner at the restaurants are not included in the workshop fee and have to be paid individually.

Dinner: Ratskeller Köpenick – August 22nd, 2012

Alt-Köpenick 21, 12555 Berlin.
`http://ratskeller-koepenick.de/`

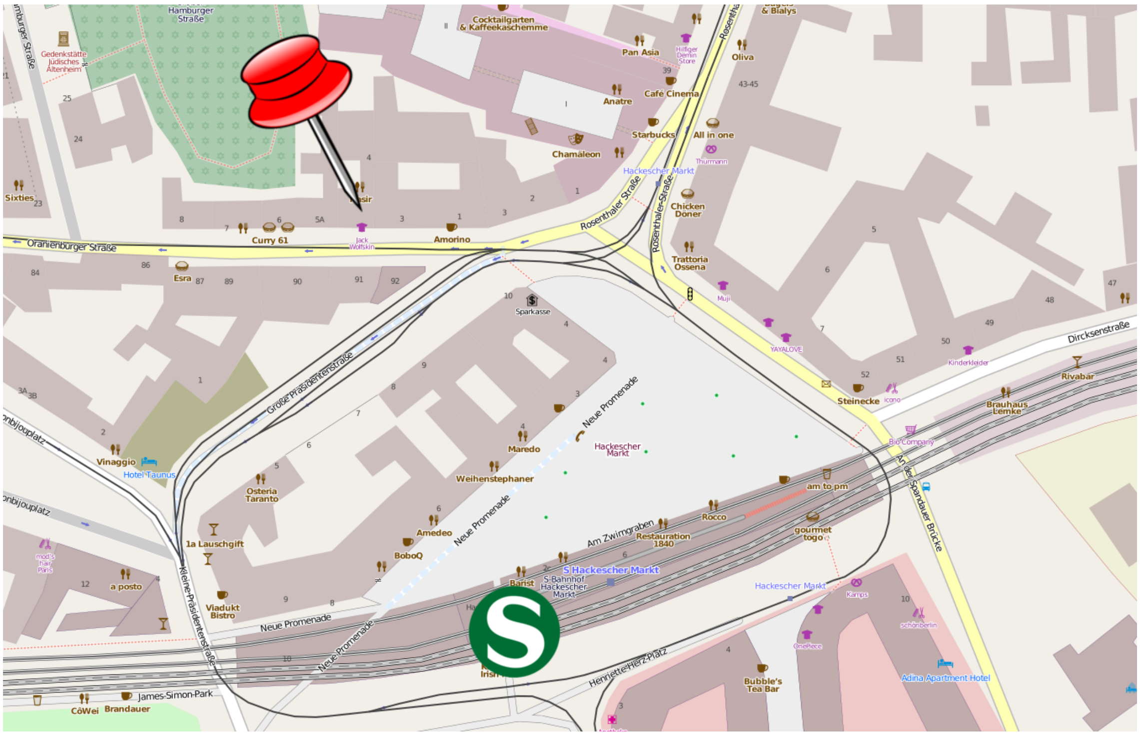
How to get there: Take the Tram 60 to "Mahlsdorf Süd" or the Tram 61 to "S Köpenick". Get out at "Schlossplatz Köpenick" and walk 4 minutes to the restaurant, see the map below.



Dinner: Hasir Mitte – August 23nd, 2012

Oranienburger Str.4, 10178 Berlin.
`http://www.hasir.de/eng/index.html`

How to get there: Walk to the train station "S Adlershof" (or alternatively take the Tram 60 or 61), take the S9 (direction: "Pankow") or the S8 (direction: "Birkenwerder") and get out at "Ostkreuz". Change to S5 (direction: "Spandau"), S75 (direction: "Westkreuz") or S7 (direction: "Potsdam") and get out at "Hackescher Markt". The walking distance to the restaurant is 5 minutes.


Hamburger Straße
Gedenkstätte Jüdisches Altenheim
Cocktailgarten & Kaffeekaschemme
Pan Asia
Oliva
Café Cinema
Anatre
Starbucks
All in one
Chamäleon
Hackescher Markt
Chicken Döner
Trattoria Ossena
Sixties
Curry 61
Amorino
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Esra
Sparkasse
Große Präsidentenstraße
Neue Promenade
Hackescher Markt
Maredo
Weihenstephaner
Osteria Taranto
Amedeo
BoboQ
1a Lauschgift
Vinaggio
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Am Zwirngraben
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gourmet togo
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S Hackescher Markt
Barist
Steinecke
Brauhaus Lemke
Rivabar
Dircksenstraße
An der Spandauer Brücke
Henriette-Herz-Platz
Bubble's Tea Bar
Adina Apartment Hotel
Kleine Präsidentenstraße
James-Simon-Park
CôWei
Brandauer

Program

Wednesday (August 22nd, 2012)

09:00-10:00	*Registration*
10:00-10:15	*Opening*
10:15-11:15	**J.M. Melenk** Numerical analysis for meshless methods: A survey
11:15-11:30	*Coffee Break*
11:30-12:30	**M.A. Schweitzer** Generalized Finite Element Methods – Enrichment, Adaptivity, Robustness
12:30-14:00	*Lunch Break*
14:00-14:30	**Y. Sudhakar**, J.P. Moitinho de Almeida, W.A. Wall A simple method for integration over enriched elements in PUM
14:30-15:00	**M. Shadi Mohamed**, M. Seaid, J. Trevelyan, O. Laghrouche Partition of unity finite element for solving time dependent heat transfer problems
15:00-15:30	**S. Mahmood**, O. Laghrouche, A. El-Kacimi, J. Trevelyan The partition of unity method for elastic wave problems in 3D
15:30-16:00	*Coffee Break*
16:00-16:30	**F.A. Faisal**, H.J. Al-Gahtani RBF meshless methods for Navier-Stokes equations
16:30-17:00	**E. Toroshchin**, O. Iliev Coupling of meshfree and finite volume discretizations for flow simulations in pleated filters
17:00-17:30	*Coffee Break*
17:30-18:30	**Y. Renard** The mathematical analysis of XFEM
19:30	*Joint Dinner at Ratskeller Köpenick*

Thursday (August 23nd, 2012)

09:00-10:00	**C.A. Duarte** The Generalized Finite Element Method as a framework for multiscale structural analysis
10:00-10:15	*Coffee Break*
10:15-11:15	**S. Bordas** Multiscale fracture, model reduction, enrichment and real-time simulations of cutting
11:15-11:30	*Coffee Break*
11:30-12:00	**P. Henning**, M. Ohlberger, B. Schweizer An adaptive multiscale finite element method
12:00-12:30	**D. Peterseim** Finite Element Computational Homogenization of Multiscale Elliptic Problems
12:30-14:00	*Lunch Break*

14:00-14:30	**M. Joulaian**, A. Düster Adaptive local enrichment for the finite cell method
14:30-15:00	**S. Amdouni**, M. Moakher, Y. Renard A local projection stabilization of fictitious domain method for elliptic boundary value problems
15:00-15:30	**A. Byfut**, A. Schröder Multi-level unsymmetric hanging nodes in *hp*-adaptive GFEM
15:30-16:00	*Coffee Break*
16:00-16:30	**K. Nissen**, V. Gravemeier, W.A. Wall Information-flux methods: Stable schemes for convection-dominated problems
16:30-17:00	**M. Winklmaier**, W.A. Wall A semi-Lagrangean time-integration approach for fixed-grid based flow problems in the XFEM
17:00-17:30	**B. Schott**, W.A. Wall A stabilized XFEM based fixed-grid approach for fluids with moving boundaries
17:30-17:45	*Coffee Break*
17:45-18:45	**J.M. Melenk** Operator adapted BEM for the Helmholtz equation
20:00	*Joint Dinner at Hasir Mitte*

Friday (August 24nd, 2012)

09:00-10:00	**S. Bordas** Simple advances in partition of unity enriched methods and implicit surface representation
10:00-10:15	*Coffee Break*
10:15-11:15	**Y. Renard** The contact condition on crack lips with XFEM
11:15-11:30	*Coffee Break*
11:30-12:00	**G. Bricteux**, E. Marchandise, J.-F. Remacle Alternative methods to represent embedded interfaces in a mesh
12:00-12:30	**Ch.B. Davis**, S.C. Brenner, L.-Y. Sung A generalized finite element method for the displacement obstacle problem of clamped Kirchhoff Plates
12:30-14:00	*Lunch Break*
14:00-14:30	**B. Dompierre**, B. Berthoul, M. Duflot, H. Minnebo Non-linear crack initiation and propagation
14:30-15:30	**M.A. Schweitzer** Partition of Unity Methods – Stability, Fast Solvers, Parallelization
15:30-15:45	*Closing*

Numerical analysis for meshless methods: A survey

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Keywords: *PUM, GFEM, XFEM, RKPM, survey*

ABSTRACT

Meshless methods such as the the PUM/gFEM, XFEM, RKPM, Particle PUM are generalizations of the classical FEM. Particular features include their ability to design approximation spaces that are tailored to a specific problem under consideration. Additionally, many meshless methods do not use explicitly use a mesh, thus circumventing the difficulties and costs associated with meshing. Many realizations of such meshless methods and their application to a variety of problem classes have demonstrated that these ideas can be successfully used in practice.

The numerical analysis for these nonstandard methods has made significant progress in recent years, include quadrature error analysis and the incorporation of essential boundary conditions. We will survey some of these results. Additionally, we will discuss the approximation properties of spaces that emmanate from moving least squares method and those generated by radial basis functions.

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Generalized Finite Element Methods Enrichment, Adaptivity, Robustness

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Keywords:

ABSTRACT

A simple method for integration over enriched elements in PUM

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Keywords: *divergence theorem, numerical integration over polygons, PUM, XFEM*

ABSTRACT

One of the crucial issues that decides the accuracy and robustness of PUM, especially while dealing with moving interfaces, is the numerical integration of shape functions over the enriched elements. Most widely, volume decomposition [1] or moment fitting methods [2] are used to address the above issue. In this talk, we present a simple method to perform integration over enriched elements using the divergence theorem. Though the divergence theorem is used extensively for integration in computational geometry, usually it is not used in FEM because the integrand of interest is not known explicitly. For a scalar function f, using the divergence theorem, the integration over $\mathcal{S} \subset \mathbb{R}^2$ whose boundary is given by $\mathcal{C}$ can be written as,

$$\int_{\mathcal{S}} f \, d\mathcal{S} = \oint_{\mathcal{C}} F \, d\mathcal{C}; \quad where \quad F = \int f dx \tag{1}$$

It is the computation of F that requires the integrand to be known explicitly. We eliminate this necessity by using 1D Gauss quadrature to evaluate F. We show that the method is very efficient for integrating polynomials over arbitrary polygons and it is extremely easy to implement. Some preliminary results in 3D are also presented.

REFERENCES

[1] N. Sukumar, N. Möes, B. Moran and T. Belytschko. Extended finite element method for three-dimensional crack modelling. *Int. J. Numer. Meth. Engng.*, Vol. **48**, 1549–1570, 2000.

[2] Y. Sudhakar and W.A.Wall. Quadrature schemes for arbitrary convex/concave volumes and integration of weak form in enriched partition of unity methods. *Comput. Methods. Appl. Mech. Eng.*, Submitted.

Partition of unity finite element for solving time dependent heat transfer problems

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Keywords: *PUFEM, time dependent heat transfer, radiative transfer*

ABSTRACT

Recovering the temperature field in an object subjected to sudden cooling, when removed out of a furnace for example, is usually a challenge for numerical methods such as the finite element method. The sudden drop of the ambient temperature while the object is still hot results in a sharp heat gradient on the outer surface, that can be hard to recover unless highly refined meshes are used. The cooling phase until this sharp gradient decreases may last for a considerable time span depending on the thermal conductivity. This can be a real burden when small time steps are needed to solve the problem. The accuracy in recovering this phase using numerical simulation can be crucial for accurately predicting different physical properties of the object. For different industries such a simulation can be highly efficient in reducing the costs.

In this work we propose to enrich the finite element solution space with exponential and hyperbolic functions to accurately recover sharp heat gradients on coarse meshes. Since two numerically different equations (energy and radiative transfer equations) are coupled to describe the problem, two different enrichments are proposed on the same mesh. This approach can reduce the computational effort whereas a similar approach with the finite element method can only be possible when two different meshes are used. However, using two different meshes may increase the computational effort due to the mapping required between the two meshes.

The Partition of Unity Method for elastic wave problems in 3D

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Keywords: *PUFEM, finite element method, plane waves, elastic waves, 3D*

ABSTRACT

Elastic wave propagation modelling arises in many engineering applications, including traffic vibrations from roads and railways, seismic induced vibrations and foundation construction, etc. The numerical modelling of these problems, in frequency domain by the conventional Finite Element Method (FEM), requires finite element grids sufficiently fine in comparison with the wavelengths, to get accurate results. When typically, the piecewise linear finite element is implemented, around ten nodal points per lower wavelength are needed, to ensure adequate resolution of the wave pattern. However, in the case of high frequency (small wavelength) and/or large domain of interest, the finite element mesh requires a large number of elements, and consequently the procedure becomes computationally expensive and impractical.

The principal objective is to develop finite elements, for three dimensional elastic wave problems, capable of containing many wavelengths per nodal spacing. This will be achieved by applying the plane wave basis decomposition to the 3D elastic wave equation. These elements will allow us to relax the traditional requirement of around ten nodal points per wavelength and therefore solve elastic wave problems without refining the mesh of the computational domain at each frequency. The accuracy and effectiveness of the proposed technique will be determined by comparing solutions for selected problems with available analytical solutions and/or to high resolution numerical solutions using conventional finite elements.

The method of plane wave basis decomposition used to develop wave finite elements for the two-dimensional elastic wave equation [1,2] will be extended to three dimensions. The governing equation is a vector equation and multiple wave speeds are present for any given frequency. In an infinite elastic medium, there are two different types of wave propagating simultaneously, the dilatation or compression wave (P), and the distortional or shear wave (S). The application of the Helmholtz decomposition theorem to the displacement field yields a scalar wave equation for the P-wave potential and a vector wave equation for the S-wave potential. The two wave equations are independent but the boundary conditions depend on both P-wave and S-wave potentials, thus coupling the associated scalar P-wave and vector S-wave equations.

REFERENCES

[1] El Kacimi A and Laghrouche O. Numerical Modelling of Elastic Wave propagation in Frequency Domain by the Partition of Unity Finite Element Method. International Journal for Numerical Methods in Engineering, 77: 1646-1669, 2009. *Int. J. Numer. Meth. Engng.*, Vol. **77**, 1646-1669, 2009.

[2] El Kacimi A and Laghrouche O. Improvement of PUFEM for the numerical solution of high frequency elastic wave scattering on unstructured triangular mesh grids. *Int. J. Numer. Meth. Engng.*, Vol. **84**, 330-350, 2010.

RBF Meshless Methods for Navier-Stokes Equations

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Keywords: *RBF, Navier-Stokes equations, Meshless Method, stream function formulation, Reynolds number*

ABSTRACT

In this study, we consider the Navier-Stokes equations (NSE) in stream function-vorticity form and stream function only form. We present a meshless RBF collocation method [1,2] for solving NSE in these two forms. We use the generalized multiquadric RBF with constant shape parameter and varying shape parameter. The method uses the technique of fundamental solutions, where the approximations have the advantage of verifying the linear part of the PDE.

The efficiency and accuracy of this method are investigated by running several experiments to two model problems: the driven cavity flow in square and a problem with known solution. Stream function contours for different values of Reynolds numbers are displayed to see the quality of the RBF solutions. The computed velocity profiles along the vertical and horizontal centerlines are in good agreement with available data in the literature. RBF solutions using uniformly spaced interior points are compared with RBF solutions using irregularly spaced Halton points. Also, we compare the effect of placing the boundary centers directly on the boundary as opposed to placement outside the domain. Several observations are made on the errors and the condition numbers and also some interesting findings are observed.

REFERENCES

[1] E. Kansa. Multiquadrics A scattered data approximation scheme with applications to computation fluid dynamics I. Surface approximations and partial derivatives estimates. *Computers and Mathematics with Applications*, 19: 127–145, 1990.

[2] G.E. Fasshauer *Meshfree Approximation Methods with MATLAB.*, World Scientific Publishers, 2007.

Coupling of Meshfree and Finite Volume Discretizations for Flow Simulations in Pleated Filters

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Keywords: *Meshfree, Finite Volume, Porous Medium*

ABSTRACT

The cylindrical pleated filter element is a common type of filters used in the industry for filtration of air, water, oil, etc. The computer simulations can significantly contribute to the design of improved/optimized filter elements, however the existing commercial and academic CFD software tools do not provide enough capability for performing such simulations. The CFD simulations for such type of filter elements are especially difficult due to several reasons, e.g., the complicated geometry of the pleated filtering medium, possible contacts between the filtering medium and the walls; possible deflection of pleats, etc.

This talk presents our current developments on coupling meshfree and Finite Volume Discretizations for flow simulations in pleated filters. The motivation for using such a discretization is as follows. The Finite Volume discretization on curvilinear boundary fitted grid suits very well the resolution of the porous filtering media and the accurate simulation of the flow through this porous media. Finite Volume discretization on regular grids might be beneficial for the subdomains with regular shape. Finally, the meshfree discretization is motivated by two issues: i) the complicated shape of the inter-pleat space; ii) the eventual deflection of the pleats would lead to degeneration of a grid, but can be easier handled by meshfree discretization.

First results on coupling meshfree and Finite Volume Discretizations will be presented. The first studies are done for a scalar elliptic equation. Next, the approach is tested on flow in a channel with and without filtering medium. Finally, simulations for a pleated filter geometry will be shown.

The mathematical analysis of XFEM

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Keywords: *XFEM, a priori error analysis.*

ABSTRACT

The aim of the presentation is to summarize the work on the mathematical analysis of the XFEM method developped in the references [1] to [7] with the co-authors E. Chahine, P. Laborde, J. Lasry, S. Nicaise, J. Pommier and M. Salaün.

The mathematical tools dedicated to the analysis of the XFEM method will be detailed. The optimality of some variants of XFEM will be analyzed and an interpretation of the mechanisms that allow XFEM to be a performing method will allow to interprete some aspects of the essence of XFEM.

REFERENCES

[1] P. Laborde, J. Pommier, Y. Renard, M. Salaün. High order extended finite element method for cracked domains. *Int. J. Numer. Meth. Engng.*, 64: 354–381, 2005.

[2] E. Chahine, P. Laborde, Y. Renard. Crack-tip enrichment in the XFEM method using a cut-off function. *Int. J. Numer. Meth. Engng.*, 75(6): 629–646, 2008.

[3] E. Chahine, P. Laborde, Y. Renard. Spider-XFEM, an extended finite element variant for partially unknown crack-tip displacement. *European Journal of Computational Mechanics*, 17(5–7): 625–636, 2008.

[4] E. Chahine, P. Laborde, Y. Renard. A reduced basis enrichment for the extended finite element method. *Math. Model. Nat. Phenom.*, 4(1): 88–105, 2009.

[5] J. Lasry, Y. Renard, M. Salan. Extended finite element method for thin cracked plates with kirchhoff-love theory. *Int. J. Numer. Meth. Engng.*, 84(9): 1115–1138, 2010.

[6] E. Chahine, P. Laborde, Y. Renard. A non-conformal extended finite element approach: Integral matching XFEM. *Applied Numerical Mathematics*, 61: 322–343, 2011.

[7] S. Nicaise, Y. Renard, E. Chahine. Optimal convergence analysis for the extended finite element method. *Int. J. Numer. Meth. Engng.*, 86: 528–548, 2011.

The Generalized Finite Element Method as a Framework for Multiscale Structural Analysis

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Keywords: *GFEM, higher-order, multiscale*

ABSTRACT

This presentation reports on recent advances of the Generalized Finite Element Method (GFEM) for problems with strong interactions between two or more scales. This method is based on the solution of interdependent global (structural) and local scale problems. The local problems focus on the resolution of fine scale features of the solution in the vicinity of, e.g., three-dimensional propagating cracks while the global problem addresses the macro scale structural behavior. The local solutions are embedded into the global solution space using the partition of unity method. The local problems are accurately solved using the *hp*-GFEM and thus the proposed method does not rely on analytical solutions. Fine scale computations can be performed at patches associated with nodes of coarse scale finite element meshes. These computations do not involve exchange of data among patches and are thus naturally parallelizable. Numerical examples demonstrating this are presented.

An a-priori error estimate for the proposed enrichment functions is presented along with numerical verification. The convergence analysis shows that the method can deliver the same accuracy as direct numerical simulations (DNS) while using meshes with elements that are orders of magnitude larger than in the DNS case. Furthermore, the method is fully compatible with the standard FEM and thus can be integrated with existing analysis software. We show an example of a non-intrusive implementation in Abaqus.

In this presentation we also report on extensions of the method to three-dimensional problems exhibiting localized nonlinear behavior. Our nonlinear model problem focuses on structures with plastic deformations at regions that are orders of magnitude smaller than the dimensions of the structural component. We show that the proposed GFEM can produce accurate nonlinear solutions at a reduced computational cost compared with available FEMs.

Simple advances in partition of unity enriched methods and implicit surface representation

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Keywords: *enriched FEM, enriched EFG, isogeometric BEM*

ABSTRACT

This talk present recent advances in partition of unity methods. It is organised in two parts. We start by discussing numerical integration techniques proposed in S. Natarajan's PhD thesis and later developed during his post-doc. We then present methods to address the deterioration of the conditioning number and blending. Finally, applications to various areas of solid mechanics within an enriched FEM and enriched EFG context (plates, gradient elasticity, etc.) and discuss a posteriori error estimation.

The second part of the talk presents new advances in implicit surface definition from parametric surfaces. First, a multi-level set method to represent arbitrary solids known by parametric representations, including sharp edges and corners is presented. Second, advances in (enriched) isogeometric and maximum entropy interpolants are discussed. In particular, a 3D isogeometric boundary element method based on T-splines is described.

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An adaptive multiscale finite element method

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Keywords: *Multiscale Finite Element, a posteriori error estimate, oversampling control*

ABSTRACT

In this presentation, we introduce an adaptive multiscale finite element method (MsFEM) for solving elliptic problems with rapidly oscillating coefficients. Starting from a general version of the MsFEM with oversampling, we present an a posteriori estimate for the H^1 -error between the exact solution of the problem and a corresponding MsFEM approximation. Our estimate holds without any assumptions on scale separation or on the type of the heterogeneity. The estimator splits into different contributions which account for the coarse grid error, the fine grid error and even the oversampling error. Based on the error estimate we construct an adaptive algorithm that is validated in numerical experiments.

Finite Element Computational Homogenization of Multiscale Elliptic Problems

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Keywords: *finite element method, a priori error estimate, convergence, multiscale method*

ABSTRACT

This talk presents a new approach for computational homogenization of elliptic problems with rough coefficients without any assumptions on scales. The new (variational) multiscale method is based on a local generalized finite element basis which consists of classical nodal basis functions and corresponding enrichment functions. The enrichment functions are solutions of the variational problem with the additional constraint that some quasi-interpolation of trial and test functions vanishes. This constraint ensures some surprising exponential decay of the enrichment functions and justifies their approximation on local vertex patches. The method represents unresolved scales in such a way that linear convergence with respect to the mesh size is preserved on arbitrary coarse meshes without any pre-asymptotic effects.

REFERENCES

[1] A. Malqvist and D. Peterseim. Localization of Elliptic Multiscale Problems. *arXiv:1110.0692v3.*

Adaptive local enrichment for the finite cell method

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Keywords: *FCM, hp-d, local enrichment, PUM*

ABSTRACT

The capability of using local enrichments is added to the standard Finite Cell Method (FCM) by means of the the *hp-d* method [1]. Following the general idea of the adaptivity, we also enrich the FCM approximation space by an appropriate enrichment space. The enrichment space can be provided either by choosing the pre-known functions that can describe the special phenomenon in the domain [2] or by a local *h*-, *p*- or *hp*-extension [3,4]. Here, irrespective of the definition of the enrichment space, additional shape functions are introduced on a superimposed mesh utilizing the *hp-d* method. This rises the possibility of using different mesh resolutions for the FCM space and enrichment space. The suggested method can be of special interest for the simulation of singularities, multi-scale problems, or in the case of heterogeneous materials where the solution exhibits weak discontinuities. Furthermore, this method enables us to take advantage of different solvers and software for each part of the approximation space. For the purpose of our discussion, difficulties in the standard FCM for the case of heterogeneous materials simulation are addressed in detail. The *hp-d* method and its implementation in the framework of the FCM are also described. Moreover, the performance of the method is investigated by several numerical examples.

REFERENCES

[1] E. Rank. Adaptive remeshing and *h-p* domain decomposition. *Comput. Methods Appl. Mech. Engrg.*, Vol. **101**, 299–313, 1992.

[2] N. Sukumar, D. L. Chopp, N. Moës, and T. Belytschko. Modeling holes and inclusions by level sets in the extended finite-element method. *Comput. Methods Appl. Mech. Engrg.*, Vol. **190**, 6183–6200, 2001.

[3] A. Düster, A. Niggl and E. Rank. Applying the *hp–d* version of the FEM to locally enhance dimensionally reduced models. *Comput. Methods Appl. Mech. Engrg.*, Vol. **196**, 3524–3533, 2007.

[4] D. Schillinger, A. Düster, and E. Rank. The *hp-d* adaptive Finite Cell Method for geometrically nonlinear problems of solid mechanics. *Int. J. Numer. Mech. Engng.*, Vol. **89**, 1171–1202, 2011.

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A local projection stabilization of fictitious domain method for elliptic boundary value problems

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Keywords: *Finite element method, Xfem, fictitious domain method, Poisson problem.*

ABSTRACT

The fictitious domain methods is a technique which allows to use regular structured meshes on a simple shaped fictitious domain containing the real domain. Generally, this technique is used for solving elliptic problems in domains with unknown or moving boundary without having to build a body fitted mesh. In this paper, we perform a study similar to [1] for a new stabilization technique applied to the fictitious domain method inspired by the X-FEM introduced in [1], [2], [3]. The principle of this new stabilization technique is to penalize the difference of the multiplier with its projection on some pre-defined patches. The advantage of this method is of at least threefold: the method is fully consistent, their is no use of another mesh than the (possibly cartesian) one of the fictitious domain and the additional term concerns only the multiplier and is not model dependent such as the Barbosa-Hughes stabilization technique. In this work we present theoretical convergence analysis of our new stabilization technique. We also validate this result by two and three-dimensional numerical experiments and we make comparison with the use of Barbosa-Hughes stabilization technique.

REFERENCES

[1] J. Haslinger and Y. Renard. A new fictitious domain approach inspired by the extended finite element method. *J. Numer. Anal.* Vol. **47**(2):1474-1499, 2009.

[2] E. Burman and P. Hansbo. Fictitious domain finite element methods using cut elements:I. A satabilized Lagrange multiplier method. *Comput. Meth. Appl. Mech. Engrg.*, Vol **199**, 2680–2686, 2010.

[3] E. Burman and P. Hansbo. Fictitious domain finite element methods using cut elements:II. A satabilized Nitsche method. *Applied Numerical Mathematics* Vol **62**(4), 328–341, 2012.

Multi-level unsymmetric hanging nodes in *hp*-adaptive GFEM

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Keywords: *GFEM, higher-order, hanging nodes, XFEM, fictitious domain method*

ABSTRACT

In this presentation, the concept of constrained approximation for standard higher-order finite element methods is carried over to the generalized finite element method (GFEM). As a consequence, meshes containing multi-level, unsymmetric hanging nodes may be employed in *h*- and *hp*-adaptive GFEM.

Considering the ability of GFEM to include almost arbitrary functions into its finite element space, one may conclude that adaptivity is generally unnecessary to approximate the solution of a given problem “well”. However, an (automatic) problem-dependent composition of a set of enrichment functions as well as an (automatic) inclusion into the finite element spaces is a non-trivial task to appropriately cover each and every possible feature of a sought solution. In modern higher-order finite element schemes, *h*- and *hp*-adaptivity based on a posteriori error control is a standard tool to (automatically) increase the approximation quality of a finite element scheme. This presentation will show, that these standard techniques may very well be applied for *h*- and *hp*-adaptive GFEM such as the XFEM or the fictitious domain method, see also [1].

For *h*- and *hp*-adaptivity, it is necessary to locally refine meshes. Whenever one mesh element is refined but at least one of its neighboring elements is not, then (multi-level) hanging nodes may occur, if they are not eliminated from the mesh using sophisticated refinement strategies. For the standard *h*- and *hp*-FEM, constrained approximation is the technique of choice to ensure the continuity of finite element shape functions associated to these hanging nodes, see also [1,2]. This presentation will show, how constrained approximation for standard FEM based on Lagrange, integrated Legendre and Gauss-Lobatto polynomials can be carried over to the GFEM, to allow for *h*- and *hp*-adaptivity with the appropriate convergence rates.

REFERENCES

[1] A. Byfut and A. Schröder. *hp*-Adaptive Extended Finite Element Method, *Int. J. Numer. Meth. Engng.*, Vol. **89**, 1392–1418, 2012.

[2] A. Schröder. Constraints Coefficients in *hp*-FEM. Numerical Mathematics and Advanced Applications, *Springer Berlin Heidelberg*, 183–190, 2008.

Information-flux methods: Stable schemes for convection-dominated problems

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Keywords: *maximum entropy, information flux, convection diffusion, Petrov-Galerkin*

ABSTRACT

Severe instabilities exhibited by standard Galerkin schemes for convection-dominated problems are well documented. For standard finite element methods, well-established stabilisation methods exist, which are however limited in their applicability to other methods. The proposed information-flux method incorporates a novel approach to stable methods. The key idea is to seperately assign the two goals of accuracy and stability to solution and weighting functions, respectively. For accuracy, the solution functions fulfil the respective consistency conditions, as usual. However, the weighting functions, dedicated to the stability of the method, should represent the information flux of the underlying physical (adjoint) problem. Hence, it is constructed to resemble fundamental solutions of the adjoint problem. Such information-flux methods are *a priori* stable methods in general, which are applicable to a large variety of approximation schemes. In our presentation, we will show the information-flux approach applied to the convection-diffusion problem with maximum-entropy basis functions; see [1] for the maximum-entropy method in general and [2,3] for its application in information-flux methods to convection-diffusion problems.

REFERENCES

[1] M. Arroyo, M. Ortiz. Local maximum-entropy approximation schemes: a seamless bridge between finite elements and meshfree methods.. *Int. J. Numer. Meth. Engng.*, Vol. **65**, 2167–2202, 2006.

[2] C.J. Cyron, K. Nissen, V. Gravemeier, W.A. Wall. Information flux maximum-entropy approximation schemes for convection-diffusion problems. *Int. J. Numer. Meth. Fluids*, Vol. **64**, 1180–1200, 2010.

[3] K. Nissen, C.J. Cyron, V. Gravemeier, W.A. Wall. Information-flux methods: a meshfree maximum-entropy Petrov-Galerkin method including stabilised finite element methods. *Comp. Meth. Appl. Mech. Engng.*, submitted, 2011.

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A semi-Lagrangean time-integration approach for fixed-grid based flow problems in the XFEM

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Keywords: *XFEM, time integration, Navier-Stokes*

ABSTRACT

Many computational flow problems like combustion, two-phase flow or fluid-structure interaction incorporate discontinuities that evolve in time. The eXtendend Finite Element Method (XFEM) is able to represent discontinuities inside elements sharply by adding degrees of freedom in the vicinity of the interface.

We address the difficulties of formulating problems involving moving domains on fixed grids and point out that this is an important issue for problems with weak discontinuous fields and crucial for problems with strongly discontinuous fields. A method using semi-Lagrangean techniques is proposed to adequately handle time integration based on finite difference schemes in the context of the XFEM [1][2]. Previous solutions are adapted to the current interface location by tracking back virtual Lagrangean particles to their previous positions and thus extrapolating within a smooth field circumventing the discontinuity.

Convergence properties in time and space of the proposed method are thoroughly studied for one-dimensional model problems for strong and weak discontinuities including a comparison with other time-integration approaches.

Numerical examples are shown for premixed combustion, where a strong discontinuity separates the burnt from the unburnt gases, and for two-phase flow, where a weak discontinuity in the velocity field and either a weak discontinuity (without surface tension) or a strong discontinuity (with surface tension) in the pressure field is present between the two fluid fields.

REFERENCES

[1] F. Henke, M. Winklmaier, W.A. Wall and V. Gravemeier. A semi-Lagrangean time-integration approach for extended finite element methods, in preparation.

[2] A. Staniforth and J. Côtè. Semi-Lagrangian Integration Schemes for Atmospheric Models - A Review, *Monthly weather review*, Vol. **119**, 2206 - 2223, 1991.

Berlin PUM Workshop 2012
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22 – 24 August 2012, Berlin, Germany

A stabilized XFEM based fixed-grid approach for fluids with moving boundaries

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Keywords: *XFEM, stability, Ghost Penalty, Navier-Stokes*

ABSTRACT

XFEM based fixed-grid approaches represent very promising approaches when dealing with moving boundaries or more complex fluid-structure interaction (FSI) applications involving large deformations of the structure. Describing the entire fluid domain by a fixed-grid Eulerian formulation using cut elements allows for large and complex changes of the physical fluid domain without fluid mesh distortion and eventually, remeshing of the fluid domain.

Hereby, high demands are made on approximation quality, stability and accuracy of the fixed-grid formulation, particularly with regard to moving boundaries or interfaces in time. Since the mesh is not fitted to the domain, boundary and coupling conditions are imposed weakly using a stabilized Nitsche type approach or stress based Lagrange multipliers [1][2]. In case of arbitrary cut elements these methods can loose stability and control of the non-physical degrees of freedom outside the physical domain. To retain accuracy in imposing boundary conditions without blowing up the condition number of the linear system Ghost Penalty stabilization approaches recover coercivity independent of the cut configuration [2]. In a similar manner, inf-sup instabilities are compensated by a gradient-penalty based edge-oriented fluid stabilization. These techniques have been extended to the incompressible Navier-Stokes equations on cut elements to obtain a robust, stable and accurate approach and an improvement of the system conditioning without element manipulation or blocking strategies of degrees of freedom.

Hence, in this talk we propose a fully stabilized XFEM-based formulation for 2D and 3D Navier-Stokes equations on non-boundary-fitted fixed grids. Results from numerical examples of fluid problems with structural interfaces will be shown and discussed in context of stability and accuracy.

REFERENCES

[1] A. Gerstenberger and W.A. Wall. An embedded Dirichlet formulation for 3d continua. *Int. J. Numer. Meth. Engng.*, Vol. **82**(5), 537-563, 2010.

[2] E. Burman and P. Hansbo. Fictitious domain finite element methods using cut elements: II. A stabilized Nitsche method. *Appl. Numer. Math.*, Vol. **62**(4), 328–341, 2012.

Berlin PUM Workshop 2012
Analysis and Application of the GFEM, XFEM, MM
22 – 24 August 2012, Berlin, Germany

Operator adapted BEM for the Helmholtz equation

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Keywords: *BEM, Helmholtz equation, high wavenumbers*

ABSTRACT

The Helmholtz equation is a fundamental equation when treating wave propagation problems in a time-harmonic setting. In particular, when the Helmholtz equation is considered in a exterior domain, as is typically the case in scattering problems, a numerical method based on an integral equation is an attractive option, leading to boundary element methods (BEMs). At high frequencies/wavenumbers, such methods face several challenges that arise from the highly oscillatory nature of the sought solution and the kernels of the integral operators. In this talk, we will discuss specialized approximation spaces that incorporate the oscillatory structure of the solution, and we will present quadrature schemes that are able to cope with highly oscillatory integrands. The resulting method lead to almost wavenumber-robust methods.

Multiscale fracture, model reduction, enrichment and real-time simulations of cutting

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Keywords: *multiscale method, extended bridging domain method, adaptive domain decomposition based model reduction*

ABSTRACT

In the first part of this presentation, we present a brief introduction to multiscale methods for fracture. We then discuss one particular method based on an extended bridging domain method (XBDM) for 3D dynamic crack propagation in brittle materials.

In a second part, motivated by the real time simulation of cutting, we address another technique to reduce computational expense in multiscale fracture, namely adaptive domain decomposition based model reduction, as an alternative to concurrent methods and enrichment. To conclude, we present briefly a method based on GPUs to simulate cutting in real time in the corotational formulations of large deformations.

The contact condition on crack lips with XFEM

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Keywords: *XFEM, contact condition*

ABSTRACT

The purpose of this presentation is to discuss on several manners, introduced in [1] to [5], to prescribe a contact condition on a crack in the framework of the extended finite element method (XFEM).

The a priori error analysis developped in [5] will be presented together with some numerical experiments.

REFERENCES

[1] J. Dolbow, N. Moës, T. Belytschko. An extended finite element method for modeling crack growth with frictional contact. *Comput. Methods Appl. Mech. Engrg.*, 190: 6825–6846, 2001.

[2] A. Khoei and M. Nikbakht. Contact friction modeling with the extended finite element method (XFEM). *Journal of Materials Processing Technology*, 177: 58–62, 2006.

[3] S. Geniaut, P. Massin, and N. Moës. A stable 3D contact formulation for cracks using XFEM. *Revue Européenne de Mécanique Numérique* , 16: 259–275, 2007.

[4] E. Pierres, M.-C. Baietto, and A. Gravouil. A two-scale extended finite element method for modeling 3D crack growth with interfacial contact. *Comput. Methods Appl. Mech. Engrg.*, 199: 1165–1177, 2010.

[5] S. Amdouni, P. Hild, V. Lleras, M. Moakher, Y. Renard. A stabilized lagrange multiplier method for the enriched finite-element approximation of contact problems of cracked elastic bodies. *ESAIM: M2AN*, 46: 813–839, 2012.

Berlin PUM Workshop 2012
Analysis and Application of the GFEM, XFEM, MM
22 – 24 August 2012, Berlin, Germany

Alternative methods to represent embedded interfaces in a mesh.

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Keywords: *embedded interface, XFEM, anisotropic mesh adaptation*

ABSTRACT

The use of levelset for representing discontinuous interfaces has become common in engineering analysis: the aim being, of course, to reduce the complexity of the mesh generation process. Here, the main advantage is that the computing mesh does not have to be conforming to the interfaces.

Even though the use of levelsets enables to reduce dramatically the mesh generation process, it does not remove all meshing issues : the computing mesh has to be sufficiently fine to resolve the geometrical complexity of the interfaces [1].

In this presentation, we focus on two different approaches that enable to deal with embedded Dirichlet boundary conditions.

The first approach makes use of Lagrange multipliers. The formulation is stabilized through some kind of SUPG term. An optimal stabilization parameter is provided that enables to achieve optimal convergence.

The second approach can be seen as a brute force approach. The mesh is adapted anisotropically close to the levelset. Dirichlet boundary conditions are applied on existing nodes of the adapted mesh. We show that this very simple approach enables to achieve optimal convergence as well.

These two approaches are compared for several non trivial benchmark problems both in solid and fluid mechanics.

REFERENCES

[1] N. Moës, M. Cloirec, P. Cartraud, J.F. Remacle. A computational approach to handle complex microstructure geometries *Comput. Methods Appl. Mech. Engrg.*, Vol. **192**, 3163–3177, 2003.

Berlin PUM Workshop 2012
Analysis and Application of the GFEM, XFEM, MM
22 – 24 August 2012, Berlin, Germany

A Generalized Finite Element Method for the Displacement Obstacle Problem of Clamped Kirchhoff Plates

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Keywords: *PUM, Obstacle Problem*

ABSTRACT

The obstacle problem for clamped Kirchhoff plates is a fourth order variational inequality. The numerical analysis of this problem is more difficult when compared to its second order counterpart. This is due to the fact that the fourth order problem does not have full elliptic regularity where the second order problem does; and hence the strong form of the variational inequality is not available for the error analysis of the fourth order problem. A unified convergence analysis for conforming finite element methods, nonconforming finite element methods, and discontinuous Galerkin methods for this problem on convex polygons with homogeneous Dirichlet boundary conditions has been developed in [1]. These results then were extended to a C^0 interior penalty method and a Morley finite element method for general polygonal domains and general Dirichlet boundary conditions in [2,3]. The goal of this work is to extend these results to a generalized finite element method [4]. In this talk I will go over the construction of the approximation space, convergence analysis, and numerical examples.

REFERENCES

[1] S.C. Brenner, L.-Y. Sung, and Y. Zhang. Finite element methods for the displacement obstacle problem of clamped plates. *Math. Comp*, Vol. **81**, 1247-1262, 2012.

[2] S.C. Brenner, L.-Y. Sung, H. Zhang, and Y. Zhang. A quadratic C^0 interior penalty method for the displacement obstacle problem of clamped Kirchhoff plates. preprint.

[3] S.C. Brenner, L.-Y. Sung, H. Zhang, and Y. Zhang. A Morley finite element method for the displacement obstacle problem of clamped Kirchhoff plates, submitted. preprint.

[4] S.C. Brenner, C.B. Davis, and L.-Y. Sung. A generalized finite element method for the displacement obstacle problem of clamped Kirchhoff plates, submitted. preprint.

Non-linear crack initiation and propagation

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Keywords: *XFEM, continuum damage mechanics, fracture mechanics*

ABSTRACT

The XFEM has been widely developed in the framework of linear elastic fracture mechanics. It enables to solve challenging problems such as damage tolerance analysis, it is already implemented in general purpose codes and used in the industry.
The present contribution proposes an adaptation of these techniques to material and geometrical non-linearities as well as crack initiation and monotonic loadings. The objective is clearly to combine Continuum Damage Mechanics and Fracture Mechanics in order to represent a complete behavior from sane material to structural failure. This can be articulated around the following steps:

- local degradation of the material based on damage,
- crack insertion without mesh modification in the critical area,
- load continuation and crack propagation under damage and stress state criteria.

This strategy will be illustrated by multiple examples, on metals (Fig. 1) and composites (Fig. 2).

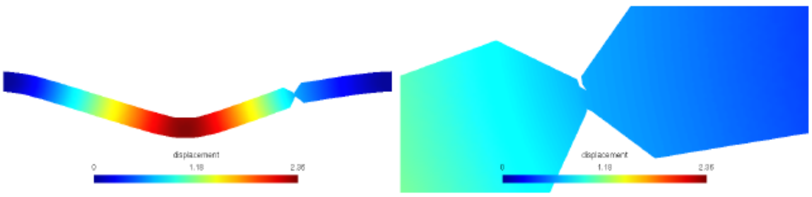

FIGURE 1. Decapping test: displacement, far view (left) and zoom (right)

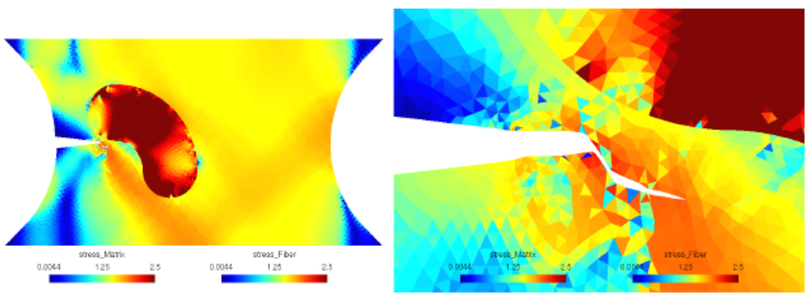

FIGURE 2. Fracture of composite with bean-shaped fiber, far view (left) and zoom (right)

Berlin PUM Workshop 2012
Analysis and Application of the GFEM, XFEM, MM
22 – 24 August 2012, Berlin, Germany

Partition of Unity Methods – Stability, Fast Solvers, Parallelization

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