Berlin PUM Workshop 2012
Analysis and Application of the GFEM, XFEM, MM
22 – 24 August 2012, Berlin, Germany

# Multi-level unsymmetric hanging nodes in *hp*-adaptive GFEM

**Andreas Byfut**
Department of Mathematics, Humboldt-Universität zu Berlin,
Unter den Linden 6, 10099 Berlin, byfut@math.hu-berlin.de

**Andreas Schröder**
Department of Mathematics, Humboldt-Universität zu Berlin,
Unter den Linden 6, 10099 Berlin, schroder@math.hu-berlin.de

**Keywords:** *GFEM, higher-order, hanging nodes, XFEM, fictitious domain method*

## ABSTRACT

In this presentation, the concept of constrained approximation for standard higher-order finite element methods is carried over to the generalized finite element method (GFEM). As a consequence, meshes containing multi-level, unsymmetric hanging nodes may be employed in *h*- and *hp*-adaptive GFEM.

Considering the ability of GFEM to include almost arbitrary functions into its finite element space, one may conclude that adaptivity is generally unnecessary to approximate the solution of a given problem "well". However, an (automatic) problem-dependent composition of a set of enrichment functions as well as an (automatic) inclusion into the finite element spaces is a non-trivial task to appropriately cover each and every possible feature of a sought solution. In modern higher-order finite element schemes, *h*- and *hp*-adaptivity based on a posteriori error control is a standard tool to (automatically) increase the approximation quality of a finite element scheme. This presentation will show, that these standard techniques may very well be applied for *h*- and *hp*-adaptive GFEM such as the XFEM or the fictitious domain method, see also [1].

For *h*- and *hp*-adaptivity, it is necessary to locally refine meshes. Whenever one mesh element is refined but at least one of its neighboring elements is not, then (multi-level) hanging nodes may occur, if they are not eliminated from the mesh using sophisticated refinement strategies. For the standard *h*- and *hp*-FEM, constrained approximation is the technique of choice to ensure the continuity of finite element shape functions associated to these hanging nodes, see also [1,2]. This presentation will show, how constrained approximation for standard FEM based on Lagrange, integrated Legendre and Gauss-Lobatto polynomials can be carried over to the GFEM, to allow for *h*- and *hp*-adaptivity with the appropriate convergence rates.

## REFERENCES

[1] A. Byfut and A. Schröder. *hp*-Adaptive Extended Finite Element Method, *Int. J. Numer. Meth. Engng.*, Vol. **89**, 1392–1418, 2012.

[2] A. Schröder. Constraints Coefficients in *hp*-FEM. Numerical Mathematics and Advanced Applications, *Springer Berlin Heidelberg*, 183–190, 2008.