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The Partition of Unity Method for elastic wave problems in 3D

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ABSTRACT

Elastic wave propagation modelling arises in many engineering applications, including traffic vibrations from roads and railways, seismic induced vibrations and foundation construction, etc. The numerical modelling of these problems, in frequency domain by the conventional Finite Element Method (FEM), requires finite element grids sufficiently fine in comparison with the wavelengths, to get accurate results. When typically, the piecewise linear finite element is implemented, around ten nodal points per lower wavelength are needed, to ensure adequate resolution of the wave pattern. However, in the case of high frequency (small wavelength) and/or large domain of interest, the finite element mesh requires a large number of elements, and consequently the procedure becomes computationally expensive and impractical.

The principal objective is to develop finite elements, for three dimensional elastic wave problems, capable of containing many wavelengths per nodal spacing. This will be achieved by applying the plane wave basis decomposition to the 3D elastic wave equation. These elements will allow us to relax the traditional requirement of around ten nodal points per wavelength and therefore solve elastic wave problems without refining the mesh of the computational domain at each frequency. The accuracy and effectiveness of the proposed technique will be determined by comparing solutions for selected problems with available analytical solutions and/or to high resolution numerical solutions using conventional finite elements.

The method of plane wave basis decomposition used to develop wave finite elements for the two-dimensional elastic wave equation [1,2] will be extended to three dimensions. The governing equation is a vector equation and multiple wave speeds are present for any given frequency. In an infinite elastic medium, there are two different types of wave propagating simultaneously, the dilatation or compression wave (P), and the distortional or shear wave (S). The application of the Helmholtz decomposition theorem to the displacement field yields a scalar wave equation for the P-wave potential and a vector wave equation for the S-wave potential. The two wave equations are independent but the boundary conditions depend on both P-wave and S-wave potentials, thus coupling the associated scalar P-wave and vector S-wave equations.

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