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A simple method for integration over enriched elements in PUM

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ABSTRACT

One of the crucial issues that decides the accuracy and robustness of PUM, especially while dealing with moving interfaces, is the numerical integration of shape functions over the enriched elements. Most widely, volume decomposition [1] or moment fitting methods [2] are used to address the above issue. In this talk, we present a simple method to perform integration over enriched elements using the divergence theorem. Though the divergence theorem is used extensively for integration in computational geometry, usually it is not used in FEM because the integrand of interest is not known explicitly. For a scalar function f, using the divergence theorem, the integration over $\mathcal{S} \subset \mathbb{R}^2$ whose boundary is given by $\mathcal{C}$ can be written as,

$$\int_{\mathcal{S}} f \; d\mathcal{S} = \oint_{\mathcal{C}} F \; d\mathcal{C}; \quad \textit{where} \quad F = \int f dx \tag{1}$$

It is the computation of F that requires the integrand to be known explicitly. We eliminate this necessity by using 1D Gauss quadrature to evaluate F. We show that the method is very efficient for integrating polynomials over arbitrary polygons and it is extremely easy to implement. Some preliminary results in 3D are also presented.

REFERENCES

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