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Multiscale Total Variation with Automated Regularization Parameter Selection for Image Restoration

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Der Wissenschaftsfonds.

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    - Degradation Model
    - Rudin-Osher-Fatemi Model
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    - Multiscale Total Variation Model
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Degradation Model

$$z = K\hat{u} + n$$

- $K \in \mathcal{L}(L^2(\Omega))$ is a blurring operator
- n represents white Gaussian noise with mean 0 and variance σ^2

Problem

- Restore $\hat{u}$ from z with n unknown
- Ill-posed problem

Degradation Model

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Rudin-Osher-Fatemi (ROF) Model (1992)

$$\min_{u \in BV(\Omega)} \int_\Omega |Du|$$

$$\text{subject to } \int_\Omega |Ku - z|^2 dx \leq \sigma^2 |\Omega|$$

- $BV(\Omega)$ denotes the space of functions of bounded variation
- $\int_\Omega |Du| = \sup \left\{ \int_\Omega u \,\mathrm{div} \vec{v} dx : \vec{v} \in (C_0^\infty(\Omega))^2, \|\vec{v}\|_\infty \leq 1 \right\}$

Rudin-Osher-Fatemi (ROF) Model (1992)

$$\min_{u \in BV(\Omega)} \int_\Omega |Du|$$

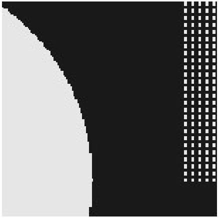
$$\text{subject to } \int_\Omega |Ku - z|^2 dx \leq \sigma^2 |\Omega|$$

- Equivalent to unconstrained minimization problem (Chambolle and Lions, 1997)

$$\min_{u \in BV(\Omega)} \int_\Omega |Du| + \frac{\lambda}{2} \int_\Omega |Ku - z|^2 dx$$

- $\lambda > 0$

Results with Different λ



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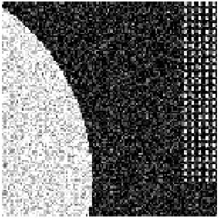

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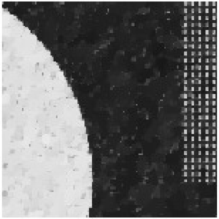
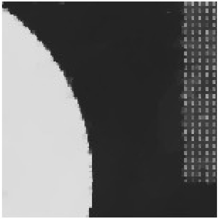


$\lambda = 20$



$\lambda = 10$

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Multiscale Total Variation Model (Rudin, 1995)

$$\min_{u \in BV(\Omega)} \int_\Omega |Du| + \frac{1}{2}\int_\Omega \lambda(x)|Ku - z|^2 dx$$

- $0 < \underline{\lambda} \le \lambda(x) \le \bar{\lambda}$ a.e. in Ω
- In multiscale total variation model, λ is spatially varying

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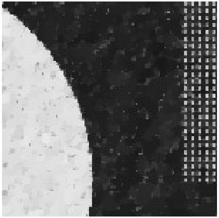
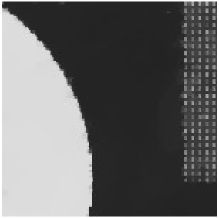
Result with Multiscale Total Variation Method

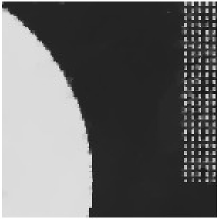


$\lambda = 20$



$\lambda = 10$

λ



Restored Image

Primal-Dual Algorithm for ROF Model (Hintermüller and Stadler, 2006)

$$\min_{u \in H_0^1(\Omega)} \frac{\mu}{2}\int_\Omega |\nabla u|_2^2 dx + \frac{\lambda}{2}\int_\Omega |Ku - z|^2 dx + \int_\Omega |\nabla u|_2 dx$$

- It is a close approximation of ROF model
- μ is helpful for function space analysis
- This algorithm uses Fenchel dual technique and semismooth Newton method
- This algorithm converges locally at a superlinear rate

Problem

$$\min_{u\in H_0^1(\Omega)} \frac{\mu}{2}\int_\Omega |\nabla u|_2^2 dx + \frac{1}{2}\int_\Omega \lambda(x)|Ku - z|^2 dx + \int_\Omega |\nabla u|_2 dx$$

Dual Problem

$$\sup_{\substack{\vec{p}\in \mathbf{L}^2(\Omega)\\ |\vec{p}(x)|_2\leq 1 \ \text{ a.e. in } \Omega}} -\frac{1}{2}|||K^*z - \mathrm{div}\vec{p}|||_{H^{-1}}^2 + \frac{1}{2}\|z\|_{L^2}^2$$

- Definition of Fenchel conjugate: $\mathcal{F}^*(v^*) = \sup_{v\in V}\{\langle v, v^*\rangle_{V,V^*} - \mathcal{F}(v)\}$
- $\inf_{v\in V}\{\mathcal{F}(v) + \mathcal{G}(\wedge v)\} = \sup_{q\in Y^*}\{-\mathcal{F}^*(\wedge^* q) - \mathcal{G}^*(-q)\}$

Problem

$$\min_{u\in H_0^1(\Omega)} \frac{\mu}{2}\int_\Omega |\nabla u|_2^2 dx + \frac{1}{2}\int_\Omega \lambda(x)|Ku - z|^2 dx + \int_\Omega |\nabla u|_2 dx$$

Dual Problem

$$\sup_{\substack{\vec{p}\in \mathbf{L}^2(\Omega)\\ |\vec{p}(x)|_2 \le 1 \text{ a.e. in } \Omega}} -\frac{1}{2}|||K^* z - \mathrm{div}\vec{p}|||_{H^{-1}}^2 + \frac{1}{2}\|z\|_{L^2}^2$$

- K^* is adjoint operator of K
- $|||v|||_{H^{-1}}^2 = \left\langle (K^*\lambda K - \mu\triangle)^{-1} v, v\right\rangle_{H_0^1, H^{-1}}$ with λ as function
- The solution of the dual problem is not unique

Dual Problem

$$\sup_{\substack{\vec{p} \in \mathbf{L}^2(\Omega) \\ |\vec{p}(x)|_2 \leq 1 \ \text{a.e. in } \Omega}} -\frac{1}{2}|||K^* z - \mathrm{div}\vec{p}|||^2_{H^{-1}} + \frac{1}{2}\|z\|^2_{L^2} - \frac{\gamma}{2}\int_\Omega \|\vec{p}\|^2_{\mathbf{L}^2}$$

Problem

$$\min_{u \in H_0^1(\Omega)} \frac{\mu}{2}\int_\Omega |\nabla u|_2^2 dx + \frac{1}{2}\int_\Omega \lambda(x)|Ku - z|^2 dx + \int_\Omega \Phi_\gamma(\nabla u) dx$$

- $\Phi_\gamma(\vec{v})(x) = \begin{cases} |\vec{v}(x)|_2 - \frac{\gamma}{2}, & \text{if } |\vec{v}(x)|_2 \geq \gamma \\ \frac{1}{2\gamma}|\vec{v}(x)|_2^2, & \text{if } |\vec{v}(x)|_2 < \gamma \end{cases}$

Optimality Condition

$$\mathrm{div}\bar{\vec{p}} \in \partial\left(\frac{\mu}{2}\int_\Omega |\nabla \bar{u}|_2^2 dx + \frac{1}{2}\int_\Omega \lambda(x)|K\bar{u} - z|^2 dx\right)$$
$$\bar{\vec{p}} \in \partial\left(\int_\Omega \Phi_\gamma(\nabla \bar{u}) dx\right)$$

Equations for the solutions

$$-\mu\triangle\bar{u} + K^*\lambda K\bar{u} - \mathrm{div}\bar{\vec{p}} = K^*\lambda z \quad \text{in } H^{-1}(\Omega)$$
$$\left.\begin{array}{rl} \gamma\bar{\vec{p}} - \nabla\bar{u} = 0 & \text{if } |\bar{\vec{p}}|_2 < 1 \\ |\nabla\bar{u}|_2\bar{\vec{p}} - \nabla\bar{u} = 0 & \text{if } |\bar{\vec{p}}|_2 = 1 \end{array}\right\} \text{in } \mathbf{L}^2(\Omega)$$

Optimality Condition

$$\mathrm{div}\vec{\bar{p}} \in \partial\left(\frac{\mu}{2}\int_\Omega |\nabla \bar{u}|_2^2 dx + \frac{1}{2}\int_\Omega \lambda(x)|K\bar{u} - z|^2 dx\right)$$

$$\vec{\bar{p}} \in \partial\left(\int_\Omega \Phi_\gamma(\nabla \bar{u}) dx\right)$$

Equations for the solutions

$$-\mu\triangle\bar{u} + K^*\lambda K\bar{u} - \mathrm{div}\vec{\bar{p}} = K^*\lambda z \quad \text{in } H^{-1}(\Omega)$$

$$\max(\gamma, |\nabla\bar{u}|_2)\vec{\bar{p}} - \nabla\bar{u} = 0 \quad \text{in } \mathbf{L}^2(\Omega)$$

ROF Model (1992)

$$\min_{u\in BV(\Omega)} \int_\Omega |Du|$$

$$\text{subject to } \int_\Omega |Ku - z|^2 dx \le \sigma^2 |\Omega|$$

Locally Constrained Model

$$\min_{u\in BV(\Omega)} \int_\Omega |Du|$$

$$\text{subject to } S_u(x) \le \sigma^2, \text{ a.e. } x \in \Omega$$

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Locally Constrained Model

$$\min_{u \in BV(\Omega)} \int_\Omega |Du|$$

$$\text{subject to } S_u(x) \le \sigma^2, \text{ a.e. } x \in \Omega$$

Local Smoothness

$$S_u(x) = w \star |Ku - z|^2(x) = \int_\Omega w(x-y)|Ku - z|^2(y)dy$$

- $w \in L^\infty(\Omega)$ is a normalized smoothing filter

Locally Constrained Model

$$\min_{u\in BV(\Omega)} \int_\Omega |Du|$$

$$\text{subject to } S_u(x) \leq \sigma^2, \text{ a.e. } x \in \Omega$$

- Consider the unconstrained minimization problem

$$\min_{u\in BV(\Omega)} \int_\Omega |Du| + \frac{\gamma}{2}\int_\Omega \max(S_u(x) - \sigma^2, 0)^2 dx$$

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Unconstrained Version of Locally Constrained Model

$$\min_{u \in BV(\Omega)} \int_\Omega |Du| + \frac{\gamma}{2} \int_\Omega \max(S_u(x) - \sigma^2, 0)^2 dx$$

Multiscale Total Variation Model

$$\min_{u \in BV(\Omega)} \int_\Omega |Du| + \frac{1}{2} \int_\Omega \lambda(x) |Ku - z|^2 dx,$$

- $\hat{\lambda}(y) = \gamma \max(S_u(y) - \sigma^2, 0)$
  $\lambda(x) = w \star \hat{\lambda}(x) = \int_\Omega w(x,y) \hat{\lambda}(y) dy$
- $\lambda > 0$

Locally Smoothing Filter

$$w(x,y) = \begin{cases} \frac{1}{w_\epsilon^2}, & \|x-y\|_\infty \leq \frac{\omega}{2} \\ \epsilon, & \text{otherwise} \end{cases}$$

- $0 < \epsilon \ll 1$
- w_ϵ such that $\int_\Omega \int_\Omega w(x,y)dydx = 1$

Locally Smoothing Filter

$$w(x,y) = \begin{cases} \frac{1}{\omega^2}, & |x-y| \in [-\frac{\omega}{2}, \frac{\omega}{2}] \times [-\frac{\omega}{2}, \frac{\omega}{2}] \\ 0, & \text{otherwise} \end{cases}$$

Local Constraint

$$S_u(x) = w \star |Ku - z|^2(x) = \frac{1}{\omega^2} \int_{\Omega_x^\omega} |Ku - z|^2 dy$$

- $\Omega_x^\omega = \{y \in \Omega : |x - y| \in [-\frac{\omega}{2}, \frac{\omega}{2}] \times [-\frac{\omega}{2}, \frac{\omega}{2}]\}$

Locally Smoothing Filter

$$w(x,y) = \begin{cases} \frac{1}{\omega^2}, & |x-y| \in [-\frac{\omega}{2}, \frac{\omega}{2}] \times [-\frac{\omega}{2}, \frac{\omega}{2}] \\ 0, & \text{otherwise} \end{cases}$$

Local Constraint

$$\frac{1}{\omega^2} \int_{\Omega_x^\omega} |Ku - z|^2 dy \le \sigma^2, \text{ a.e. } x \in \Omega$$

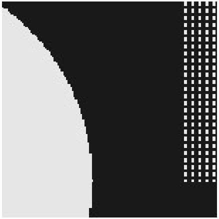
- $\Omega_x^\omega = \{y \in \Omega : |x-y| \in [-\frac{\omega}{2}, \frac{\omega}{2}] \times [-\frac{\omega}{2}, \frac{\omega}{2}]\}$

Local Variance Estimator

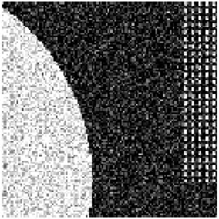
$$S_{i,j}^{\omega} = \frac{1}{\omega^2} \sum_{(s,t)\in\Omega_{i,j}^{\omega}} \left((K\tilde{u})_{s,t} - z_{s,t}\right)^2$$

- $\tilde{u}$ is the restored image by solving the classical ROF model with a relatively small λ
- $\Omega_{i,j}^{\omega} = \left\{(s+i, t+j) : -\left\lfloor \frac{\omega}{2} \right\rfloor \leq s, t \leq \left\lfloor \frac{\omega}{2} \right\rfloor\right\}$ is the set of coordinates in a ω-by-ω window centered at (i, j)

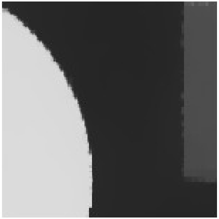
Example 1



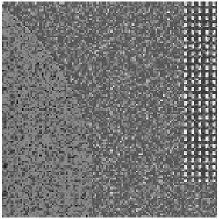
Original Image



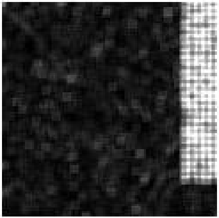
Noisy Image



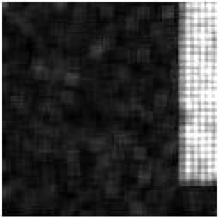
Restored Image



Residual

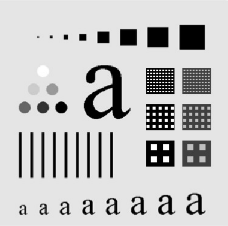


S^5

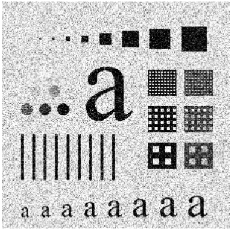


S^7

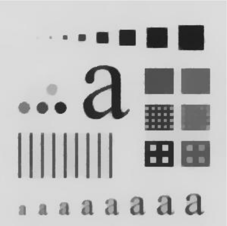
Example 2



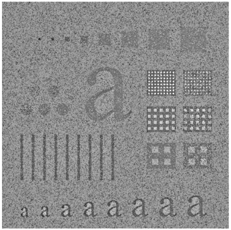
Original Image



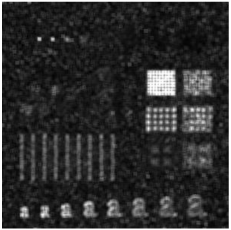
Noisy Image



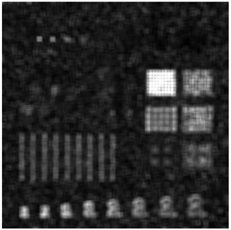
Restored Image



Residual



S^5



S^7

$$T_{i,j}^{\omega} = \frac{1}{\sigma^2} \sum_{(s,t)\in\Omega_{i,j}^{\omega}} (n_{s,t})^2$$

- $T_{i,j}^{\omega}$ has the χ^2-distribution with ω^2 degrees of freedom; i.e., $T_{i,j}^{\omega} \sim \chi^2_{\omega^2}$
- If $u = \hat{u}$ satisfies $n = z - K\hat{u}$, then

$$S_{i,j}^{\omega} = \frac{1}{\omega^2} \sum_{(s,t)\in\Omega_{i,j}^{\omega}} (z_{s,t} - (K\hat{u})_{s,t})^2 = \frac{\sigma^2}{\omega^2} T_{i,j}^{\omega}$$

- If the residual image $z - K\tilde{u}$ contains details, we expect

$$S_{i,j}^{\omega} = \frac{1}{\omega^2} \sum_{(s,t)\in\Omega_{i,j}^{\omega}} (z_{s,t} - (K\tilde{u})_{s,t})^2 > \frac{\sigma^2}{\omega^2} T_{i,j}^{\omega}$$

What is a suitable bound B such that $S_{i,j}^{\omega} > B$ implies that in the residual there are some details left in $\Omega_{i,j}^{\omega}$?

The bound should relate to the maximum of the m^2 random variables $\frac{\sigma^2}{\omega^2} T_k^{\omega}$, $k = 1, \dots, m^2$. We propose the following bound

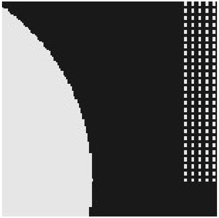
$$B^{\omega,m} := \frac{\sigma^2}{\omega^2} \left(\mathfrak{E}\left(\max_{k=1,\dots,m^2} T_k^{\omega}\right) + \mathfrak{d}\left(\max_{k=1,\dots,m^2} T_k^{\omega}\right) \right)$$

- $m \times m$ is the image size
- $\mathfrak{E}$ represents the expected value of a random variable
- $\mathfrak{d}$ represents the variance of a random variable

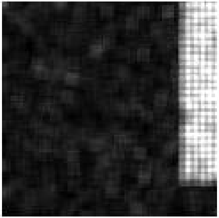
Improved Local Variance Estimator

$$\tilde{S}_{i,j}^{\omega} := \begin{cases} \frac{1}{\omega^2} \sum\limits_{(s,t)\in\Omega_{i,j}^{\omega}} (z_{s,t} - (K\tilde{u})_{s,t})^2 & \text{if } S_{i,j}^{\omega} \geq B^{\omega,m}, \\ \sigma^2 & \text{otherwise} . \end{cases}$$

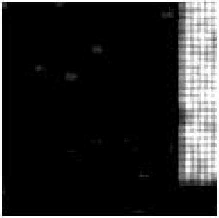
Example 1



Original Image

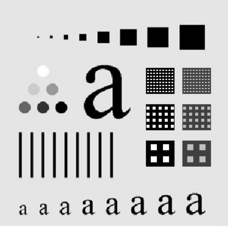


S^7



$\tilde{S}^7$

Example 2



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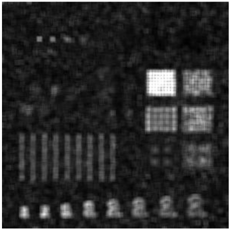
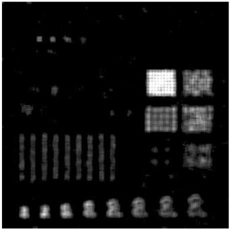


S^7



$\tilde{S}^7$

Selection of the Parameter λ

$$(\hat{\lambda}_{k+1})_{i,j} = (\hat{\lambda}_k)_{i,j} + \rho\big((\tilde{S}_k^\omega)_{i,j} - \sigma^2\big)$$

$$(\lambda_{k+1})_{i,j} = \frac{1}{\omega^2} \sum_{(s,t)\in\Omega_{i,j}^\omega} (\hat{\lambda}_{k+1})_{s,t}$$

- $\rho = \|\hat{\lambda}_k\|_\infty/\sigma$ in order to keep the new $\hat{\lambda}_{k+1}$ at the same scale as $\hat{\lambda}_k$

Basic MTV Algorithm

1: Initialize $u_0 = 0 \in \mathbb{R}^{m\times m}$, $p_0 = 0 \in (\mathbb{R}^{(m\times m)})^2$, $\lambda_0 = \hat{\lambda}_0 \in \mathbb{R}_+^{m\times m}$ and $k = 0$.

2: Solve the discrete version

$$u_k = \arg \min_{u\in BV(\Omega)} \int_\Omega |Du| + \frac{1}{2}\int_\Omega \lambda_k (Ku - z)^2 dx.$$

3: Based on u_k, update λ_{k+1} as

$$(\hat{\lambda}_{k+1})_{i,j} = (\hat{\lambda}_k)_{i,j} + \rho\big((\tilde{S}_k^\omega)_{i,j} - \sigma^2\big),$$

$$(\lambda_{k+1})_{i,j} = \frac{1}{\omega^2} \sum_{(s,t)\in\Omega_{i,j}^\omega} (\hat{\lambda}_{k+1})_{s,t}.$$

4: Stop, or set $k := k+1$ and return to step 2.

Tadmor-Nezzar-Vese(TNV) Algorithm (2004, 2008)

1: Initialize $u_0 = 0 \in \mathbb{R}^{m\times m}$, $p_0 = 0 \in (\mathbb{R}^{m\times m})^2$, $\lambda_0 \in \mathbb{R}_+$ and $k = 0$.

2: Calculate $v_k = z - Ku_k$. Then, solve the minimization problem

$$\min_{u\in BV(\Omega)} \int_\Omega |Du| + \frac{\lambda_k}{2}\int_\Omega |Ku - v_k|^2 dx,$$

and get $\tilde{u}$.

3: Update $u_{k+1} = u_k + \tilde{u}$.

4: Based on u_{k+1}, update $\lambda_{k+1} = 2\cdot\lambda_k$.

5: Stop; or set $k := k+1$ and go to step 2.

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Selection of the Parameter λ

$$(\hat{\lambda}_{k+1})_{i,j} = 2 \cdot \min\left((\hat{\lambda}_k)_{i,j} + \rho \left(\sqrt{(\tilde{S}_k^\omega)_{i,j}} - \sigma \right), L \right)$$

$$(\lambda_{k+1})_{i,j} = \frac{1}{\omega^2} \sum_{(s,t)\in\Omega_{i,j}^\omega} (\hat{\lambda}_{k+1})_{s,t}$$

- L is a large positive value to ensure $\hat{\lambda}_k \in L^\infty(\Omega)$

SA-TV Algorithm

1: Initialize $u_0 = 0 \in \mathbb{R}^{m\times m}$, $p_0 = 0 \in (\mathbb{R}^{(m\times m)})^2$, $\lambda_0 = \hat{\lambda}_0 \in \mathbb{R}_+^{m\times m}$ and $k = 0$.

2: Calculate $v_k = z - Ku_k$. Then, solve the minimization problem

$$\min_{u\in H_0^1(\Omega)} \frac{\mu}{2}\int_\Omega |\nabla u|_2^2 dx + \frac{1}{2}\int_\Omega \lambda_k(x)|Ku - v_k|^2 dx + \int_\Omega |\nabla u|_2 dx$$

by primal-dual algorithm, and get $\tilde{u}$ and p_{k+1}.

3: Update $u_{k+1} = u_k + \tilde{u}$.

4: Based on u_{k+1}, update

$$(\hat{\lambda}_{k+1})_{i,j} = 2\cdot \min\left((\hat{\lambda}_k)_{i,j} + \rho\left(\sqrt{(\tilde{S}_k^\omega)_{i,j}} - \sigma\right), L\right)$$

$$(\lambda_{k+1})_{i,j} = \frac{1}{\omega^2}\sum_{(s,t)\in\Omega_{i,j}^\omega} (\hat{\lambda}_{k+1})_{s,t}$$

5: Stop; or set $k := k+1$ and go to step 2.

Restoration of Noisy Images

Original Image

Noisy Image

ROF (λ=11)

Bregman (λ_0=2.5)

TNV (λ_0=2.5)

SA-TV (λ_0=2.5)

Restoration of Noisy Images

Barbara

Part of Barbara

Restoration of Noisy Images

Original Image

Noisy Image

ROF (λ_0=14)

Bregman (λ_0=2.5)

TNV (λ_0=2.5)

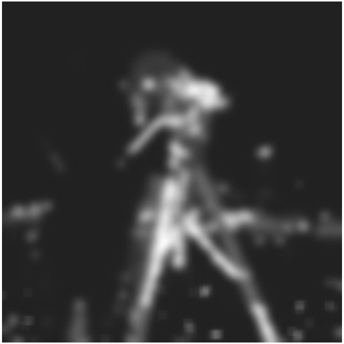
SA-TV (λ_0=2.5)

Final Value of λ

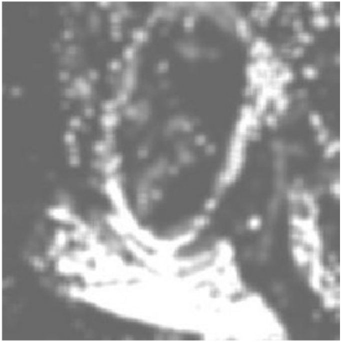
Cameraman

Part of Barbara

Final Value of λ

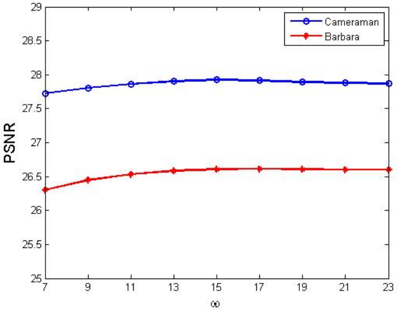


Cameraman



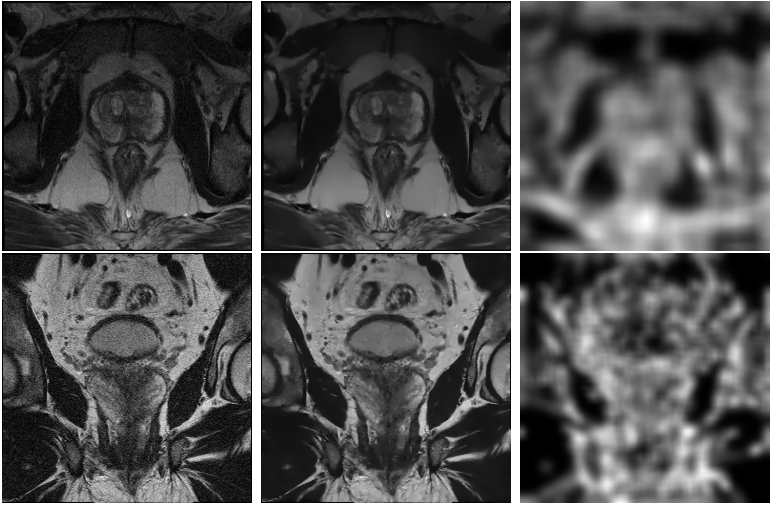
Part of Barbara

PSNR of Restored Images by Our Method for Different ω



- Since the confidence interval technique from statistics is introduced in the local variance estimator, λ can be adjusted automatically based on the size of the windows Ω^{ω}. This yields a parameter-free method.

Restoration of MRI



Noisy images Restored images λ

Restoration of MR Images with Non-Uniform Noise

$$\min_{u \in BV(\Omega)} \int_\Omega |Du|$$

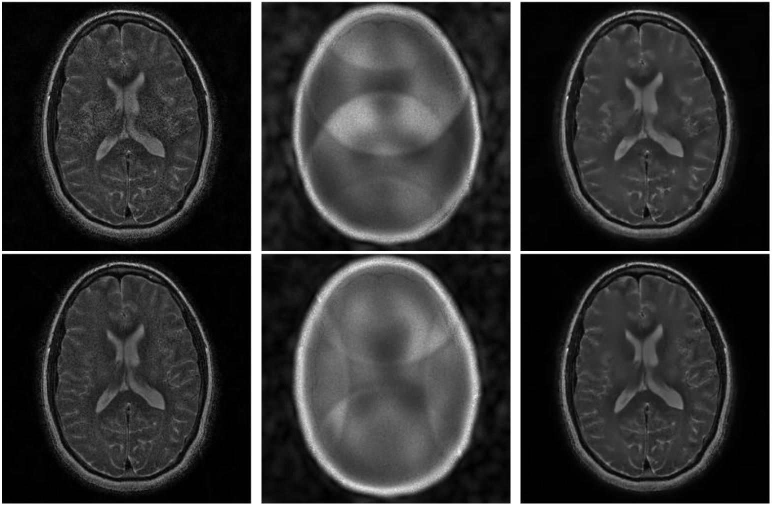
$$\text{subject to } S_u(x) \leq \sigma^2(x), \text{ a.e. } x \in \Omega$$

- σ^2 is not a scalar but related to the position in the image
- Selection of λ becomes

$$(\hat{\lambda}_{k+1})_{i,j} = 2 \cdot \min\left((\hat{\lambda}_k)_{i,j} + \rho \left(\sqrt{(\tilde{S}_k^\omega)_{i,j}} - \sigma_{i,j} \right), L \right)$$

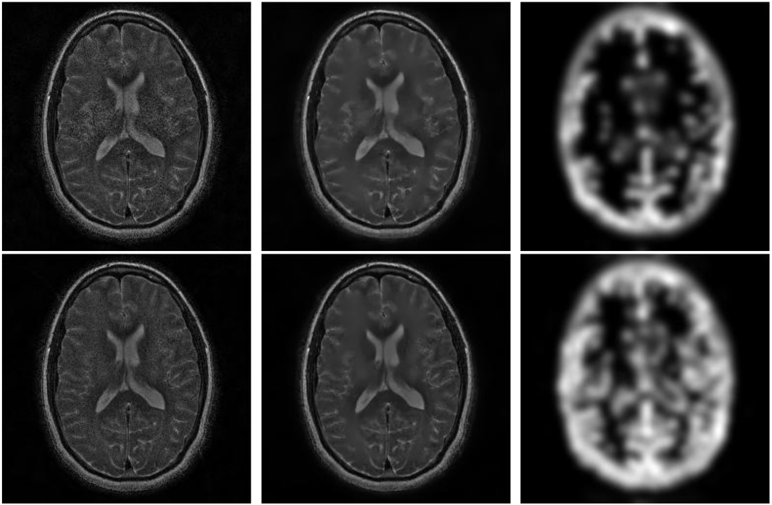
$$(\lambda_{k+1})_{i,j} = \frac{1}{\omega^2} \sum_{(s,t)\in\Omega_{i,j}^\omega} (\hat{\lambda}_{k+1})_{s,t}$$

Restoration of MR Images with Non-Uniform Noise



Noisy images   Noise ratios   Restored images

Restoration of MR Images with Non-Uniform Noise



Noisy images   Restored images λ

Degradation Model for Color Images

$$\mathbf{z} = K\hat{\mathbf{u}} + \mathbf{n}$$

- $\hat{\mathbf{u}}, \mathbf{z} : \Omega \to \mathbb{R}^M$ are vector-valued functions
- $K \in \mathcal{L}(L^2(\Omega; \mathbb{R}^M))$ is a cross-channel blurring operator
- M is the number of channels in the color model

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MTV Method
MTV Model
Algorithm for MTV
λ Selection
Local Constraint
Detail Detector
λ Selection
SA-TV Algorithm
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MVTV Model
Conclusion

Degradation Model for Color Images

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Multiscale Vectorial TV Model

$$\min_{\mathbf{u}\in \mathbf{BV}(\Omega)} \int_\Omega |D\mathbf{u}| + \frac{1}{2}\int_\Omega \lambda(x)|K\mathbf{u}-\mathbf{z}|_2^2 dx$$

- $\int_\Omega |D\mathbf{u}| = \sup\left\{\int_\Omega \mathbf{u}\cdot \mathrm{div}\vec{\mathbf{v}}\, dx : \vec{\mathbf{v}} \in C_c^1(\Omega, \mathbb{R}^{M\times 2}), |\vec{\mathbf{v}}|_F \leq 1 \text{ in } \Omega\right\}$
- $\lambda \in L^\infty(\Omega)$ with $0 < \underline{\lambda} \leq \lambda(x) \leq \bar{\lambda}$ for almost all $x \in \Omega$

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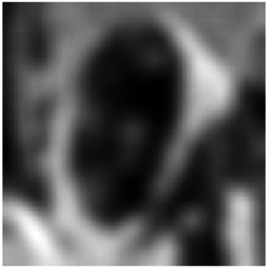
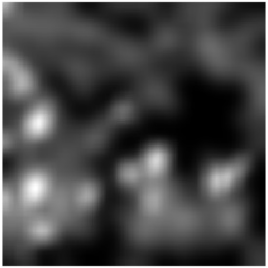
Original Image

Restoration of Noisy Images

Noisy Images　　VTV method　　SA-TV method

Final Value of λ

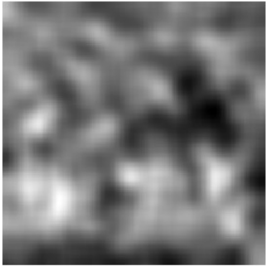




Restoration of Blurred Noisy Images

Noisy Images VTV method SA-TV method

Final Value of λ



Conclusion

- Based on spatially dependent parameter selection, our method is able to preserve the details during noise removal.
- With confidence interval technique from statistics, our method is parameter-free.
- In our method, a superlinearly convergent algorithm based on Fenchel-duality and inexact semismooth Newton techniques is used to solve the multiscale total variation model.

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Thank you!