Stochastic Processes II Summer term 2008
(Stochastische Analysis)
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Exercises, 11th June

9.1 (1+1+1 points) Prove the following propeties of the covariation $\langle M, N\rangle$ of two continuous local martingales M and N:

a) $\langle M, N\rangle = \langle N, M\rangle$,

b) $\langle \alpha M, N\rangle = \alpha\langle M, N\rangle, \quad \alpha \in \mathbb{R}$.

c) If N_1 and N_2 are two continuous local martingales, then $\langle M, N_1 + N_2\rangle = \langle M, N_1\rangle + \langle M, N_2\rangle$.

9.2 (8 points) Let M be a continuous local martingale. Prove that the sets

$$\left\{\lim_{t\to\infty} M_t \text{ exists and is finite}\right\} \quad \text{and} \quad \{\langle M\rangle_\infty < \infty\}$$

coincide P-a.s..

9.3 (3 points) Let $M, N \in \mathcal{H}^2$ with $M_0 = N_0 = 0$. The martingales M and N are called *weakly orthogonal* if $E[M_s N_t] = 0$ for every s and $t \geq 0$. Prove that the following properties are equivalent:

a) M and N are weakly orthogonal,

b) $E[M_s N_s] = 0$ for every $s \geq 0$,

c) $E[\langle MN\rangle_s] = 0$ for every $s \geq 0$,

d) $E[M_\tau N_s] = 0$ for every $s \geq 0$ and every stopping time $\tau \geq s$.

9.4 (4 points) Use the Itô-isometry to compute the variance of

$$\int_0^t |B_s|^p dB_s \quad \text{and} \quad \int_0^t (B_s + s)^2 dB_s \quad (t, p \geq 0)$$

for a standard Brownian motion B.

The problems 9.1 -9.4 should be solved at home and delivered at Wednesday, the 18th June, before the beginning of the tutorial.