Final Exam–Sample Test

The Final Test will consist of a subset of 8 problems similar (but not equal) to one of the 13 problems on that sample. Expect 2 out of (1), 3 out of (2)-(4), and one problem out of each of (5),(6) and (7).

1. Describe all solutions of the following differential equations for functions $y = y(x)$. Use an appropriate method.

$$(y/x + 6x) + (\log x - 2)y' = 0$$
$$t^3 y' + 4t^2 y = e^{-t}, \quad y(-1) = 0$$
$$y' = x^2/(y(1 + x^3))$$

2. Find the Wronskian of two solutions of the following differential equation.

$$t^2 y'' - t(t + 2)y' + (t + 2)y = 0.$$

3. Find all solutions of the following differential equations. You are free to use either the method of undetermined coefficients (if it is appropriate) or variation of parameters.

$$y'' + 2y' + y = 3e^{-t}$$
$$y'' + 4y = t^2 + 3e^t$$
$$y'' + 4y = 1$$
$$x^2 y'' - 3xy' + 4y = 0, \quad x > 0.$$

4. Solve the given differential equation by means of power series about $x_0 = 0$

$$y'' - xy' - y = 0.$$

5. For the following system of differential equations

$$x' = x - x^2 - xy$$
$$y' = 3y - xy - 2y^2$$

(a) determine all critical points,
(b) find the corresponding linear system at each critical point,
(c) find the eigenvalues and eigenvectors of each of these linear systems and draw a coclusion about the behavior of solutions with initial values close to the critical point,
(d) sketch a phase portrait using nullclines.

6. Show that the following functions are Liapunov functon for the critical point $(0, 0)$ and analyze the stability properties for each of following

systems of differential equations near $(0,0)$:
(a) $V(x,y) = x^4 + y^4$ for

$$x' = -x^3 + y^3$$
$$y' = -x^3 - y^3$$

(b) $V(x,y) = x^4 - y^4$ for

$$x' = -x^3 + y^3$$
$$y' = x^3 + y^3.$$

7. The following autonomous system of differential equations is expressed in polar coordinates. Determine all periodic solutions, all limit cycles, and determine their stability properties:

$$r' = r(r-1)(r-2)$$
$$\theta' = -1.$$