Homework 12

Topology II

Winter 2016/17

Review in tutorial on 13.2. and in class on 15.2. if requested

Problem 1
Compute homology and cohomology with various coefficients of the following pairs of spaces and show that they are not homotopy equivalent:
(a) $\mathbb{R}P^2 \vee S^3, \mathbb{R}P^3$
(b) $\mathbb{C}P^3, S^4 \times S^2$

Problem 2
Show that any continuous map $f : S^{k+\ell} \to S^k \times S^\ell$ induces a trvial map $f_* : H_{k+\ell}(S^{k+\ell}) \to H_{k+\ell}(S^k \times S^\ell)$ as long as $k, \ell > 0$. Is the same true for all continuous maps $g : S^k \times S^\ell \to S^{k+\ell}$?

Problem 3
Let $d \in \mathbb{N}$. $d > 0$. For the map $f_d : \mathbb{C}P^n \to \mathbb{C}P^n$ given by $f_d([z_0 : ... : z_n]) := [z_0^d : ... : z_n^d]$ compute $f_d^* : H^*(\mathbb{C}P^n; \mathbb{Z}) \to H^*(\mathbb{C}P^n; \mathbb{Z})$.

Problem 4
(a) Repeat the statement of Seifert and van Kampens Theorem as formulated in Hatchers book (several open sets covering X).
(b) Hatcher pg. 52,53: problems 2.,3.,4.,9
(c) Compute fundamental groups of a surface of genus $g = 1, 2, ...$, $\mathbb{R}P^2$, Klein bottle, finite connected sums of $\mathbb{R}P^2$.
(d) Hatcher pg. 54,55: problems 17.,20.
(e) fundamental groups of knot complements: Hatcher pg. 55, problem 22.