Homework 3

Topology II

Winter 2016/17

Problem 1
Let M be the Möbius band und ∂M its boundary.
(i) Show that $(M, \partial M)$ is a *good pair* and the quotient $M/\partial M$ is homeomorphic to $\mathbb{R}P^2$.
(ii) Determine all groups and maps of the long exact sequence of $(M, \partial M)$.
(iii) Use (i) and (ii) to compute the homology of $\mathbb{R}P^2$.

Problem 2
(i) Let X be a topological space, $p \in X$ a point. Denote by CX the cone over X given by $CX := X \times [0,1]/X \times \{1\}$ equipped with a topology induced by product and quotient topology and similarly by $\Sigma X := CX/X \times \{0\}$. Compute the singular homologies of CX, (CX, Cp), ΣX and $(\Sigma X, \Sigma p)$, possibly in dependence of the homology of X.
(ii) Show that $\Sigma S^n \cong S^{n+1}$ are homeomorphic.

Problem 3
(i) Let X, Y be topological spaces, $p \in X$ and $q \in Y$. Denote by $X \vee Y$ the space $X \coprod Y/\{p; q\}$. Compute the homology of $X \vee Y$.
(ii) Show that $X \vee Y$ is homeomorphic to $X \times \{q\} \cup Y \times \{p\}$. Denote by $X \wedge Y = X \times Y/X \vee Y$ the wedge of X and Y. Compute the homology of it.

Problem 4
Compute the homology of the torus and the Klein bottle using their compinatorial descriptions via squares and identification of their points on edges. Hint: Consider a closed disk or a square in the interior of the original square and apply excision. Then either apply Hurewicz' theorem in the presence of continuous maps or represent homology classes via cycles and show that a certain map on first homology vanishes.