Homework 9

Topology II

Winter 2016/17

Problem 1
(a) Discuss the definition of $\text{Ext}^n_R(G, H)$ of R-modules via free resolutions and present the necessary proofs. R is assumed to be a principal domain.
(b) Compute $\text{Ext}^n_{\mathbb{Z}_4}(\mathbb{Z}_2, \mathbb{Z}_2)$.
(c) Compute $\text{Ext}(\mathbb{Q}, \mathbb{Z})$ and $\text{Ext}(\mathbb{Z}_p, \mathbb{Z}_q)$ for prime numbers p and q.
(d) Show that $\text{Ext}^n_R(G \oplus G', H) = \text{Ext}^n_R(G, H) \oplus \text{Ext}^n_R(G', H)$.

Problem 2
(a) Show for a field F of characteristic 0 that $\text{Ext}(G, F) = 0$.
(b) Show that if $H_*(X)$ is finitely generated than for any field F of characteristic 0 for the Euler characteristic

$$\chi(X) = \sum_{k=0}^{\infty} (-1)^k \dim_F(H^k(X; F)).$$

(c) Try to extend the claim to any field.

Problem 3
Show that if the reduced cohomology groups $\tilde{H}^k(X; \mathbb{Z}_p)$ and $\tilde{H}^k(X; \mathbb{Q})$ vanish for all primes p, then also $\tilde{H}^k(X; \mathbb{Z})$ vanishes.