DISTRIBUTION SENSITIVITY FOR A CHANCE CONSTRAINED MODEL OF OPTIMAL LOAD DISPATCH

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Abstract:

We present an optimal-load-dispatch-model with considering the demand as a random vector and putting the equilibrium between total generation and demand as a probabilistic constraint. Motivated by the fact that the probability distribution μ for the demand is not exactly available, we study the stability of the model with respect to perturbations of μ.

1. Modelling

The problem of optimal load dispatch consists of allocating amounts of electric power to generation units such that the total generation costs are minimal while an electric power demand is met and certain additional constraints are satisfied. Our purpose is to obtain an optimal production policy for an energy production system consisting of thermal power stations, pumped storage plants and an energy contract for a time period up to one day with a discretization into hourly or half-hourly intervals. Unit commitment and network questions are excluded. The equilibrium between total generation and demand is modeled as a probabilistic (or chance-) constraint, thus obtaining a high reliability for the equilibrium to hold if the demand is considered as a random vector.
Other aspects of modelling, model behaviour and computation in scheduling optimization are stressed in the related papers [2], [3], [8] and in the book [14].

Let K and M denote the number of thermal power stations resp. pumped storage plants the system comprises and N be the number of subintervals in the discretization of the time period. The (unknown) levels of production in the thermal power stations and the pumped storage plants are y_r^i $(i=1,\dots,K;\ r=1,\dots,N)$, s_r^j $(j=1,\dots,M;\ r=1,\dots,N)$ (generation mode) and w_r^j $(j=1,\dots,M;\ r=1,\dots,N)$ (pumping mode). By z_r $(r=1,\dots,N)$ we denote the (unknown) amounts for energy purchased or sold according to the contract.
The total generation costs are given by the fuel costs of the thermal power stations (which are assumed to be a strictly convex quadratic function of the generated power, cf. [13]) plus the costs (resp. takings) according to the energy contract (which are a linear function of the power). In our model the pumped storage plants do not cause costs, which, of course, simplifies the matter. We refer to [2] for a model incorporating costs for the pumped storage plants. Hence the objective of the model becomes

$$(1.1)\qquad y^THy + h^Ty + g^Tz$$

where $y \in \mathbb{R}^{KN}$, $z \in \mathbb{R}^N$, $H \in L(\mathbb{R}^{KN}, \mathbb{R}^{KN})$ - positive definite, diagonal, $h \in \mathbb{R}^{KN}$ and $g \in \mathbb{R}^N$.
According to the discretization of the time period we have a demand vector d (of dimension N) which is understood as a random vector with distribution $\mu \in \mathcal{P}(\mathbb{R}^N)$ - the set of all Borel probability measures on $\mathbb{R}^N$. Claiming that a generation (y,s,w,z) fulfils the demand with probability $p_o \in (0,1)$ then means that

$$(1.2)\qquad \mu(\{d \in \mathbb{R}^N:\ \sum_{i=1}^{K} y_r^i + \sum_{j=1}^{M}(s_r^j - w_r^j) + z_r \geq d_r,\ r=1,\dots,N\}) \geq p_o .$$

In addition to this probabilistic constraint we take into account conditions which characterize the operation of the different plants:

$$(1.3)\qquad \underline{a}_1 \leq y \leq \bar{a}_1 ,\ 0 \leq s \leq \bar{a}_2 ,\ 0 \leq w \leq \bar{a}_3 ,\ \underline{a}_4 \leq z \leq \bar{a}_4 ;$$

$$(1.4)\qquad S_j^{oo} - S_j^{o} \leq \sum_{r=1}^{\tau}(s_r^j - \eta_j w_r^j) \leq S_j^{oo} ,\quad j=1,\dots,M,\ \tau=1,\dots,N;$$

$$(1.5)\qquad \sum_{r=1}^{N}(s_r^j - \eta_j w_r^j) = b_1 ,\quad \sum_{r=1}^{N} z_r = b_2 .$$

Restrictions for the power output are modeled in (1.3). The inequalities (1.4) reflect the balance between generation and pumping (measured in energy) in the pumped storage plants, S_j^{oo} and S_j^{o} denote the initial resp. maximal stock in the upper dam and η_j are the pumping efficiencies which we put as the quotients of the maximal stocks (in energy) in the upper and in the lower dam. Here we assume that the maximal stocks (in water) in the upper and in the lower dam are equal and that no additional inflow resp. outflow occurs. The equations (1.5) are balances over the whole time period for the pumped storage plants resp. according to the energy contract. The model can be supplemented by further linear (non-probabilistic) constraints, for instance those reflecting fuel quotas in the thermal power stations.

From the formal point of view our model can be expressed as

$$(1.6)\qquad \min\{\ f(x)\ :\ x \in X_o ,\ \mu(\{d:\ Ax \geq d\}) \geq p_o\}$$

where $x = (y,s,w,z) \in \mathbb{R}^m$ $(m=N(K+2M+1))$, $f(x)$ is defined by (1.1), $X_o \subset \mathbb{R}^m$ is the bounded convex polyhedron given by (1.3)-(1.5), μ is the probability distribution of the demand and $A \in L(\mathbb{R}^m, \mathbb{R}^N)$ is a suitable matrix.

2. Sensitivity Analysis

Let us consider the following more general chance constrained model

(2.1) $\min \{ f(x) : x \in \mathbb{R}^m, \mu(\{z \in \mathbb{R}^s : x \in X(z)\}) \geq p_o \}$

where f is a real-valued function defined on $\mathbb{R}^m$, X is a set-valued mapping from $\mathbb{R}^s$ into $\mathbb{R}^m$, $p_o \in [0,1]$ is a prescribed probability level and μ is a probability distribution on $\mathbb{R}^s$. For basic results on chance constrained problems consult [4], [15] and the references therein. We are going to study the behaviour of (2.1) with respect to (small) perturbations of the probability distribution μ. Our approach relies on stability results for parametric optimization problems with parameters varying in metric spaces (see e.g. [1], [6], [10]). As parameter space we consider the space $\mathcal{P}(\mathbb{R}^s)$ of all Borel probability measures on $\mathbb{R}^s$ equipped with a suitable metric. Because of its central place in the convergence theory for probability measures it seems appropriate to metrize the topology of weak convergence on $\mathcal{P}(\mathbb{R}^s)$ by a suitable distance function. This has been done in the stability analysis carried out in [5] (using the results of [10]). An example in [12] indicates that stability of (2.1) with respect to the topology of weak convergence can not be expected in general without additional smoothness assumptions on the measure μ. It turned out in [11], [12] that the distance

(2.2) $\alpha_{\mathcal{B}}(\mu,\nu) := \sup \{ |\mu(B) - \nu(B)| : B \in \mathcal{B} \}$

$(\mu, \nu \in \mathcal{P}(\mathbb{R}^s))$, where $\mathcal{B}$ is a proper subclass of Borel sets in $\mathbb{R}^s$, is a suitable metric on $\mathcal{P}(\mathbb{R}^s)$ for the sensitivity analysis of (2.1). In the following, $\mathcal{B}$ will be chosen such that $\alpha_{\mathcal{B}}$ forms a metric on $\mathcal{P}(\mathbb{R}^s)$ and that it contains all the pre-images $X^-(x) := \{z \in \mathbb{R}^s : x \in X(z)\} (x \in \mathbb{R}^m)$.
Next we introduce some basic concepts and notations which are used throughout. For $\nu \in \mathcal{P}(\mathbb{R}^s)$ we denote by F_ν the distribution function of ν and set for $p \in [0,1]$

$C_p(\nu) := \{ x \in \mathbb{R}^m : \nu(X^-(x)) \geq p \}$, hence problem (2.1) becomes $\min \{ f(x) : x \in C_{p_o}(\mu) \}$. Given $V \subseteq R^m$ and $\nu \in \mathcal{P}(\mathbb{R}^s)$ we denote

$\varphi_V(\nu) := \inf \{ f(x) : x \in C_{p_o}(\nu) \cap \text{cl } V \}$ and

$\psi_V(\nu) := \{ x \in C_{p_o}(\nu) \cap \text{cl } V : f(x) = \varphi_V(\nu) \}$,

where we employ the abbreviation cl for closure. Following [10], [6] we call a nonempty subset M of R^m a complete local minimizing set (CLM set) for f on $C_{p_o}(\nu)$ if there is an open set $Q \supset M$ such that $M = \psi_Q(\nu)$. Examples for CLM sets are the set of global minimizers (which we shall denote by $\psi(\nu)$ and, accordingly, the global optimal value by $\varphi(\nu)$) or strict local minimizing points.
We call a multifunction Γ from a metric space T to $\mathbb{R}^m$ closed resp. upper semicontinuous (usc) at $t_o \in T$ according to the usual definitions (cf. [1]), Γ is said to be pseudo-Lipschitzian at $(x_o, t_o) \in \Gamma(t_o) \times T$ if there are neighbourhoods $U = U(t_o)$, $V = V(x_o)$ and a positive constant L such that

$\Gamma(t') \cap V \subset \Gamma(t'') + L d(t',t'') B_m$

whenever $t', t'' \in U$, here d denotes the metric in T and B_m is the closed unit ball in $\mathbb{R}^m$.
The following theorem asserts in a fairly general frame sensitivity properties for solutions of a parametric chance constrained problem. The proof which relies on stability results for abstract parametric programming problems obtained by D. Klatte in [6] can be found in [11](Th. 5.4.).

Theorem 2.1.:

Let in (2.1) $\mu \in \mathcal{P}(\mathbb{R}^s)$, $p_o \in (0,1)$ and $\{ X^-(x) : x \in \mathbb{R}^m \} \subset \mathcal{B}$. Let further X be a closed multifunction and f be locally Lipschitzian. Assume that there exists a bounded open set $V \subset \mathbb{R}^m$ such that $\psi_V(\mu)$ is a CLM set for f on $C_{p_o}(\mu)$. Let the multifunction $p \longmapsto C_p(\mu)$ be pseudo-Lipschitzian at each $(x_o, p_o) \in \psi_V(\mu) \times \{p_o\}$. Then ψ_V is usc at μ with respect to the metric $\alpha_{\mathcal{B}}$ on $\mathcal{P}(\mathbb{R}^s)$ and there exist constants $L > 0$ and $\delta > 0$ such that $\psi_V(\nu)$ is a CLM set for f on $C_{p_o}(\nu)$ and

$$|\varphi_V(\mu) - \varphi_V(\nu)| \leq L \alpha_{\mathcal{B}}(\mu,\nu)$$

whenever $\alpha_{\mathcal{B}}(\mu,\nu) < \delta$.

If $\alpha_{\mathcal{B}}(\mu_n, \mu) \longrightarrow 0$ holds for every sequence (μ_n) converging weakly to μ, then the class $\mathcal{B}$ is called a μ-uniformity class. As an example we consider $\bar{\mathcal{B}} := \{ \emptyset, (-\infty, z] : z \in \mathbb{R}^s \}$ which is a μ-uniformity class if F_μ is a continuous function. The metric $\alpha_{\bar{\mathcal{B}}}$ then is the Kolmogorov distance, i.e. $\alpha_{\bar{\mathcal{B}}}(\mu,\nu) = \sup \{ |F_\mu(z) - F_\nu(z)| : z \in \mathbb{R}^s \}$ (consult [12] for a more detailed exposition).
To obtain an implication of Theorem 2.1. that finally may be applied to the above load dispatch model we assume that the distribution μ of the unperturbed problem is logarithmic concave, i.e. $\mu(\lambda B_1 + (1-\lambda)B_2) \geq (\mu(B_1))^\lambda (\mu(B_2))^{1-\lambda}$ holds for all $\lambda \in [0,1]$ and all Borel sets B_1, B_2 in $\mathbb{R}^s$ ([7]). It is known that μ is logarithmic concave if it has a density f_μ such that $\ln f_\mu$ is a concave function on $\mathbb{R}^s$ ([7]). Note that, hence, e.g. the (non-degenerate) multivariate normal distribution is logarithmic concave. Furthermore, we note that for a logarithmic concave $\mu \in \mathcal{P}(\mathbb{R}^s)$ the function $-\ln F_\mu$ is convex.
A second prerequisite is the following lemma which is an immediate consequence of Corollary 2 in [9].

Lemma 2.2.:

Let the multifunction Γ (from $\mathbb{R}$ to $\mathbb{R}^m$) be given by $\Gamma(t) := \{ x \in X_o : g(x) \leq t \}$ where $X_o \subset \mathbb{R}^m$ is a nonempty closed convex set and g is a convex function from $\mathbb{R}^m$ to $\mathbb{R}$. Let $x_o \in \Gamma(t_o)$ and assume that there exists $\bar{x} \in X_o$ such that $g(\bar{x}) < t_o$ (Slater point).
Then Γ is pseudo-Lipschitzian at (x_o, t_o).

For convex chance constraints we now have the following corollary to Theorem 2.1.

Corollary 2.3.:

Let in (2.1) $\mu \in \mathcal{P}(\mathbb{R}^s)$ be logarithmic concave, $p_o \in (0,1)$, f be locally Lipschitzian and X be a multifunction from $\mathbb{R}^s$ to $\mathbb{R}^m$ given by $X(z) := \{x \in X_o : Ax \geqslant z\}$ with a nonempty, compact, convex set $X_o \subset \mathbb{R}^m$ and $A \in L(\mathbb{R}^m, \mathbb{R}^s)$. Assume that there exists $\bar{x} \in X_o$ such that $F_\mu(A\bar{x}) > p_o$.

Then ψ is usc at μ with respect to the metric $\alpha_{\mathfrak{B}}$ on $\mathcal{P}(\mathbb{R}^s)$ and there exist constants $L > 0$ and $\delta > 0$ such that $\psi(\nu) \neq \emptyset$ and $|\varphi(\mu) - \varphi(\nu)| \leqslant L\alpha_{\mathfrak{B}}(\mu,\nu)$ whenever $\alpha_{\mathfrak{B}}(\mu,\nu) < \delta$.

Proof:

We write (2.1) in the equivalent form

(2.3) $\min\{f(x) : x \in X_o, -\ln F_\mu(Ax) \leqslant -\ln p_o\}$.

According to the assumptions the set $\psi(\mu)$ of global minimizers to (2.3) is nonempty and compact. Hence the assumptions in Theorem 2.1. concerning the CLM set may be fulfilled with a bounded open set $V \subset X_o$ (recall that X_o is bounded) and the mappings ψ and ψ_V resp. φ and φ_V coincide. Since μ is logarithmic concave the function $g(x) := -\ln F_\mu(Ax)$ is convex, and strict monotonicity of the logarithm implies $g(\bar{x}) < -\ln p_o$. Hence Lemma 2.2. and the fact that the logarithm is Lipschitzian on bounded sets provide that the multifunction $p \longmapsto C_p(\mu)$ is pseudo-Lipschitzian at each (x_o, p_o) belonging to $\psi(\mu) \times \{p_o\}$. Obviously, the multifunction X is closed and $\{X^-(x) : x \in \mathbb{R}^m\} \subset \mathfrak{B}$. The assertion now follows from Theorem 2.1. □

We remark that Corollary 2.3. applies to the load dispatch model (1.6) if one assumes that the probability distribution μ for the demand is logarithmic concave and that there exists a generation policy $\bar{x} \in X_o$ such that $F_\mu(A\bar{x}) > p_o$, i.e. a feasible generation policy that strictly fulfills the chance constraint (1.2). As one conclusion of Corollary 2.3. we then obtain that the optimal generation costs behave Lipschitzian if one replaces μ by approximations of sufficient accuracy.

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