Stability and other Reminiscences

(Rüdiger's early years in stochastic optimization)

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http://www.math.hu-berlin.de/~romisch

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MATHEMATICAL RESEARCH | MATHEMATISCHE FORSCHUNG

Parametric Optimization and Related Topics

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ON APPROXIMATIONS AND STABILITY IN STOCHASTIC PROGRAMMING*)

Peter Kall**)

ABSTRACT

It has become an accepted approach to attack stochastic programming problems by approximating the given probability distribution in various ways. After sketching one of these approaches for recourse problems, the stability problem with respect to the probability measure, involved with those approximations as well as with inexact information in applied problems, is discussed for recourse and chance constrained models.

1. INTRODUCTION

In mathematical programming we are used to deal with problems of the type

$$\left.\begin{array}{ll} \min & f(x) \\ \text{s.t.} & g(x) \geq 0 \\ & x \in X, \end{array}\right\} \qquad (1.1)$$

where $X \subset \mathbb{R}^n$, $f:X \to \mathbb{R}$ and $g:X \to \mathbb{R}^m$ are given and various assumptions are imposed on X (e.g. convexity, compactness, polyhedrality) and on f and g, respectively (e.g. continuity, convexity, differentiability, linearity). The crucial hypothesis for dealing with (1.1) is that f and g are given deterministic functions. Since the constraints $g(x) \geq 0$ in (1.1) in applications mean, for instance, production requirements, capacity restrictions etc., in many cases the constraint functions are very likely to be affected by a random parameter ξ. A similar observation can be

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STABILITY IN TWO-STAGE STOCHASTIC PROGRAMMING

Stephen M. Robinson
University of Wisconsin-Madison

and

Roger J.-B. Wets
International Institute for Applied Systems Analysis (IIASA)
and University of California, Davis

ABSTRACT

We analyze the effect of changes in problem functions and/or distributions in certain two-stage stochastic programming problems with recourse. Under reasonable assumptions the locally optimal value of the perturbed problem will be continuous and the corresponding set of local optimizers will be upper semicontinuous with respect to the parameters (including the probability distribution in the second stage).

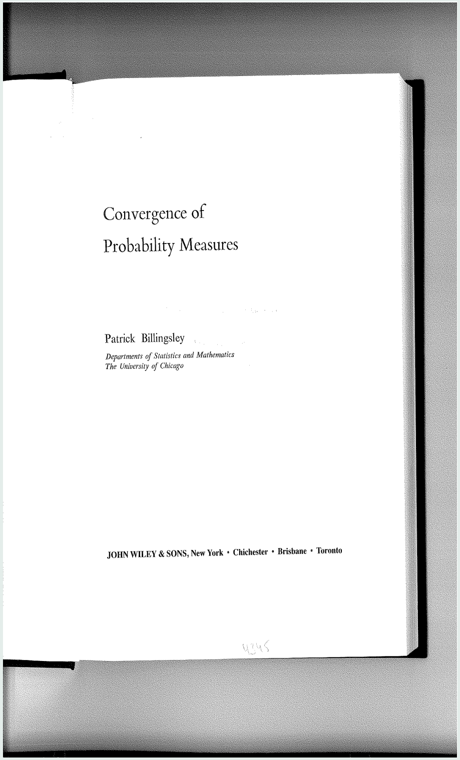
AMS(MOS) Subject Classifications: 90C42, 90C31

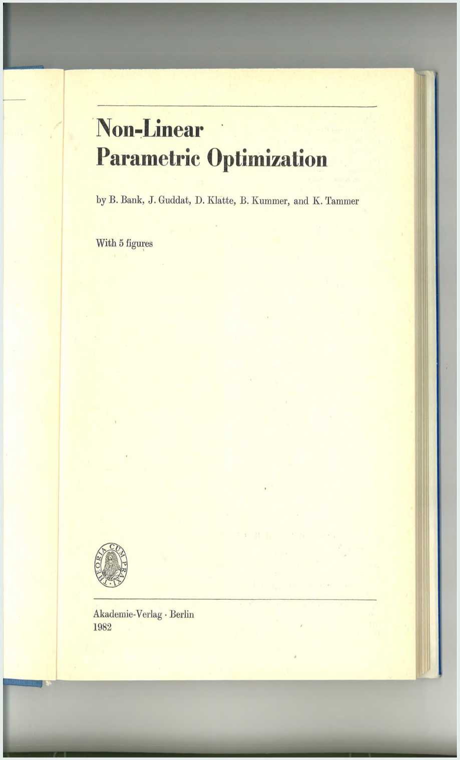
Short Title: STABILITY IN TWO-STAGE STOCHASTIC PROGRAMMING

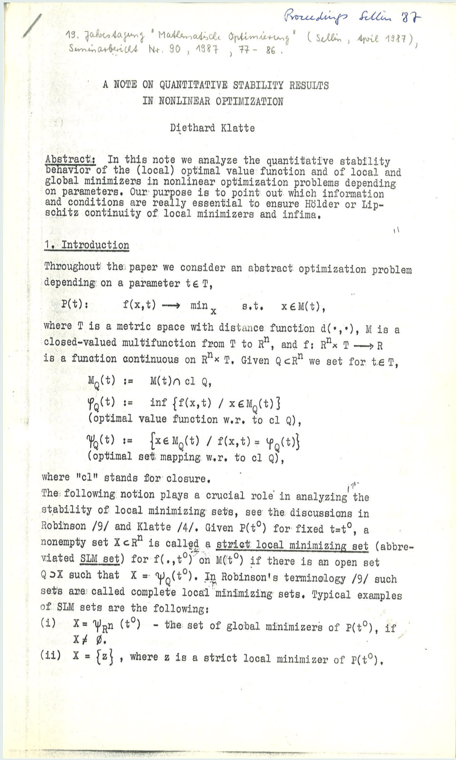
Keywords: Stochastic programming, recourse, stability, sensitivity analysis, weak convergence.

Sponsored by the National Science Foundation under Grants DCR-8502202 and ECS-8542328, and the United States Army under Contract No. DAAG29-80-C-0041. An earlier version of this work was presented by invitation at the IIASA Workshop on Numerical Methods for Stochastic Optimization, December 1983.

Two bibles and one research paper







Proceedings Sellin '87

19. Jahrestagung "Mathematische Optimierung" (Sellin, April 1987), Seminarbericht Nr. 90, 1987, 77 - 86.

A NOTE ON QUANTITATIVE STABILITY RESULTS IN NONLINEAR OPTIMIZATION

Diethard Klatte

Abstract: In this note we analyze the quantitative stability behavior of the (local) optimal value function and of local and global minimizers in nonlinear optimization problems depending on parameters. Our purpose is to point out which information and conditions are really essential to ensure Hölder or Lipschitz continuity of local minimizers and infima.

1. Introduction

Throughout the paper we consider an abstract optimization problem depending on a parameter $t \in T$,

$$P(t): \quad f(x,t) \longrightarrow \min_x \quad \text{s.t.} \quad x \in M(t),$$

where T is a metric space with distance function $d(\cdot,\cdot)$, M is a closed-valued multifunction from T to R^n, and $f: R^n \times T \longrightarrow R$ is a function continuous on $R^n \times T$. Given $Q \subset R^n$ we set for $t \in T$,

$$M_Q(t) := M(t) \cap \text{cl } Q,$$

$$\varphi_Q(t) := \inf \{f(x,t) \,/\, x \in M_Q(t)\}$$

(optimal value function w.r. to cl Q),

$$\psi_Q(t) := \{x \in M_Q(t) \,/\, f(x,t) = \varphi_Q(t)\}$$

(optimal set mapping w.r. to cl Q),

where "cl" stands for closure.

The following notion plays a crucial role in analyzing the stability of local minimizing sets, see the discussions in Robinson /9/ and Klatte /4/. Given $P(t^o)$ for fixed $t=t^o$, a nonempty set $X \subset R^n$ is called a strict local minimizing set (abbreviated SLM set) for $f(.,t^o)$ on $M(t^o)$ if there is an open set $Q \supset X$ such that $X = \psi_Q(t^o)$. In Robinson's terminology /9/ such sets are called complete local minimizing sets. Typical examples of SLM sets are the following:

(i) $X = \psi_{R^n}(t^o)$ - the set of global minimizers of $P(t^o)$, if $X \neq \emptyset$.

(ii) $X = \{z\}$, where z is a strict local minimizer of $P(t^o)$.

Weak convergence of probability measures in $\mathcal{P}(\mathbb{R}^d)$:

$$P_n \to^w P \quad \text{iff} \quad \lim_{n\to\infty} \int_{\mathbb{R}^d} f(\xi) P_n(d\xi) = \int_{\mathbb{R}^d} f(\xi) P(d\xi) \quad (\forall f \in C_b(\mathbb{R}^d))$$

Metrization:

Prokhorov metric:

$$\rho(P,Q) = \inf \left\{ \varepsilon > 0 : P(A) \leq Q(A^\varepsilon) + \varepsilon \text{ for all closed } A \subseteq \mathbb{R}^d \right\},$$

where $A^\varepsilon = \{y \in \mathbb{R}^d : d(y,A) < \varepsilon\}$ is the open ε-enlargement of A.

Dudley's bounded Lipschitz metric:

$$\beta(P,Q) = d_{\mathrm{BL}}(P,Q) = \sup_{\|f\|_{\mathrm{BL}} \leq 1} \left| \int_{\mathbb{R}^d} f(\xi) P(d\xi) - \int_{\mathbb{R}^d} f(\xi) P(d\xi) \right|,$$

where $\mathrm{BL}(\mathbb{R}^{\mathrm{d}}, \mathbb{R})$ is the linear space of real-valued bounded and Lipschitz continuous functions on $\mathbb{R}^d$ with norm

$$\|f\|_{\mathrm{BL}} = \sup_{\xi \in \mathbb{R}^d} |f(\xi)| + \sup_{\substack{\xi, \tilde{\xi} \in \mathbb{R}^d \\ \xi \neq \tilde{\xi}}} \frac{|f(\xi) - f(\tilde{\xi})|}{\|\xi - \tilde{\xi}\|}.$$

Properties:

- $\frac{2}{3}(\rho(P,Q))^2 \leq \beta(P,Q) \leq 2\rho(P,Q)$, $\forall P,Q \in \mathcal{P}(\mathbb{R}^d)$.
- $P_n \to^w P$ and $\mathcal{F}$ uniformly bounded and equicontinuous implies
$$\lim_{n\to\infty} \sup_{f\in\mathcal{F}} \left| \int_{\mathbb{R}^d} f(\xi)P_n(d\xi) - \int_{\mathbb{R}^d} f(\xi)P(d\xi) \right| = 0\,.$$
.
- $P_n \to^w P$ implies $\limsup_{n\to\infty} P_n(A) \leq P(A)$ if $A \subseteq \mathbb{R}^d$ is closed.
$P_n \to^w P$ implies $\lim_{n\to\infty} P_n(A) = P(A)$ if A is closed and $P(\partial A) = 0$.

Problems: Continuity properties of the mappings

- $P \mapsto \int_{\mathbb{R}^d} f(\xi)P(d\xi)$ if f is locally Lipschitz continuous on $\mathbb{R}^d$.
- $P \mapsto \int_{\mathbb{R}^d} \mathbf{1}_A(\xi)P(d\xi) = P(A)$ if A belongs to a subclass of all convex subsets of $\mathbb{R}^d$.

We consider the stochastic program

$$\min\left\{\int_\Xi f_0(x,\xi)P(d\xi) : x \in X\right\}.$$

With $v(P)$ and $S(P)$ denoting its optimal value and solution set it holds

$$|v(P)-v(Q)| \;\leq\; \sup_{x\in X}\left|\int_\Xi f_0(x,\xi)P(d\xi) - \int_\Xi f_0(x,\xi)Q(d\xi)\right|$$

$$\emptyset \neq S(Q) \;\subseteq\; S(P) + \Psi_P^{-1}\left(\sup_{x\in X}\left|\int_\Xi f_0(x,\xi)P(d\xi) - \int_\Xi f_0(x,\xi)Q(d\xi)\right|\right),$$

where X is assumed to be compact, Q is a probability distribution approximating P and Ψ_P is the growth function of the objective near the solution set, i.e.,

$$\Psi_P(t) := \inf\left\{\int_\Xi f_0(x,\xi)P(d\xi) - v(P) : x \in X, d(x,S(P)) \geq t\right\}.$$

Hence, the distance $d_{\mathcal{F}}$ with $\mathcal{F} := \{f_0(x,\cdot) : x \in X\}$ becomes important

$$d_{\mathcal{F}}(P,Q) := \sup_{f\in\mathcal{F}}\left|\int_\Xi f(\xi)P(d\xi) - \int_\Xi f(\xi)Q(d\xi)\right|.$$

Two-stage stochastic programs:

$$\min\left\{\langle c,x\rangle+\int_{\Xi}\Phi(q(\xi),h(\xi)-T(\xi)x)P(d\xi):x\in X\right\},$$

where $c\in\mathbb{R}^m$, Ξ and X are polyhedral subsets of $\mathbb{R}^d$ and $\mathbb{R}^m$, respectively, P is a probability measure on Ξ and the $s\times m$-matrix $T(\cdot)$, the vectors $q(\cdot)\in\mathbb{R}^{\overline{m}}$ and $h(\cdot)\in\mathbb{R}^s$ are affine functions of ξ.

The function Φ denotes the parametric infimum function of the linear second-stage program

$$\Phi(u,t)=\inf\{\langle u,y\rangle:Wy=t,y\in Y\},$$

which is finite and continuous on $\mathcal{D}\times W(Y)$, where $\mathcal{D}$ is the dual feasibility set

$$\mathcal{D}=\left\{u\in\mathbb{R}^{\overline{m}}:\left\{z\in\mathbb{R}^s:W^\top z-u\in Y^\star\right\}\neq\emptyset\right\},$$

where W is the $s\times\overline{m}$ recourse matrix, $W^\top$ the transposed of W and $Y^\star$ the polar cone to the polyhedral cone Y in $\mathbb{R}^{\overline{m}}$.

The function Φ is concave-convex polyhedral, hence, locally Lipschitz continuous with linearly growing local Lipschitz moduli on $\mathcal{D}\times W(Y)$ and it holds

$$d_{\mathcal{F}}(P,Q)\leq C\left(1+\int_{\mathbb{R}^d}\|\xi\|^{2p}(P+Q)(d\xi)\right)\beta(P,Q)^{1-\frac{1}{p}}\quad(p>1).$$

Faculty of Mathematics and Physics
Charles University

International Conference
on Stochastic Programming

Abstracts
Instructions for the authors

Prague, September 15 – 19, 1986

ESTIMATES FOR OPTIMAL SOLUTIONS OF APPROXIMATIONS IN STOCHASTIC LINEAR PROGRAMMING WITH RECOURSE

Rüdiger Schultz, Humboldt-Universität, Sektion Mathematik,
DDR-1086 Berlin, PSF 1297

Starting from known quantitative stability results concerning optimal values of stochastic programming problems we derive convergence rates for optimal solution points of approximations (e.g. via conditional expectations) in stochastic linear programming with recourse.
Special attention is paid to the case of simple recourse.

Mathematical Programming 50 (1991) 197–226
North-Holland

Distribution sensitivity in stochastic programming

Werner Römisch and Rüdiger Schultz
Sektion Mathematik, Humboldt-Universität Berlin, O-1086 Berlin, Germany

Received 31 December 1987
Revised manuscript received 20 June 1989

In this paper, stochastic programming problems are viewed as parametric programs with respect to the probability distributions of the random coefficients. General results on quantitative stability in parametric optimization are used to study distribution sensitivity of stochastic programs. For recourse and chance constrained models quantitative continuity results for optimal values and optimal solution sets are proved (with respect to suitable metrics on the space of probability distributions). The results are useful to study the effect of approximations and of incomplete information in stochastic programming.

AMS 1980 Subject Classifications: 90C15, 90C31.

Key words: Stochastic programming, quantitative stability, recourse problem, chance constrained problem, probability metric.

1. Introduction

In the present paper we study the behaviour of stochastic programming problems with respect to (small) perturbations of the underlying probability distributions. Emphasis is placed on quantitative stability results for optimal values and sets of optimal solutions to stochastic programs. To explain our aim, let us consider the following rather general stochastic programming model

$$\min\left\{\int_Z f(z, x)\mu(\mathrm{d}z)\colon x \in \mathbb{R}^m,\ \mu(\{z \in Z\colon x \in X(z)\}) \geqslant p_0\right\} \tag{1.1}$$

where $Z \subset \mathbb{R}^s$ is a Borel set, f is a function from $Z \times \mathbb{R}^m$ to $\mathbb{R}$, X is a set-valued mapping from Z into $\mathbb{R}^m$, $p_0 \in [0, 1]$ is a prescribed probability level and μ is a probability distribution on Z. Note that stochastic programs with (linear and quadratic) recourse (cf. (3.3) and (3.4)) and programs with probabilistic (or chance) constraints (cf. (5.1)) fit into (1.1).

Motivated by a number of applications, it seems particularly desirable to establish the stability of stochastic programs with respect to perturbations of the underlying distribution μ in the sense of the topology of weak convergence on $\mathscr{P}(Z)$ — the

This research was presented in parts at the 4th International Conference on Stochastic Programming held in Prague in September 1986.

WORKING PAPER

DISTRIBUTION SENSITIVITY FOR A CHANCE CONSTRAINED MODEL OF OPTIMAL LOAD DISPATCH

Werner Römisch
Rüdiger Schultz

November 1989
WP-89-090

IIASA
International Institute
for Applied Systems Analysis

JOURNAL OF OPTIMIZATION THEORY AND APPLICATIONS: Vol. 71, No. 3, DECEMBER 1991

Distribution Sensitivity for Certain Classes of Chance-Constrained Models with Application to Power Dispatch[1,2]

W. RÖMISCH[3] AND R. SCHULTZ[4]

Communicated by A. V. Fiacco

Abstract. Using results from parametric optimization, we derive for chance-constrained stochastic programs quantitative stability properties for locally optimal values and sets of local minimizers when the underlying probability distribution is subjected to perturbations in a metric space of probability measures. Emphasis is placed on verifiable sufficient conditions for the constraint-set mapping to fulfill a Lipschitz property which is essential for the stability results. Both convex and nonconvex problems are investigated. For a chance-constrained model of power dispatch, where the power demand enters as a random vector with incompletely known probability distribution, we discuss consequences of our general results for the stability of optimal generation costs and optimal generation policies.

Key Words. Parametric optimization, chance-constrained stochastic programming, sensitivity analysis, optimal power dispatch.

1. Introduction

To ensure a certain level of reliability for the solutions to optimization problems containing random data, it has become an accepted approach to

[1]The authors thank P. Kleinmann (formerly with the Humboldt-Universität, Berlin, Germany) for his active cooperation in designing the power dispatch model and J. Mayer (MTA SZTAKI, Budapest, Hungary) for his insight into energy optimization. Further thanks are due to the referees for their constructive criticism.

[2]This research was developed in the course of a contract study between the International Institute for Applied Systems Analysis, Laxenburg, Austria and the Humboldt-Universität, Berlin, Germany.

[3]Associate Professor, Humboldt-Universität, Sektion Mathematik, Berlin, Germany.

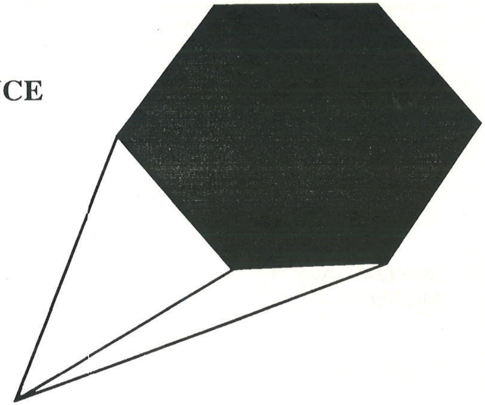
[4]Research Assistant, Humboldt-Universität, Sektion Mathematik, Berlin, Germany.

W. Römisch

FIFTH INTERNATIONAL

CONFERENCE

ON



STOCHASTIC PROGRAMMING

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FINAL PROGRAM

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Ann Arbor, Michigan, USA
August 13–18, 1989

Sponsors:

The National Science Foundation

IBM Corporation

AT&T

Operations Research Society of America

The Institute of Management Sciences

Department of Industrial and Operations Engineering and

College of Engineering, The University of Michigan

The Committee for Stochastic Programming, Mathematical Programming Society

Multisage Stochastic Optimization

R.T. Rockafellar
University of Washington

Most of the computational efforts so far in stochastic programming have revolved around two-stage models, but problems with more than two stages have been studied theoretically and will surely be of increasing interest. Major challenges are to set up the problem structure so as to be amenable to computation, and to understand the connections with the models offered in dynamic programming and stochastic control. This talk will report on joint work in these directions with Roger Wets.

For problems akin to multistage linear or convex programming, duality can be developed in the form of saddle point representations of optimal solutions. Such a representation opens up new numerical approaches for which experiments are now going on. The associated dual problem has a dynamical structure that goes backward in time. The most interesting feature is that in order to bridge between stochastic programming, as conceived until now, and other forms of dynamic stochastic optimization, one needs to introduce both primal and dual information structures.

The primal information structure expresses limitations on how decisions can be made in the primal problem, while the dual information structure expresses limitations on how costs can be charged and therefore on the way prices can evolve. The underlying idea is that the costs added at time t may be based on more information than is available for making the decision that is necessary in the primal problem at time t, or it may necessarily be based on less. The latter case corresponds to a primal constraint structure involving conditions on the expected values of certain quantities.

Stability Analysis for Stochastic Programs

W. Römisch and R. Schultz
Humboldt-Universität Berlin

This paper continues our (quantitative) stability analysis for stochastic programs both with (linear) complete recourse and with probabilistic constraints. We study the effect of perturbations of the underlying probability distribution on the optimal value and optimal solution set.

For linear stochastic programs with complete recourse and random right-hand sides we show that, under reasonable assumptions on the data and the original distribution , the Hausdorff distance of the solution sets behaves (locally) Holder continuous at μ (with exponent 1/2) with respect to a Wasserstein metric on the space of probability measures. Examples show that this result becomes wrong when the constraint set is no longer polyhedral and that the exponent 1/2 is optimal. A key part in our approach is to derive conditions under which complete recourse functionals are C(1,1) and strongly convex. The conditions for strong convexity consist of an interplay between algebraic assumptions on the complete recourse matrix and analytical assumptions on the density of μ. These conditions are verified for recourse functionals with separability structure and for certain recourse functionals which are non- separable.

For stochastic programs with several joint probabilistic constraints we identify a suitable probability metric alpha (a so-called discrepancy) on the space of all (Borel) probability measures for which (quantitative) stability results can be proved. The central result asserts upper semicontinuity of (local) minimizing sets at the (unperturbed) distribution and a Lipschitz property of (locally) optimal values at both with respect to the metric alpha. Emphasis is placed on establishing verifiable sufficient conditions for stability. Especially, we consider first the case, where μ satisfies a convexity property, and secondly the case of one differentiable (joint) probabilistic constraint. Finally we point out that the results apply to a number of practical models known from the literature.

Parallel Decomposition of Linear Stochastic Control Problems

Andrzej Ruszczyński
Wydzial Elektroniki, Institut Automatyki, Warsaw, Poland

We consider a finite horizon linear stochastic control problem with discrete time, described by the equations

$$x_t = A_t x_{t-1} + B_t u_t + d_t,\ t = 1, \dots, T,$$

where x_t denotes the state vector, u_t is the control vector and $s_t = (A_t, B_t, d_t)$, $t = 1, \dots, T$, is a sequence of discretely distributed random variables. We assume that at each time instant t we observe s_t. The problem is to determine the *policy* $u_t(s_{t-1}, s_1, \dots, s_t)$ so as to satisfy additional constraints

$$(u_t, x_t) \in X_t,\ t = 1, \dots, T,$$

where X_t is a convex polyhedron, and to minimize the functional

$$E\{\sum_{t=1}^{T} c_t x_t + q_t u_t\}.$$

To this end we define a tree of subproblems, corresponding to the tree of realizations of the process s_t. In each subproblem we minimize in u_t a local cost function composed of the corresponding parts of the objective and of an estimate of the *cost-to-go* function at the current stage. The subproblems generate cutting planes for improving the estimates used by their predecessors and trial points used as previous states by their successors.

We show that the subproblems can operate in a parallel and asynchronous mode and that the whole method is finitely convergent under the assumption that the problem is bounded. We discuss the use of regularization in the subproblems and its impact on behavior of the method. Finally we show on a simple example that our parallel method can be substantially faster than any serial algorithm based on nested decomposition.

Convergence of Infima, Especially Stochastic Infima

Gabriella Salinetti and Roger J-B Wets
Universite "La Sapienza" di Roma/University of California-Davis

We analyze the convergence of optimal values and solutions of a sequence of a sequence of optimization problems

STABILITY ANALYSIS FOR STOCHASTIC PROGRAMS

Werner RÖMISCH and Rüdiger SCHULTZ
Humboldt-Universität Berlin, Sektion Mathematik, P.O. Box 1297, D-1086 Berlin, Germany

For stochastic programs with recourse and with (several joint) probabilistic constraints, respectively, we derive quantitative continuity properties of the relevant expectation functionals and constraint set mappings. This leads to qualitative and quantitative stability results for optimal values and optimal solutions with respect to perturbations of the underlying probability distributions. Earlier stability results for stochastic programs with recourse and for those with probabilistic constraints are refined and extended, respectively. Emphasis is placed on equipping sets of probability measures with metrics that one can handle in specific situations. To illustrate the general stability results we present possible consequences when estimating the original probability measure via empirical ones.

Keywords: Stochastic programs with recourse, stochastic programs with probabilistic constraints, distribution sensitivity, probability metrics.

1. Introduction

When formulating a stochastic programming model, one tacitly assumes the underlying probability distribution to be given. In practical situations, however, this is rarely the case; moreover, one often has to live with incomplete information and approximations. Furthermore, also under full information about the underlying distributions one is led to approximations, since exact computation of expectations and probabilities typically arising in stochastic programming is beyond the present numerical capabilities for a large class of distributions (e.g. multivariate continuous ones). These circumstances motivate a stability analysis for optimal values and optimal solutions to stochastic programs with respect to perturbations of the underlying probability distributions (cf. [10,17,31,33,43]). In the present paper, we pursue this for two basic problem classes in stochastic programming – for stochastic programs with recourse and for stochastic programs with probabilistic (or chance) constraints. We lay stress on structural properties of expectation functionals and of certain multifunctions defined by probabilities, on implications of these properties with respect to stability, on a proper selection of metrics in spaces of probability measures to guarantee the structural properties, on the one hand, and to be able to compute (or to estimate) distances of probability measures in specific situations, on the other hand.

ABSTRACTS

GAMM/IFIP-WORKSHOP

Stochastic Programming:
Stability, Numerical Methods and Applications

Gosen near Berlin
Germany
March 23-27, 1992

The Workshop is sponsored
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Mathematik und Mechanik (GAMM),
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and by the Humboldt-Universität Berlin

Strong Convexity and a Regularization Scheme for Two-Stage Stochastic Programs

Rüdiger Schultz
Humboldt-Universität Berlin
Fachbereich Mathematik

Abstract:

We investigate the structure of two-stage stochastic programs with linear complete recourse, random right-hand sides and random technology matrix. Emphasis is placed on sufficient conditions to end up with strong convexity of the expected-recourse functional. The latter leads to a good conditioning of the model (the solution set is non-empty and a singleton) and has consequences for the stability behaviour of the optimal solutions when perturbing the underlying probability distribution. The sufficient conditions for strong convexity are developed from structural results for two-stage models with random right-hand side and fixed technology matrix. As a by-product of our analysis we obtain a scheme to regularize (by artificial randomization) stochastic programs with fixed technology matrix.

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Strong convexity in stochastic programs with complete recourse ☆

Rüdiger Schultz

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ABSTRACTS

SIXTH INTERNATIONAL CONFERENCE

on

STOCHASTIC PROGRAMMING

September 14 - 18, 1992
CISM, Udine
Italy

VI INTERNATIONAL CONFERENCE ON STOCHASTIC PROGRAMMING

TENTATIVE PROGRAM

(AS OF AUGUST 26th)

MONDAY (SEPTEMBER 14th, 1992):

8:30 **Registration (CISM)**

Tutorials (Room D)

9:30 - 11:00. P.KALL (University of Zürich): *A general overview of stochastic programming,*

11:30 - 13:00. A.PRÉKOPA (RUTGERS, New Brunswick): *Probabilistic constrained programming,*

15:00 - 16:30. J.BIRGE (University of Michigan, Ann Arbor): *Multistage stochastic programming,*

17:00 - 18:30. A.KING (IBM, Yorktown Heights): *Statistical approaches to stochastic programming.*

TUESDAY (SEPTEMBER 15th, 1992):

8:30 **Registration (CISM)**

10:00 - 11:15. Opening session (Room K) G.ANDREATTA, G.SALINETTI and P.SERAFINI: *Welcoming Remarks,*
R.WETS: *Opening Lecture.*

11:45 - 12:30. **Tutorial (K)** Chair: K. FRAUENDORFER
A.A.GAIVORONSKI (Italtel, Milano): *Software for stochastic optimization problems.*

14:30 - 16:00. **Algorithms 1 (D)** Chair: Y.ERMOLIEV
S.M.ROBINSON, B.J.CHUN, B.R.FU, R.SURI (University of Wisconsin, Madison): *Bundle-based Methods for Stochastic Optimization,*
G.INFANGER (Stanford University): *Solving Large-Scale Multi-Stage Stochastic Linear Programs,*
S.D.FLÅM (University of Bergen) and R.SCHULTZ (Humboldt University, Berlin): *A New Approach to Stochastic Linear Programming.*

Engineering Applications (U) Chair: A.GAIVORONSKI
T.CHAKRABARTI (University College of Science, Calcutta): *Stochastic Transportation Problem,*
C.KAO (National Cheng Kung University, Taiwan): *Determination of Optimal Shipping Policy under Stochastic Shipping Time,*
E.MESSINA (State University of Milan), A.A.GAIVORONSKI (Italtel, Milano) and A.SCIOMACHEN (State University of Milan): *A Stochastic Optimization Approach for Robot Scheduling.*

16:30 - 18:00. **Approximations 1 (D)** Chair: S. SEN
J.BIRGE and D.HOLMES (University of Michigan, Ann Arbor): *The Value of the Approximate Solution in Stochastic Programming,*
S.E.WRIGHT (IBM, Yorktown Heights): *Primal-Dual Aggregation and Disaggregation for Stochastic Linear Programs,*
R.LEPP (Estonian Academy of Sciences, Tallinn): *Approximate solution of Stochastic Programs - The discretization Approach.*

Integer Stochastic Programming 1 (U) Chair: S.W.WALLACE
S.SHIODE (Osaka University) and H.ISHII (Okayama University): *Stochastic Bottleneck Spanning Tree problems,*
R.N.SEN (University of Calcutta): *On some Multicommodity Network Flows with Probabilistic Conversions,*
R.SCHULTZ (Humboldt University, Berlin): *Structure and Stability in Stochastic Programs with complete Integer Recourse.*

18:00 **Reception (CISM)**

18:30 - 19:00. **COSP Meeting.**

CONTINUITY AND STABILITY IN TWO-STAGE STOCHASTIC INTEGER PROGRAMMING

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ABSTRACT: For two-stage stochastic programs where the optimization problem in the second stage is a mixed–integer linear program continuity of the expectation of second–stage costs jointly in the first–stage strategy and the integrating probability measure is derived. Then, regarding the two–stage stochastic program as a parametric program with the underlying probability measure as parameter, continuity of the locally optimal value and upper semicontinuity of the corresponding set of local solutions are established.

1 Introduction

In this paper, we will analyse parameter dependent two–stage stochastic optimization problems of the type

$$P(\mu) \qquad \min\{f(x) + Q(x,\mu) : x \in C\},$$

where

$$Q(x,\mu) = \int_{\mathbf{R}^s} \Phi(z - Ax)\mu(dz) \tag{1.1}$$

and

$$\Phi(b) = \min\{q^T y + q'^T y' \ : \ Wy + W'y' = b,\ y' \geq 0,\ y \geq 0,\ y \in \mathbf{Z}^s\} \tag{1.2}$$

Here we assume that f is a continuous real–valued function on $\mathbf{R}^m$, $C \subset \mathbf{R}^m$ non–empty, closed, $z \in \mathbf{R}^s$, $A \in L(\mathbf{R}^m, \mathbf{R}^s)$, $q \in \mathbf{R}^{\bar{s}}$, $q' \in \mathbf{R}^{s'}$, $W \in L(\mathbf{R}^{\bar{s}}, \mathbf{R}^s)$, $W' \in L(\mathbf{R}^{s'}, \mathbf{R}^s)$, $b \in \mathbf{R}^s$. By $\mathbf{Z}^{\bar{s}}$ we denote the subset of vectors in $\mathbf{R}^{\bar{s}}$ having only integral components. Throughout, we assume that W and W' are rational

1. Allgemeine Angaben

Neuantrag auf Gewährung einer Sachbeihilfe

1.1 Antragsteller

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1.2 Thema

Approximation und Stabilität stochastischer Optimierungsprobleme

1.3 Kennwort

Stochastische Optimierung

1.4 Fachgebiet und Arbeitsrichtung

Angewandte Mathematik; stochastische Optimierung, parametrische und nichtglatte Optimierung, stochastische dynamische Optimierung und Steuerung, numerische Analysis.

1.5 Voraussichtliche Gesamtdauer

5 Jahre

1.6 Antragszeitraum

2 Jahre

1.7 Gewünschter Beginn der Förderung

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MATHEMATICAL PROGRAMMING

Mathematical Programming 70 (1995) 73–89

On structure and stability in stochastic programs with random technology matrix and complete integer recourse [1]

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ELSEVIER

Minisymposium
on
Stochastic Programming

January 22 - 23, 1994
Berlin

Scientific Program
and
Abstracts.

Organizers: W. Römisch and R. Schultz

Supported by the Deutsche Forschungsgemeinschaft (DFG)
Schwerpunktprogramm "Anwendungsbezogene Optimierung und Steuerung"

Titles of talks

Artstein, Z.:	Limit laws for variable decisions in stochastic programming
Dempster, M. A. H.:	Hierarchical models for telecommunications network planning
Dupaćová, J.:	Analysis of the mean value solution and the worst case analysis under relaxed convexity assumptions
Flåm, S.:	Learning to play Nash equilibrium
Frauendorfer, K.:	Computational issues for analyzing barycentric scenario trees
Henrion. R.	Topological characterization of the approximate subdifferential in the finite-dimensional case
Higle, J.	Inexact subgradient methods with applications in stochastic programming
Kaňková, V.	A note on sensitivity analysis for stochastic programming problems
Lepp, R.	Discrete approximation of SLP problems with recourse - the case of unbounded domains
Marti, K.	Differentiation of probability functions
Norkin, V. I.:	The comparison of stochastic integer programming methods on a recourse allocation stochastic problem
Pflug, G.:	Minimum distance tests for convex hypotheses
Rockafellar, R. T.:	Cost-to-go in multistage stochastic programming
Römisch, W.:	Differentiability of solution sets in two-stage stochastic programming
Ruszczynski, A.:	Strategic pricing in markets with a conformity effect
Schultz, R.:	Algebraic prerequisites for stochastic integer programming
Sen, S.:	Epigraphical nesting and convergence of a stochastic decomposition algorithm for multi-stage stochastic programs
Stougie, L.:	On the convex hull of the simple integer recourse objective function
van der Vlerk, M. H.:	Solving stochastic integer programs with complete recourse
Vogel, S.:	Continuous convergence and epi-convergence of random functions - sufficient conditions
Wets, R. J.-B.:	N. N.

Mixed-integer two-stage stochastic programs:

$$\min\left\{\langle c,x\rangle + \int_{\mathbb{R}^d}\Phi(q(\xi),h(\xi)-T(\xi)x)P(d\xi) : x\in X\right\},$$

where Φ denotes the parametric infimal function of the second-stage program

$$\Phi(u,t) := \inf\left\{\langle u_1,y_1\rangle + \langle u_2,y_2\rangle : W_1y_1 + W_2y_2 \le t, y_1\in\mathbb{R}^{m_1}, y_2\in\mathbb{Z}^{m_2}\right\}$$

for all $(u,t)\in\mathbb{R}^{m_1+m_2}\times\mathbb{R}^s$, and $c\in\mathbb{R}^m$, a closed subset X of $\mathbb{R}^m$, (s,m_1) and (s,m_2)-matrices W_1 and W_2, affine functions $T(\xi)\in\mathbb{R}^{s\times m}$, $q(\xi)\in\mathbb{R}^{m_1+m_2}$, $h(\xi)\in\mathbb{R}^s$, and a probability measure P on $\mathbb{R}^d$. We introduce

$$\begin{aligned}\mathcal{T} &= \{t\in\mathbb{R}^r : \exists(y_1,y_2)\in\mathbb{R}^{m_1}\times\mathbb{Z}^{m_2} \text{ such that } W_1y_1+W_2y_2\le t\}\\ \mathcal{U} &= \left\{u=(u_1,u_2)\in\mathbb{R}^{m_1+m_2} : \exists v\in\mathbb{R}^r_- \text{ such that } W_1^\top v = u_1,\, W_2^\top v = u_2\right\}.\end{aligned}$$

the primal and dual feasible right-side sets and assume:

(B1) The matrices W_1 and W_2 have only rational elements.

(B2) The cardinality of the set

$$\bigcup_{t\in\mathcal{T}}\{y_2\in\mathbb{Z}^{m_2} : \exists y_1\in\mathbb{R}^{m_1} \text{ such that } W_1y_1+W_2y_2\le t\}$$

is finite, i.e., the number of integer decisions is finite.

Proposition:

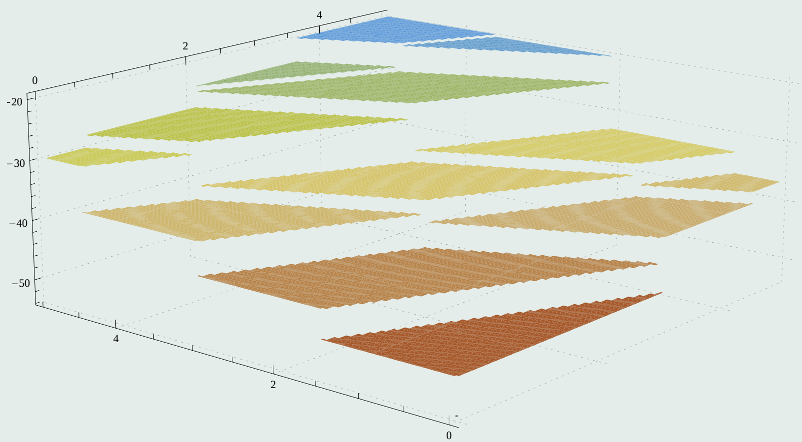
Assume (B1) and (B2). The function Φ is finite and lower semicontinuous on $\mathcal{U}\times\mathcal{T}$ and there exists a finite decomposition of $\mathcal{U}\times\mathcal{T}$ consisting of Borel sets $U_\nu\times B_\nu$, $\nu\in\mathcal{N}$, such that their closure is convex polyhedral and Φ is bilinear in (u,t) on each $U_\nu\times B_\nu$. Φ may have kinks and discontinuities at the boundaries of $U_\nu\times B_\nu$.

Example: (Schultz-Stougie-van der Vlerk 98)

$m = d = s = 2$, $m_1 = 0$, $m_2 = 4$, $c = (0,0)$, $X = [0,5]^2$, $h(\xi) = \xi$, $q(\xi) \equiv q = (-16,-19,-23,-28)$, $y_i \in \{0,1\}$, $i = 1,2,3,4$, $P \sim \mathcal{U}(5,10,15\}$ (discrete)

Second stage problem: MILP with 1764 binary variables and 882 constraints.

$$T = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \qquad W = \begin{pmatrix} 2 & 3 & 4 & 5 \\ 6 & 1 & 3 & 2 \end{pmatrix}$$



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RATES OF CONVERGENCE IN STOCHASTIC PROGRAMS WITH COMPLETE INTEGER RECOURSE*

RÜDIGER SCHULTZ†

Abstract. The stability of stochastic programs with mixed-integer recourse and random right-hand sides under perturbations of the integrating probability measure is considered from a quantitative viewpoint. Objective-function values of perturbed stochastic programs are related to each other via a variational distance of probability measures based on a suitable Vapnik–Červonenkis class of Borel sets in a Euclidean space. This leads to Hölder continuity of local optimal values. In the context of estimation via empirical measures the general results imply qualitative and quantitative statements on the asymptotic convergence of local optimal values and optimal solutions.

Key words. stochastic integer programming, parametric integer programming, Hölder continuity, stability, variational distance of probability measures, Vapnik–Červonenkis class, law of the iterated logarithm

AMS subject classifications. 90C15, 90C11, 90C31

1. Introduction. Consider the following two-stage stochastic integer program:

$$\min\{f(x) + Q(x,\mu) : x \in C\}, \tag{1}$$

where

$$Q(x,\mu) = \int_{\mathbb{R}^s} \Phi(z - Ax)\mu(\mathrm{d}z) \tag{2}$$

and

$$\Phi(b) = \min\{q^T y + q'^T y' : Wy + W'y' = b,\ y' \geq 0,\ y \geq 0,\ y' \in \mathbb{R}^{s'},\ y \in \mathbb{Z}^{\bar{s}}\}. \tag{3}$$

Generally, we assume that $f : \mathbb{R}^m \to \mathbb{R}$ is continuous; $C \subset \mathbb{R}^m$ nonempty, closed; $q \in \mathbb{R}^{\bar{s}}$; $q' \in \mathbb{R}^{s'}$; that $W \in L(\mathbb{R}^{\bar{s}}, \mathbb{R}^s)$, $W' \in L(\mathbb{R}^{s'}, \mathbb{R}^s)$ are matrices with rational entries; and that μ belongs to $\mathcal{P}(\mathbb{R}^s)$, the set of all Borel probability measures on $\mathbb{R}^s$. Throughout, $\mathbb{Z}$ denotes the set of integers.

The model (1) arises from a minimization problem with uncertain constraint parameters whose realizations are not known when having to fix the (first-stage) decision variable x. Infeasibilities b occurring after the realization of the uncertain parameters can be compensated at cost $\Phi(b)$ by the second-stage optimization procedure (3). Altogether, (1) aims at finding a first-stage decision x such that the sum of the first-stage costs $f(x)$ and the expected compensation (or recourse) costs $Q(x,\mu)$ become minimal. Of course, for the latter to be well defined one needs (at least) a probability distribution of the uncertain parameters.

The above model essentially differs from traditional two-stage stochastic programs (cf. [12], [40]) by the integrality constraints in the second stage. Whereas integrality in the first stage can be dealt with by fairly conventional means [41], its presence in the second stage is much more cumbersome since the integrand Φ in (2) is discontinuous. However, there are several examples in the literature showing that integrality of

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Abschlußbericht

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קרית-הטכניון, חיפה 32000

7th INTERNATIONAL CONFERENCE ON STOCHASTIC PROGRAMMING

June 26–29, 1995
Nahariya, ISRAEL

PROGRAMME

16:00-17:30 **Session E1 - Algorithmic Analysis** (Carmel)

Chair: **T. Szántai** (Technical University of Budapest, Hungary)

16:00-16:30 **O. Fiedler** (Freie Universität Berlin, Germany)
ON A DUAL METHOD FOR A SPECIALLY STRUCTURED LINEAR PROGRAMMING PROBLEM

16:30-17:00 **D. Dentcheva** (Humboldt-Universität Berlin, Germany)
DIFFERENTIAL STABILITY OF TWO-STAGE STOCHASTIC PROGRAMS

17:00-17:30 **T. Szántai** (Technical University of Budapest, Hungary)
BOOLE-BONFERRONI TYPE BOUNDS FOR SYSTEM-FAILURE PROBABILITY

Session E2 - Algorithmic Analysis (Carmel)

Chair: **Hercules Vladimirou** (University of Cyprus)

16:00-16:30 **A.I. Kibzun** (Moscow Aviation Institute, Russia)
CONTINUITY, CONVEXITY, AND DIFFERENTIABILITY OF THE QUANTILE FUNCTION

16:30-17:00 **R. Schultz** (Konrad-Zuse-Zentrum Berlin, Germany)
STRONG CONVEXITY IN TWO-STAGE LINEAR STOCHASTIC PROGRAMS WITH PARTIALLY RANDOM RIGHT-HAND SIDE

17:00-17:30 **H. Vladimirou** (University of Cyprus, Cyprus)
INVESTIGATING RECOURSE ROBUSTNESS IN STOCHASTIC PROGRAMS

17:30-18:30 Meeting of the Committee on Stochastic Programming (Gallery)

19:30 **Arabian Nights Dinner**

Happy birthday, Rüdiger !