On Helson's conjecture

The L^q norm of a Dirichlet polynomial $F(s) = \sum_{n=1}^{N} a_n n^{-s}$ is defined as

$$\|F\|_q := \left(\lim_{T\to\infty} \frac{1}{T} \int_0^T |F(it)|^q \, dt \right)^{1/q}$$

for $1 \le q < \infty$. We are particularly interested in the Dirichlet polynomial

$$D_N(s) := \sum_{n=1}^{N} n^{-s}$$

and the following conjecture of Helson:

$$\|D_N\|_1 = o(\sqrt{N})$$

when $N \to \infty$.

Although our research has led us to doubt that the conjecture can be true, but we have only been able to establish the following nontrivial estimate.

Theorem 1 *We have*

$$\|D_N\|_1 \gg \sqrt{N} (\log N)^{-0.02152}.$$

This is a joint work with Kristian Seip.