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Zahlentheorie

SoSe 2019

Exercise Sheet 6

If you want your solutions to be corrected, you should hand them in by Monday, May 27.
Please write your name and immatriculation number on top of every exercise

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Exercise 6.1 (3+3+3 points)

a) Let A be a UFD. Show that irreducible elements are prime elements.

b) Show that in a Noetherian ring every element can be written (non-uniquely) as product of finitely many irreducible elements.

c) Let A be a Dedekind domain. Show that if A is a UFD then A is a PID.

Exercise 6.2 (4 points) Is the ring $\mathbb{Z}[X]$ a Dedekind domain? Motivate your answer.

Exercise 6.3 (3+3 points) Let A be a Dedekind domain. Let $I = \mathfrak{p}_1^{n_1} \dots \mathfrak{p}_r^{n_r}$ and $J = \mathfrak{q}_1^{m_1} \dots \mathfrak{q}_s^{m_s}$ be two nonzero ideals, written in terms of their unique factorization into prime ideals.

a) Write the unique factorization of the ideal $I + J$.

b) Write the unique factorization of the ideal $I \cap J$.