CONFORMAL GEOMETRY AND SPECTRAL THEORY, BERLIN 11-13.11.2016

TITLES AND ABSTRACTS

Bernd Ammann *The Yamabe invariant*

Abstract: The (conformal) Yamabe constant of a compact riemannian manifold (M, g_0) is defined as

$$Y(M, [g_0]) := \inf \int_M \mathrm{scal}^g \, dv^g$$

where the infimum runs over all metrics g of volume 1 in $[g_0]$. The (smooth) Yamabe invariant of M is then defined as

$$\sigma(M) := \sup Y(M, [g_0])$$

where the supremum runs over all conformal classes $[g_0]$ on M.

The Yamabe invariant $\sigma(M)$ is positive if and only if M carries a metric of positive scalar curvature. Despite of its simple definition, the Yamabe invariant is extremely difficult to calculate, and it is only known for very few manifolds.

I want to give an overview over the knowledge about the invariant. In particular I discuss the behavior of the Yamabe invariant under surgeries, which is joint work with Mattias Dahl and Emmanuel Humbert. Using bordism theory we can see e.g. that the Yamabe invariant of a simply connected compact spin manifold of dimension 5 is between 45 and 79.

Andreas Čap *From holonomy reductions of Cartan geometries to geometric compactifications*

Abstract: (based on joint work with A. Rod Gover and M. Hammerl.)

Poincaré–Einstein metrics are special cases of conformally compact (pseudo–)Riemannian metrics. Starting from the description of Poincaré–Einstein metrics via reductions of conformal holonomy, I will describe similar reductions of projective and c–projective holonomy. Generalizing the structures obtained from these reductions leads to the concepts of projective and c–projective compactness. On a manifold with boundary, these concepts give rise to different geometric structures on the interior and on the boundary, which are tied together by a "background structure" on the whole manifold with boundary.

Matthias Fischmann *Bernstein-Sato identities for generalized Riesz distributions*

Abstract: We present three Bernstein-Sato identities for three classes of generalized Riesz distributions; scalar, differential forms and spinor version. As application, they are used to find meromorphic extensions of corresponding Riesz distributions.

Charles Frances *Dynamics and topology for 3-dimensional Lorentz manifolds*

Abstract: It is a well known phenomenon that in contrast to what happens for Riemannian manifolds, compact Lorentz structures might have a noncompact group of isometries. Such a property of the isometry group generally has strong consequences both on the geometry, and on the topology of the manifold. The aim of the talk is to review old, and present new results on the subject, with an emphasis on closed 3-dimensional manifolds.

Robin Graham *Higher-dimensional Willmore energies via minimal submanifold asymptotics*

Abstract: This talk will describe a derivation of a conformally invariant energy for an even-dimensional submanifold of a Riemannian manifold generalizing the Willmore energy of a surface. The energy and its associated Euler-Lagrange equation both arise naturally upon considering the asymptotics of minimal submanifolds in asymptotically Poincaré-Einstein spaces associated to the background conformal manifold. Particular attention will be paid to the case of a submanifold of dimension 4.

Matthias Hammerl *The Fefferman-Graham ambient spaces of conformal Patterson-Walker metrics*

Abstract: Situations in which Fefferman-Graham ambient metrics are known to exist as smooth Ricci flat metrics and not just as an infinite order expansion are rare, and situations in which they can be described explicitly are even rarer. While global existence and an explicit formula has been known for conformally Einstein structures for a long time, only recently have other classes of conformal structures been shown to allow an explicit global ambient metric. The topic of the present talk are conformal Patterson-Walker metrics. Patterson-Walker metrics are generically not conformally Einstein, but the ambient metric exists to all orders and can be realized in a natural way. As an application of the concrete ambient metric realization it is shown that Patterson-Walker metrics have vanishing Q-curvature. This talk is based on joint work with Katja Sagerschnig (Polytecnico di Torino) and Josef Šilhan, Arman Taghavi-Chabert and Vojtěch Žádník (Masaryk University Brno).

Paweł Nurowski *Conformal structures with linear Fefferman-Graham equations*

Abstract: I will discuss classes of conformal structures, which have representatives for which the Fefferman-Graham equations for the ambient metric become linear. An attempt of explanation of the appearence of this phenomenon will be given. My talk will be based on a joint work with Ian Anderson, Thomas Leistner and Andre Lischewski.

Martin Olbrich *Invariant currents on limit sets of groups acting on complex hyperbolic spaces*

Abstract: It is often useful to study hyperbolic n-manifolds M via the conformal action of their fundamental groups on the $(n-1)$-dimensional sphere. For instance, for convex-cocompact quotients of the hyperbolic space, invariant k-currents on the sphere that are supported on the limit set of that action give (almost) canonical representatives of de Rham cohomology classes of M of degree $n-k$. After having explained this relation the talk will concentrate on a similar kind of Hodge theory for convex cocompact quotients of complex hyperbolic spaces. Here the action of the fundametal group on the boundary sphere is no longer conformal but CR, and instead of currents we have to consider distributional sections of Rumin's complex.

Bent Ørsted *Riesz distributions for differential forms*

Abstract: The classical Riesz distributions on Euclidian space are complex powers of the Euclidian norm function, and they are important for harmonic analysis, in particular for the study of the Laplace operator. In this lecture, based on joint work with Matthias Fischmann, we shall introduce a similar natural family of distributions, acting on differential forms on Euclidian space. Acting by convolution, they provide explicit versions of Knapp-Stein intertwining operators; and we give their meromorphic contiuations, Fourier transforms, and relations to conformal geometry.

Michael Pevzner *Symmetry breaking operators for reductive pairs.*

Abstract: Understanding of intertwining operators arising in branching laws of infinite-dimensional representations of reductive Lie groups is a subtle problem mixing Combinatorics, Geometry, complex and real Analysis. Sometimes it turn out that such

"symmetry breaking operators" are realized by differential operators on appropriate functional spaces. A fundamental example of this phenomenon is given by the celebrated Rankin-Cohen brackets. We explain the algebraic and geometric nature of these operators and present a general method for their effective construction. We will illustrate these ideas with some concrete examples.

Jan Slovák *Calculus on symplectic and conformal Fedosov manifolds*

Abstract: In the talk, I shall explain the alternatives to the de Rham complex on symplectic manifolds. The motivation comes from the geometric analysis on the complex projective space and we shall build a series of complexes reminding the Berstein-Gelfand-Gelfand resolutions in parabolic geometries. All this will be a report on ongoing joint work with Michael Eastwood.