Monday, June 6

9:30	*Registration*	
10:30	*Coffee break*	
11:00	**Kudla**	Modularity of generating series for special divisors on arithmetic ball quotients I
12:00	**Ullmo**	Flows on abelian varieties and Shimura varieties
13:00	*Lunch break*	
15:00	**Bruinier**	Modularity of generating series for special divisors on arithmetic ball quotients II
16:00	*Coffee break*	
16:30	**Müller-Stach**	Arakelov inequalities for special subvarieties in Mumford–Tate varieties

Tuesday, June 7

9:30	**Jorgenson**	Dedekind sums associated to higher order Eisenstein series
10:30	*Coffee break*	
11:00	**Zhang**	Congruent number problem and BSD conjecture
12:00	**İmamoğlu**	Another look at the Kronecker limit formulas
13:00	*Lunch break*	
15:00	**Michel**	Moments of L-functions and exponential sums
16:00	*Coffee break*	
16:30	**Soulé**	Asymptotic semi-stability of lattices of sections

Wednesday, June 8

9:30	**Freixas**	Generalizations of the arithmetic Riemann–Roch formula
10:30	*Coffee break*	
11:00	**Faltings**	Arakelov theory on degenerating curves
12:00	**Gillet**	The fiber of a cycle class map
13:00	*Lunch break*	
15:00	**Burgos**	The singularities of the invariant metric of the Poincaré bundle
16:00	*Coffee break*	
16:30	**Bost** *Special lecture*	Theta series, euclidean lattices, and Arakelov geometry

Thursday, June 9

9:30	**Edixhoven**	Gauss composition on primitive integral points on spheres, partly following Gunawan
10:30	*Coffee break*	
11:00	**Gubler**	On the pointwise convergence of semipositive model metrics
12:00	**Viazovska**	The sphere packing problem in dimensions 8 and 24
13:00	*Lunch break*	
15:00	**van der Geer**	Modular forms of low genus
16:00	*Coffee break*	
16:30	**Salvati Manni**	On the 2-torsion points of the theta divisor

Abstracts

Jean-Benoît Bost
Theta series, euclidean lattices, and Arakelov geometry
Université Paris-Sud, Orsay, France

A euclidean lattice is the data $(E, \|.\|)$ of some $\mathbb{Z}$-module E isomorphic to $\mathbb{Z}^r$, $r \in \mathbb{N}$, and of some Euclidean norm $\|.\|$ on the associated $\mathbb{R}$-vector space $E_{\mathbb{R}} \simeq \mathbb{R}^r$. To any Euclidean lattice $(E, \|.\|)$, one may attach its theta series

$$\theta(t) := \sum_{v \in E} e^{-\pi t \|v\|^2}$$

and the non-negative real number

$$h^0_\theta(E, \|.\|) := \log \theta(1) = \log \sum_{v \in E} e^{-\pi \|v\|^2}.$$

This talk will explain why this invariant $h^0_\theta(E, \|.\|)$ is a "natural" one, from diverse points of view, including large deviations and the thermodynamic formalism, and the classical analogy between number fields and function fields and its modern developments in Arakelov geometry.
I will also discuss some extensions of the invariant h^0_θ to certain infinite dimensional generalizations of euclidean lattices, and will present some applications to transcendence theory and to algebraization theorems in Diophantine geometry.

Jan Hendrik Bruinier
Modularity of generating series for special divisors on arithmetic ball quotients II
Technische Universität Darmstadt, Germany

The second lecture (for the first lecture, see Kudla) will sketch the proof of the main theorem, which uses the Borcherds modularity criterion. The essential point is to determine the divisor of the unitary group Borcherds forms on the integral model. This depends on an analysis of the first Fourier–Jacobi coefficient of such a form. Time permitting, the second part of the lecture will give a brief description of how the main theorem can be used to establish new cases of the Colmez conjecture.

José Ignacio Burgos Gil
The singularities of the invariant metric of the Poincaré bundle
Instituto de Ciencias Matemáticas, Madrid, Spain

With J. Kramer and U. Kühn we have studied the singularities of the line bundle of Jacobi forms on the universal elliptic curve. The natural generalization of this work is to study the singularities of the invariant metric on the Poincaré bundle on a family of abelian varieties and their duals.
The singularities of the invariant metric are an avatar of the height jump phenomenon discovered by R. Hain. In a joint work with D. Holmes and R. de Jong, we prove a particular case of a conjecture by Hain on the positivity of the height jump.
Moreover, in joint work with O. Amini, S. Bloch and J. Fresán, we show that the asymptotic behaviour of the height pairing of zero cycles of degree zero on a family of smooth curves, when one approaches a singular stable curve, is governed by the Symanzik polynomials of the dual graph of the singular curve. This fact is related to the physical intuition that the asymptotic behaviour of string theory when the length parameter goes to zero is governed by quantum field theory of particles.
In this talk I will review the joint work with Jürg as well as the generalizations.

Bas Edixhoven
Gauss composition on primitive integral points on spheres, partly following Gunawan
Universiteit Leiden, Netherlands

Gauss has given formulas for the number of primitive integral points on the 2-sphere of radius squared n. These formulas are in terms of class numbers of imaginary quadratic orders of discriminants closely related to n. It makes one wonder if this is explained by a free and transitive action of the Picard group of this order on the set of such primitive integral points up to global symmetry $\mathrm{SO}_3(\mathbb{Z})$. This is indeed the case, and the action can be made explicit. The tool that is used is group schemes over $\mathbb{Z}$, which is more direct than Galois cohomology plus adèles, and surprisingly elementary. In fact, Bhargava and Gross ask for such an approach in their article "Arithmetic invariant theory".

Reference: https://openaccess.leidenuniv.nl/handle/1887/38431

Gerd Faltings
Arakelov theory on degenerating curves
Max-Planck-Institut für Mathematik, Bonn, Germany

For a semistable family of curves we investigate the asymptotics of the Arakelov metric. As an application we get results on the asymptotic of the delta-function.

Gerard Freixas i Montplet
Generalizations of the arithmetic Riemann–Roch formula
Centre National de la Recherche Scientifique, Paris, France

In this talk I review recent extensions of the arithmetic Riemann–Roch theorem, related to the program of Burgos–Kramer–Kühn about extending arithmetic intersection theory, in order to deal with toroidal compactifications of Shimura varieties. Precisely, I will first state a Riemann–Roch isometry for the trivial sheaf on a cusp compactification of a hyperbolic orbicurve. As a consequence I will give the special value at one of the derivative of the Selberg zeta function of $\mathrm{PSL}_2(\mathbb{Z})$, that turns out to "contain" the Faltings heights of the CM elliptic curves corresponding to the elliptic fixed points. This first part is joint work with Anna von Pippich. Then I will report on joint work with Dennis Eriksson and Siddarth Sankaran, whose aim is to explore what can be done in higher dimensions. We focus on Hilbert modular surfaces. We give a possible definition of the holomorphic analytic torsion of the trivial sheaf, and by using the Jacquet–Langlands correspondence we prove it fits in a weak form of a conjectural arithmetic Riemann–Roch formula. This formula should contain a boundary contribution given by derivatives at 0 of Shimizu L-functions, in analogy with Hirzebruch's computations in the geometric case.

Gerard van der Geer
Modular forms of low genus
Universiteit van Amsterdam, Netherlands

We give a survey of recent work on Siegel modular forms for genus 2 and 3. This is based on joint work with Bergstroem and Faber and with Clery.

Henri Gillet
The fiber of a cycle class map
University of Illinois at Chicago, USA

I shall discuss how one might use Milnor K-theory and Nash functions to attempt to understand the fiber of (a version of) the cycle class map.

Walter Gubler
On the pointwise convergence of semipositive model metrics
Universität Regensburg, Germany

At a non-archimedean place, we will show that pointwise convergence of semipositive model metrics to a model metric yields that the limit metric is semipositive. Using multiplier ideals, this was shown by Boucksom, Favre and Jonsson under the assumption that the residue characteristic is zero. We will prove the claim in general using a different strategy. This is joint work with Florent Martin.

Özlem İmamoḡlu
Another look at the Kronecker limit formulas
Eidgenössische Technische Hochschule Zürich, Switzerland

Classical Kronecker limit formulas lead to some beautiful relations between quadratic fields and L-functions. In this talk I will use them to motivate the study of a new geometric invariant associated to an ideal class of a real quadratic field. This is joint work with W. Duke and A. Toth.

Jay Jorgenson
Dedekind sums associated to higher order Eisenstein series
The City College of New York, USA

Higher order Eisenstein series are, roughly speaking, defined as a series similar to non-holomorphic Eisenstein series with the inclusion of factors involving periods of certain holomorphic forms. In this talk we describe results yielding generalizations of Kronecker's limit formula as well as Dedekind sums associated to the Kronecker limit function. Specific evaluations are obtained for certain arithmetic groups. The work is joint with Cormac O'Sullivan and Lejla Smajlovic.

Stephen Kudla
Modularity of generating series for special divisors on arithmetic ball quotients I
University of Toronto, Canada

In this pair of talks we will report on joint work with B. Howard, M. Rapoport and T. Yang on the modularity of the generating series for special divisors on integral models of certain ball quotients.

The first lecture (for the second lecture, see Bruinier) will introduce the basic objects: regular integral models of Shimura varieties associated to $\mathrm{U}(n-1,1)$ and their compactifications, special divisors and their extensions to the compactifications. Then the construction of Green functions for these divisors by means of Borcherds lifts of harmonic weak Maass forms will be described. Finally, the generating series will be defined and the main theorem concerning its modularity will be stated.

Philippe Michel
Moments of L-functions and exponential sums
École Polytechnique Fédérale de Lausanne, Switzerland

In this talk, we explain a solution to the long-standing problem of evaluating asymptotically with a power saving error term, the second moment of the central value of L-functions of a modular form twisted by Dirichlet characters of prime modulus. For Eisenstein series, the problem amounts to evaluating the fourth moment of central values of Dirichlet L-functions and was solved by Young a few years ago. We will discuss the more recent and difficult case of cusp forms whose proof is a combination of various ingredients including the analytic theory of automorphic forms, analytic number theory, and l-adic cohomology. These are joint works with Blomer, Fouvry, Kowalski, Milicevic, and Sawin.

Stefan Müller-Stach
Arakelov inequalities for special subvarieties in Mumford–Tate varieties
Johannes Gutenberg-Universität Mainz, Germany

We characterize positive-dimensional special subvarieties in Mumford–Tate varieties by certain new Arakelov inequalities. As a consequence, we get an effective version of the André–Oort conjecture in this more general case.

Riccardo Salvati Manni
On the 2-torsion points of the theta divisor
La Sapienza – Università di Roma, Italy

I will present some recent results, obtained in collaboration with Pirola and Auffarth, on the bound for the number of 2-torsion points that lie on a given theta divisor. I will present also several approaches to attack a conjectural result.

Christophe Soulé

Asymptotic semi-stability of lattices of sections

Institut des Hautes Études Scientifiques Paris, France

This is joint work with T. Chinburg and Q. Guignard. Let L be a hermitian line bundle on an arithmetic surface. We consider the lattice of global sections of a high power n of L, equipped with the sup-norm. We ask whether this lattice becomes semi-stable when n goes to infinity. We get a positive answer when working with capacities and, in the Arakelov set-up, we obtain an interesting inequality, too weak to imply semi-stability.

Emmanuel Ullmo

Flows on abelian varieties and Shimura varieties

Institut des Hautes Études Scientifiques Paris, France

I will discuss several questions and some results about algebraic flows, o-minimal flows and holomorphic flows on abelian varieties and Shimura varieties.

Maryna Viazovska

The sphere packing problem in dimensions 8 and 24

Humboldt-Universität zu Berlin, Germany

In this talk we will show that the sphere packing problem in dimensions 8 and 24 can be solved by a linear programing method. In 2003 N. Elkies and H. Cohn proved that the existence of a real function satisfying certain constraints leads to an upper bound for the sphere packing constant. Using this method they obtained almost sharp estimates in dimensions 8 and 24. We will show that functions providing exact bounds can be constructed explicitly as certain integral transforms of modular forms. Therefore, we solve the sphere packing problem in dimensions 8 and 24.

Shou-Wu Zhang

Congruent number problem and BSD conjecture

Princeton University, USA

A thousand years old problem is to determine when a square free integer n is a congruent number, i.e., the areas of right angled triangles with sides of rational lengths. This problem has some beautiful connection with the BSD conjecture for elliptic curves $E_n : ny^2 = x^3 - x$. In fact by BSD, all $n = 5, 6, 7 \bmod 8$ should be congruent numbers, and most of $n = 1, 2, 3 \bmod 8$ should not be congruent numbers. Recently, Alex Smith has proved that at least 41.9% of $n = 1, 2, 3 \bmod 8$ satisfy (refined) BSD in rank 0, and at least 55.9% of $n = 5, 6, 7 \bmod 8$ satisfy (weak) BSD in rank 1. This implies in particular that at least 41.9% of $n = 1, 2, 3 \bmod 8$ are not congruent numbers, and 55.9% of $n = 5, 6, 7 \bmod 8$ are congruent numbers. I will explain the ingredients used in Smith's proof: including the classical work of Heath-Brown and Monsky on the distribution of the $\mathbb{F}_2$-rank of the Selmer group of E_n, the complex formula for the central value and the derivative of L-functions of Waldspurger and Gross–Zagier and their extension by Yuan–Zhang–Zhang, and their mod 2 version by Tian–Yuan–Zhang.