

Adapted meshes near corners of the domain preserve superconvergence properties in an elliptic optimal control problem

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The paper is concerned with the finite element solution of an optimal control problem with an elliptic state equation and pointwise bounds of the control variable. The underlying domain is polygonal, and our interest is in nonconvex domains.

We follow the approach of Meyer and Rösch [1] who discretize the control by piecewise constant functions, and the state and the adjoint state by piecewise linears. The approximate control is superclose to the interpolate of the exact optimal control, namely of second order in the L2-norm. Only this allows to prove second order convergence of the approximate state and dual state. Moreover, the supercloseness result is the basis for a postprocessing technique, where a postprocessed control converges with second order to the exact optimal control.

In the paper mentioned, the theory was developed for convex domains, where quasi-uniform finite element meshes are sufficient. In nonconvex domains the approximation deteriorates due to corner singularities. Therefore we introduce an appropriate a priori mesh grading which allows us to prove the estimate from above also in the non-convex case. Numerical examples are presented.

[1] C. Meyer, A. Rösch: Superconvergence properties of optimal control problems. SIAM J. Control and Optimization, 43(2004), 970-985.