Supercovergence and a Posteriori Error Estimates of Recovery Type for FEM of Optimal Control Problems Governed by Stokes Equations

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Consider the following optimal control problem:

$$\begin{cases} \min\limits_{\vec{u}\in K}\{\dfrac{1}{2}\|\vec{y}-\vec{y}_0\|^2_{0,\Omega}+\dfrac{1}{2}\|\vec{u}\|^2_{0,\Omega_U}\} \\ -\Delta\vec{y}+\nabla p=\vec{f}+B\vec{u}, \qquad \mathrm{div}\vec{y}=0, \qquad \text{in } \Omega \\ \vec{y}|_{\partial\Omega}=0, \quad \int_\Omega p=0, \end{cases}$$

where $\Omega, \Omega_U \subset \mathcal{R}^2$ are bounded domains, $\vec{f}$, $\vec{y}_0 \in (L^2(\Omega))^2$, B is a continuous linear operator from $U=(L^2(\Omega_U))^2$ to $H=(L^2(\Omega))^2$, $K=\{\vec{v}\in U:\ v_i\geq 0, i=1,2\}$.

In the talk, we present two types of the superconvergence analysis for the finite element approximation for above distributed convex optimal control problems governed by Stokes equations. It can be provided that if the solution is smooth (there is no requirement for the uniform meshes), the superconvergent error order can be proved. Based on the superconvergence analysis, the recovery type a posteriori error estimator can be constructed. It is proved that in many cases, the a posteriori error estimator is asymptotically exact approximation to the exact error. Such estimates, which are apparently not available in the literature, can be used to construct adaptive finite element approximation schemes and as an error bound in reliability analysis for the optimal control problems governed by Stokes equations.