Superconvergence in finite element methods for singularly perturbed problems

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In recent years it has become quite popular to use layer adapted meshes to handle boundary and interior layers; for a recent introduction into that subject see [1], for a survey [2].

Today the case of exponential layers is particularly well understood, but for the case of parabolic layers, which are in practice more important, the situation is much less clear. Consider the model problem

$$-\epsilon\triangle u + b_1 u_x + b_2 u_y + cu = f \quad \text{in}\, \Omega = (0,1)^2$$

with, for simplicity, homogeneous Dirichlet boundary conditions.

Let us discretize the problem with the streamline-diffusion finite element method or with discontinuous Galerkin and linear or bilinear elements on layer adapted meshes and ask for optimal error estimates say in the ϵ-weighted H^1-norm, the L_2-norm or the special norms related to the kind of stabilization.

Actually the following ingredients are used to prove error estimates:

- Shishkin type solution decompositions
- interpolation error estimates on layer adapted meshes
- supercloseness results

In the lecture we shall discuss the importance of supercloseness to obtain optimal error estimates uniformly with respect to the perturbation parameter.

REFERENCES

[1] M. Stynes “Steady state convection-diffusion problems”, *Acta Numerica* , (2005), 445-508.

[2] H.-G. Roos, M. Stynes and L. Tobiska (in preparation), “Numerical Methods for Singularly Perturbed Differential Equations”, second edn., Springer Series in Computational Mathematics, 2006.