Superconvergence of a 3D FEM for the incompressible Navier-Stokes Equation

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For the Poisson equation on uniform meshes it is well-known that the piecewise linear conforming finite element solution approximates the interpolant to a higher order than the solution itself. In this paper, this type of superclose property is established for a special interpolant of the $Q_2 - P_1^{disc}$ element applied to the stationary Stokes and Navier--Stokes problem, respectively. Moreover, applying a $Q_3 - P_2^{disc}$ post-processing technique, we can also state a superconvergence property for the discretisation error of the post-processed discrete solution to the solution itself. Finally, we show that inhomogeneous boundary values can be approximated by the Lagrange Q_2-interpolation without influencing the superconvergence property. Numerical experiments verify the predicted convergence rates. Moreover, a cost-benefit analysis between the two third-order methods, the post-processed $Q_2 - P_1^{disc}$ discretisation and the $Q_3 - P_2^{disc}$ discretisation, is carried out.