Boundary Element Methods and Efficient Applications to Problems in Electrostatics and Solid Mechanics

Wolfgang L. Wendland

Symmetric Galerkin boundary element methods are used for solving mixed boundary value problems of the Laplace equation and of the Lam'e system. When iterative procedures are employed for solving the enormous linear systems fast multipole approximations are used. While the single layer potential operator of the Laplacian can be treated straight forwardly as in particle simulation, the double layer potential and its adjoint in the farfield are approximated by applying normal derivatives to the multipole series of the single layer potential. The Galerkin discretization of the hypersingular kernel is reduced via Houde Han's integration by parts. For the elastostatic problems, all boundary integral operators are reduced to those of the Laplacian. Our technique results in a fast multipole performance of the matrix times vector multiplication. Together with appropriate preconditioning techniques we obtain a very efficient tool for the simulation of electrostatic and elastic stress fields with applications in engineering and industry. Finally, we show latest modifications that lead to corresponding efficient domain decomposition methods based on symmetric Galerkin boundary element methods.