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Stability and Convergence of Finite Element Approximation Schemes for Harmonic Maps

Harmonic maps are stationary points of the Dirichlet energy among vector fields with values in the unit sphere. Owing to this nonconvex constraint, harmonic maps are non-unique and fail to admit higher regularity properties. Moreover, the constraint prohibits the use of standard tools for the numerical approximation. In this talk we discuss stability and weak convergence of three numerical schemes. The first scheme consists in the minimization of the Dirichlet energy over suitable tangent spaces and a renormalization of the update in each iteration. The second approach penalizes the constraint and leads to a time-dependent Ginzburg-Landau equation. A projection method for the discretization of the harmonic map heat flow is the basis for the third approach. Besides stating sufficient conditions for stability and weak convergence to an exact solution, we indicate generalizations to the approximation of p -harmonic maps. Applications include liquid crystal theory, image processing, and micromagnetics.