Review on Error Reduction in Adaptive Finite Element Methods

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Typical adaptive mesh-refining algorithms for first-oder (conforming) finite element methods consist of a sequence of the following steps:

$$\text{SOLVE} \Rightarrow \text{ESTIMATE} \Rightarrow \text{MARK} \Rightarrow \text{COARSEN/REFINE}$$

Unlike uniform mesh-refinements, the goal of adaptive finite element methods (AFEM) is to omit some basis functions in oder to save degrees of freedom and so reduce computational costs. Thus, the sequence of generated subspaces in an AFEM is on purpose *not* necessarily dense and hence the question of strong convergence has a priori *no* trivial affirmative answer.

This review summarises the arguments from the literature on the error reduction such as reliability, bulk criterion, discrete local efficiency, Galerkin orthogonality. In the proof of the discrete local efficiency is shown for the first time how to remove the inner-node refinement. This indicates that the well-established counter example in non-generic. An overview over Rob Stevenson's arguments for control of number of degrees of freedom is included as well.

Another new result is the equivalence of a discrete residual control with error reduction and with the reliability of hierachical error estimators.

The second aim of this presentation is to discuss conditions sufficient for convergence for a class of adaptive finite element methods applied to convex minimization problems. Newer applications include relaxed models in optimal design tasks, 2-well problems allowing for microstructures, or Hencky elastoplasticity.

References

[C] C. Carstensen: *Convergence of adaptive finite element methods for convex minimisation problems.* In preparation (2004).

[MNS] P. Morin, R.H. Nochetto, and K.G. Siebert: Convergence of adaptive finite element methods. *SIAM Review* **44** (2003) 631–658.

[1] A. Veeser (2002). *Convergent adaptive finite elements for the nonlinear Laplacian.* Numer. Math., **92**, 4, 743–770.