SUPERCONVERGENCE OF TIME-SPACE DISCONTINUOUS FINITE ELEMENTS FOR FIRST ORDER HYPERBOLIC PROBLEM

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Consider the first order hyperbolic problem:

$$u_t + au_x + bu = f, \quad a > 0, \quad in \;\; \Omega = \{0 < x < 1, 0 < t < T\}$$

with the initial value $u(0,x) = u_0(x)$ and the boundary value $u(t,0) = g(t), g(0) = u_0(0)$. The domain is subdivided into many small rectangular elements $\tau_{ij} = I_i \times J_j, i = 1,...,N, j = 1,...,M$, where the subintervals $I_i = (x_{i-1}, x_i), J_j = (t_{j-1}, t_j)$ and the step-lengths $h_i = x_i - x_{i-1}, k_j = t_j - t_{j-1}$. Assume that $h = \max h_i, k = \max k_j$ and the subdivision is quasiuniform. Denote by $S^{k,h}_{m,n}$ the discontinuous finite element space which is m-degree in time and n-degree in space polynomials in each element. Denote the limit values by $U^{\pm}_i(t) = U(t, x_i \pm 0)$ and $U^{\pm}_j = U(t_j \pm 0, x)$, and the jumps by $[U_j] = U(t_j + 0, x) - U(t_j - 0, x), < U_i >= U(t, x_i + 0) - U(t, x_i - 0)$. Define the time-space discontinuous finite element solution $U(t,x) \in S^{k,h}_{i,j}$ satisfying the following orthogonal relation

$$\int_{\tau_{ij}} (U_t + aU_x + bU - f)v dxdt + \int_{I_i} [U_{j-1}(x)] v^{+}_{j-1}(x) dx$$

$$+ \int_{J_j} < aU_{i-1} > v^{+}_{i-1}(t) dt = 0, \quad v \in S^{k,h}_{n,m}.$$

We shall show that under certain regularities the discontinuous finite element solution has superconvergence at $(n+1) \times (m+1)$ order Radau's points (t_α, x_β) in each element τ_{ij},

$$\{\sum_{\tau_{ij}} \sum_{(t_\alpha, x_\beta) \in \tau_{ij}} h_i k_j |(u - U)(t_\alpha, x_\beta)|^2\} = O(h^{n+2}) + O(k^{m+2}), \quad n > 0, m > 0.$$

The conclusion is also verified by our numerical experiments.