Adaptive Methods for Global Operators

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ABSTRACT

Adaptive discretizations form a key methodology for treating large scale numerical simulation problems. This talk addresses recent developments concerning adaptive wavelet methods for boundary integral equations of potential type. Aside from a possible ill-conditioning when dealing with operators of nonzero order and the fact that one has to deal with densely populated matrices, the global nature of such operators poses further obstructions to the understanding of adaptive techniques. Our approach is based on a recent new paradigm for the adaptive solution of variational problems which covers so far indefinite as well as nonlinear problems involving differential operators [1,2] and has been extended to optimal control problems with PDE constraints. The key idea is to transform the original variational problem into an equivalent one in terms of wavelet coordinates. Combining the well-posedness of the variational problem with the norm equivalences induced by a suitable wavelet basis for the energy space, the transformed problem can be shown to be well conditioned. Hence one can devise an (ideal iteration) for the full infinite dimensional problem formulated in the space ℓ_2 of square summable sequences that exhibits a guaranteed error reduction per step. The idea is then to mimic this ideal iteration numerically by an adaptive application of the involved operator represented in wavelet coordinates with suitable dynamically updated error tolerances. The application scheme is adaptive in that it exploits a priori knowledge about the quasi sparsity of the operator in wavelet coordinates together with the a-posteriori information about the significant coefficients of the current input vector. It is indicated in this talk that for boundary integral operators of potential type this adaptive application can be arranged so as to realize an overall asymptotically optimal computational complexity of the whole solution process [4]. This means that whenever the (unknown) solution can be approximated by N terms from the underlying wavelet basis within an accuracy rate of N^{-r} then the scheme is shown to recover the solution within any tolerance $\varepsilon > 0$ in the energy norm at a computational expense that grows at most at the (optimal) rate $\varepsilon^{-1/r}$ for a range r that is limited only by the order of the basis. The main ingredients of the analysis are concepts from nonlinear approximation, matrix compression [3] and adaptive hp-quadrature. The theoretical results are illustrated by numerical examples.