Upscaling of a class of nonlinear parabolic equations for the flow transport in heterogeneous porous media

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Abstract

We develop an upscaling method for the nonlinear parabolic equation

$$\partial_t b(u_\varepsilon) - \nabla \cdot (\mathbf{g}^\varepsilon(x, u_\varepsilon) + \mathbf{a}^\varepsilon(x, u_\varepsilon)\nabla u_\varepsilon) = f(x, t),$$

which stems from the applications of the flow transport in porous media. Our direct motivation is the Richards equation which models the flow transport in unsaturated porous media. We provide a detailed convergence analysis of the method under the assumption that the oscillating coefficients are periodic. While such a simplifying assumption is *not* required by our method, it allows us to use homogenization theory to obtain the asymptotic structure of the solutions. Numerical experiments are carried out for the Richards equation of exponential model with periodic and randomly generated log-normal permeability to demonstrate the efficiency and accuracy of the proposed method.