The Structure and Stability of Stationary Solutions of Doi-Onsager Equation

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We study the structure and stability of stationary solutions to the Doi-Onsager equation with Maier-Saupe potential on the sphere, which arises in the modelling of rigid rod-like molecules of polymers. The stationary solutions are shown to be necessarily a set of axially symmetric functions, and a complete classification of parameters for phase transitions to these stationary solutions is obtained. It is shown that the number of stationary solutions hinges on whether the potential intensity crosses two critical values $\alpha_1 \approx 6.731393$ and $\alpha_2 = 7.5$. Furthermore, we present explicit formulas and linearized stabilities for all stationary solutions.