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Book of Abstracts

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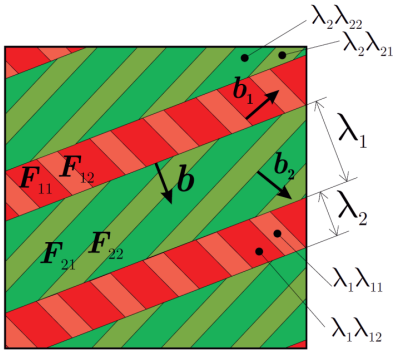

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Contents

Abstracts

Aspect-ratio dependence of energy lower bounds for a shear experiment in single-crystal plasticity

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(joint work with Patrick Dondl)

Consideration is given to a non-convex variational model for a shear experiment in the framework of single-crystal linearised plasticity with cross-hardening. The rectangular shear sample is clamped at each end, and is subjected to a prescribed horizontal shear, modelled by a hard Dirichlet condition. We ask: how much energy is required to impose such a shear, and how does it depend on the aspect ratio? Assuming that just two slip systems are active, we show that there is a critical aspect ratio, above which the energy is strictly positive, and below which it is zero.

A phase-transformation-plasticity model for polycrystalline materials based on representative crystal orientations

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The initiation and evolution of microstructure highly affects the macroscopic behaviour of solid materials. Such effects can be brought about deliberately — as in the case of TRIP steels or SMAs — or are rather undesired since they may yield a deterioration of the material's properties and functionality. On the one hand, it is thus necessary to develop micromechanically motivated material models (for single crystals) which allow for a physical explanation

of the respective material response. On the other hand, the investigation of, e.g., the effect of machining processes on the microstructure, and hence the material properties of the underlying workpiece, requests fast algorithms in the context of large scale finite element simulations.

In this contribution, we present a large strain material model for elasto-plastic polycrystalline austenitic steels which may undergo martensitic phase transformations during mechanical and thermal loading. The model is designed to combine the above mentioned requirements, i.e. to be suitable for large scale FE simulations while maintaining micromechanical aspects of martensitic phase transitions. To be more precise, we first consider a two-phase mixture of austenite and (overall) martensite in terms of a convexified energy density. The martensite "phase" is considered as an averaged quantity derived from several representative crystal orientations. These representative orientations are characterised by several chosen "directors" as well as rotations around these directors. With this, the effective quantities of martensite, i.e. the coordinates of the elasticity tensor and the transformation strain tensor, are determined via numerical integration where additional weighting factors for each orientation are used. In contrast to the constant and a priori chosen directors and rotations, these weighting factors are precalculated to that the analytical solution for the polycrystal elasticity tensor according to the Reuss assumption is captured in the case of equally distributed martensitic volume fractions. The numerical examples shall underline the capabilities of the proposed model, where emphasis is laid, among other aspects, on the transformation induced anisotropy evolution as well as on the phase-transformation-plasticity-interaction.

Relaxation via Young measures reflecting the $\det \neq 0$ **constrain**

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We investigate a new tool to study minimization problems for integral functionals defined over matrix-valued mappings that take values *only* in the set of invertible matrices. Typical examples are found, e.g., in non-linear elasti-

city where static equilibria are minimizers of the elastic energy, i.e., one is led to solve

$$\text{minimize } J(y) := \int_\Omega W(\nabla y(x))\,\mathrm{d}x\ , \qquad (*)$$

where $\Omega \subset \mathbb{R}^n$ denotes the reference configuration of the material. If $(*)$ does not have a solution, for example because of *microstructure formation*, relaxing the problem via Young measures proved a very useful tool in modelling. However, usually, the constraint that the gradients of the minimizing sequences should have a positive or at least a non-negative determinant is often neglected.

Motivated by the idea that the conditions on the non-negativity of the deformation gradient is posed by the invertibility in elasticity, we concentrate on bounded sequences in $W^{1,\infty}(\Omega;\mathbb{R}^n)$ where the inverted gradients are also bounded in $L^\infty(\Omega;\mathbb{R}^{n\times n})$. In such a case, if additionally $y \in C^1(\Omega;\mathbb{R}^n)$, y is bilipschitz, i.e., y as well as y^{-1} are both Lipschitz maps defining homeomorphisms between Ω and $y(\Omega)$. This, in the sense, means that the requirement on the bounded inverse gives a better quality to the inverse mapping.

We then completely and exlicitely describe Young measures generated by the sequences described above (i.e. bounded sequences in $W^{1,\infty}(\Omega;\mathbb{R}^n)$ where the inverted gradients are also bounded in $L^\infty(\Omega;\mathbb{R}^{n\times n})$). This extends the original results due to D. Kinderlehrer and P. Pedregal [1].

The main results of the talk can be found in [2].

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Statistically similar RVEs for the FE^2-simulation of 3D microstructures

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Nowadays, one of the most important ambitions of structural engineers is the optimization of stability of structures and likewise the reduction of weight. Especially for automotive applications and lightweight constructions the usage of advanced high strength steels achieves these aspects due to their multi-phase microstructure. The interaction of the constituents of the microstructure governs the properties of the steel, but also leads to a complex macroscopic mechanical behavior. For a reliable numerical simulation the microscopic behavior must be taken into account and this can be done by the application of a direct micro-macro transition procedure such as the FE^2 method, see e. g. [1]. A main problem of direct homogenization methods applied to large random microstructures is the high computational cost with respect to both, the amount of memory and the computation time. The efficiency of the procedure is increased by constructing statistically similar RVEs (SSRVEs), cf. [2], [3], which are much less complex than usual RVEs but which represent the mechanical response of the microstructure rather precisely. The SSRVEs are obtained from a minimization process of a least-square functional considering statistical measures computed for the real microstructure and the SSSRVE.
Here we analyze the construction of three-dimensional SSRVEs considering the volume fraction, the spectral density and the lineal-path function as statistical measures for the morphological characterization. The reconstruction of a real dual-phase steel microstructure serves as a target structure, which is obtained from EBSD-FIB measurements, cf. [4].

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Homogenization of some singular nonlinear elliptic problems

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The results presented in this talk are contained in [2], in collaboration with D. Giachetti of the University of Rome La Sapienza.

We consider the following nonlinear elliptic Dirichlet problem:

$$\begin{cases} -\text{div } (A^\varepsilon \nabla u_\varepsilon) + \lambda u_\varepsilon = \dfrac{b_\varepsilon(x, \nabla u_\varepsilon)}{(u_\varepsilon)^k} + f(x) & \text{in } \Omega, \\ u_\varepsilon = 0 & \text{on } \partial\Omega, \end{cases} \qquad (*)$$

where Ω is an open bounded set of $\mathbb{R}^N$, $N \geq 2$, f is a nonnegative function in $L^m(\Omega)$ with $m > \frac{N}{2}$, $0 < k < 1$, $\lambda \geq 0$ and $\varepsilon > 0$ tends to zero.

The functions $b_\varepsilon(x, \xi)$ are nonnegative Carathéodory functions on $\Omega \times \mathbb{R}^N$, quadratically growing in the ξ variable.

We are interested in the asymptotic behaviour, as $\varepsilon \to 0$, of (bounded) solutions u_ε of this problem, when $A_\varepsilon(x)$ is a matrix field in $M(\alpha, \beta, \Omega)$, which H-converges to a matrix field $A_0(x)$ (in the sense of Murat-Tartar, see [5]). The existence of bounded solutions, also for growth $k \geq 1$, has originally been proved in [3] for a bounded data f and a strictly positive λ, and extended in [4] to $f \in L^m(\Omega)$, $m > \frac{N}{2}$, and $\lambda \geq 0$.

As motivations for this kind of problems, let us mention a connection with boundary blow-up solutions, the Euler equations of some simple functional

like $\int_\Omega(\sqrt{u}|Du|^2 - fu)\,dx$. Also, some stationary models of growth in porous media by a change of unknown give an equation with a singular behaviour of the solution.

Here, we show that, up to a subsequence, any sequence $\{u_\varepsilon\}$ of solutions of $(*)$ weakly converges in $H_0^1(\Omega)$ to a function u_0, solution to

$$\begin{cases} -\text{div}\ (A_0\nabla u_0) + \lambda u_0 = \dfrac{b_0(x,\nabla u_0)}{(u_0)^k} + f(x) & \text{in } \Omega, \\ u_0 = 0 & \text{on } \partial\Omega, \end{cases}$$

where $b_0(x,\xi)$ is a Carathéodory function with quadratic growth in the ξ variable, constructed from the corresponding terms $b_\varepsilon(x,\xi)$ by using the corrector C_ε associated to the H-converging sequence $\{A^\varepsilon\}$.

As it was done in [1] for the nonsingular case, we prove that the corrector for our nonlinear singular problem is again that associated to the linear one.

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Micromechanical modeling of bainitic phase transformation for multi-variant polycrystalline low alloy steels

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Metal forming processes are important technologies for the production of engineering structures. In order to optimize the resulting material properties, it becomes necessary to simulate the entire forming process by taking into account physical effects such as phase transformations.

In our work we develop a micromechanical material model for the phase transformation from austenite to bainite in a polycrystalline low alloy steel. In this material (e.g. 51CrV4) the phase changes from austenite to perlite-ferrite, bainite or martensite, respectively. The presentation is concerned with phase transformation between austenite and n-bainite variants in N differently orientated grains. In contrast to the pure displacive martensitic phase transformation, the characteristic features of bainite formation are the combination of time-dependent transformation kinetics and lattice shearing in the microstructure. These effects are considered on the microscale and transferred to the polycrystalline macroscale by means of homogenisation of stochastically orientated grains. The effect of Transformation Induced Plasticity (TRIP) consisting of a visco-plastic contribution (Greenwood-Johnson-effect) and an irreversible length-change due to the lattice-shearing during the transformation (Magee- or Orientation-effect) is presented.

The numerical implementation of our model is done in form of a two-step staggered algorithm. In the first step a projected Newton algorithm combined with an active-set-strategy, based on the algorithm in [2], is used for the calculation of variants. In the second step, the simulation of the inelastic behaviour is performed with help of a Newton algorithm.

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Homogenization in linear elasticity: a contemplative revisiting

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Of course, linear elasticity has been dealt with very early on from a homogenization standpoint. In this talk, I propose to obfuscate this simple problem by pointing to a few results (mostly by others) that seem to increase complexity.

This is because it might prove fruitful to view homogenization in linear elasticity as a particular case of non-convex homogenization.

Derivation of an effective damage model with non-periodic evolving micro-structure

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In this talk rate-independent damage models for elastic materials are considered. The aim is the derivation of an effective damage model by investigating the limit process of damage models with evolving micro-defects. In all presented models the damage is modeled via a unidirectional change of the material tensor. With progressing time this tensor is only allowed to decrease in the sense of quadratic forms. The magnitude of the damage is given by comparing the actual material tensor with two reference configurations, denoting completely undamaged material and maximally damaged material (no complete damage).

The starting point is a microscopic model, where the underlying micro-defects, describing the distribution of either undamaged material or maximally damaged material (but nothing in between), are of a given shape but of different time-dependent sizes. Scaling the micro-structure of this microscopic model by a parameter $\varepsilon > 0$ the limit passage $\varepsilon \to 0$ is preformed via two-scale convergence techniques. Therefore, a regularization approach for piecewise constant functions is introduced to guarantee enough regularity for identifying the limit model. In the limit model the material tensor depends on a damage variable $z : [0, T] \to \mathrm{W}^{1,p}(\Omega)$ taking values between 0 and 1 such that, in contrast to the microscopic model, some kind of intermediate damage for a material point $x \in \Omega$ is possible. Moreover, this damage variable is connected to the material tensor via an explicit formula, namely, a unit cell formula known from classical homogenization results.

On the structure of the quasiconvex hull in planar elasticity

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Let L_1 and L_2 be compact sets of real 2×2 matrices with positive determinant. Suppose that both sets are frame invariant, meaning invariant under the left action of the special orthogonal group. Then we give an algebraic characterization for L_1 and L_2 to be incompatible for homogeneous gradient Young measures. If we apply this to planar elasticity, we can thus decide whether or not there exists deformation microstructure which is simultaneously supported on L_1 and L_2. The analysis relies on results by Faraco and Székelyhidi [1].

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On one extension of Decomposition Lemma dealing with weakly converging sequences of gradients with application to nonconvex variational problems

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We deal with the variant of Decomposition Lemma due to Kinderlehrer and Pedregal asserting that an arbitrary bounded sequence of gradients of Sobolev mappings $\{\nabla u_k\} \subseteq L^p(\Omega, \mathbf{R}^{m\times n})$, where $p > 1$, can be decomposed into a sum of two sequences of gradients of Sobolev mappings: $\{\nabla z_k\}$ and $\{\nabla w_k\}$, where $\{\nabla z_k\}$ is equintegrable and carries the same oscillations, while $\{\nabla w_k\}$ carries the same concentrations as $\{\nabla u_k\}$. We additionally impose the general trace condition "$u_k = u$" on F, where F is given closed subset of $\bar{\Omega}$. We show that under this assumption the sequence $\{z_k\}$ in decomposition can be chosen to satisfy also the trace condition $z_k = u$ a.e. on F. The result is applied to nonconvex variational problems to regularity results for sequences minimizing functionals. As the main tool we use DiPerna Majda measures.

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Investigation of formation of different lamination configurations within the orientation space

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The formation of laminated microstructures is only favorable under certain conditions in which single slip domains occur based on infinite latent hardening [1, 2]. Experimental investigations give an insight into the mechanisms of the lamination evolution influenced by dislocation interactions. In this study we perform shear experiments on copper single crystals of different orientations to explore the formation of laminated microstructure. In our investigations the shear experiments were conducted along with the digital image correlation which made it possible to determine the exact microscopic state of the deformed samples. The band-like microstructures have been investigated using high-resolution electron back scatter diffraction (EBSD) method. The topography of the dislocation structure within the orientation patterned areas was observed using the electron channeling contrast imaging (ECCI) [3, 4]. The study of the dislocation structure gives us an insight into the mechanisms of dislocation walls generation and their relation to the formation of laminates.

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Micromechanical modeling of size effects in amorphous materials

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The present contribution is concerned with the modeling and computation of size effects in amorphous materials. For the underlying model description, we resort to a thermodynamically consistent approach. A non-local material model is formulated in this framework [1]. The influence of the material lengthscale is investigated and it is shown that with decreasing sample size a delay of the shear localization is obtained in amorphous materials. The model formulation is able to model the stable growth of shear localization in sub-micron samples. In addition, the tension-compression asymmetry observed in experiments is captured by the proposed model. Further, the rate-dependent behavior as well as the influence of the results to initial local defects are investigated. The position of the initial defect(s) does not affect the macroscopic behavior - only the direction of the shear band evolution. It is shown that the inclusion of several defects leads to an imbalance in the atomic structure resulting in an acceleration of the shear band formation.

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Sequential weak continuity of null Lagrangians at the boundary

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We show weak* in measures on $\bar{\Omega}$/weak-L^1 sequential continuity of $u \mapsto$

$f(x, \nabla u) : W^{1,p}(\Omega; \mathbb{R}^m) \to L^1(\Omega)$, where $f(x, \cdot)$ is a null Lagrangian for $x \in \Omega$, it is a null Lagrangian at the boundary for $x \in \partial\Omega$ and $|f(x, A)| \leq C(1 + |A|^p)$. We also give a precise characterization of null Lagrangians at the boundary in arbitrary dimensions. Our results explain, for instance, why $u \mapsto \det \nabla u : W^{1,n}(\Omega; \mathbb{R}^n) \to L^1(\Omega)$ fails to be weakly continuous. Further, we state a new weak lower semicontinuity theorem for integrands depending on null Lagrangians at the boundary. The talk closes with an example indicating that a well-known result on higher integrability of determinant by S. Müller need not necessarily extend to our setting. The notion of quasiconvexity at the boundary due to J.M. Ball and J. Marsden is central to our analysis.

Stress constraints in large strain regime

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In accepting in continuum mechanics of simple bodies a polyconvex structure for the energy, we consider it – everybody knows that – as a convex function of the triplet including deformation gradient, its cofactor, and its determinant. We can take the elements of that triplet as separate entities or, else, we may notice that they determine a 3-vector over the ambient space. In this case we define the energy over the whole space of 3-vectors. Their use permits also the definition of the functional class of weak diffeomorphisms, which appears an appropriate ambient for describing types of transplacements that we imagine intuitively when we use the adjective elastic. To be defined, weak diffeomorphisms need the notion of current. Two questions then arise: (1) Has the notion of current a clear physical interpretation? (2) Does the extension of the energy to the whole space of 3-vectors give us advantages in describing some mechanical phenomena?

With the aim of proposing some answers, I show how the mechanics of simple bodies can be naturally described in terms of forms. From this viewpoint, the first Piola-Kirchhoff stress shows a multiplicative decomposition into a third-rank skew-symmetric tensor ω, a 3-covector indeed, and a fifth-rank one which is skew-symmetric in the first three indices, a geometric projector not related with the energy and depending only on the transplace-

ment. Within this setting I discuss boundary value problems in which ω is constrained to belong to a convex set indicating admissible and critical states. Existence of minimizers of a certain energy can be proven in a certain space the elements of which are compatible with the constraint on ω, excluding at the same time fractures.

The stress is represented naturally by a vector-valued measure on a three-dimensional surface in the six dimensional product of reference and actual spaces. Resorting to that surface appears natural when the transplacement is no more a one-to-one map everywhere, as constraints may imply. The stress measure corresponds to the standard stress in the reference place in the region where the surface just mentioned is the graph of a map. The rest is concentrated on a set having zero volume measure in the reference place. It may describe concentrated actions along dislocations which can be nucleated once the stress reaches the admissibility threshold. Their presence – if it occurs – is the harbinger for possible phase transitions toward the elastic-plastic behavior which has to be described, however, by resorting to an enriched framework which can be also discussed from a covariant point of view (I shall try to sketch even this last aspect).

Variational gradient crystal plasticity for modeling of size effects coupled with laminate microstructure evolution

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With the ongoing trend of miniaturization and nanotechnology, the predictive modeling of size effects and microstructure formation play an increasingly important role in metal plasticity. In this talk, we outline a particular formulation of gradient plasticity which allows the analysis of both effects.

To this end, we first propose a suitable framework for variational gradient plasticity based on ideas presented in [1,2]. The key ingredients are (a) a new viscous regularization that allows to overcome the difficulty of determining the active sets within full multislip scenarios of crystal plasticity, (b) a characteristic separation into long- and short-range variables that is systematically exploited in the algorithmic treatment and (c) a gradient con-

tribution based on the geometric dislocation density tensor. The formulation is shown to be fully variational in nature, governed by rate-type continuous and incremental algorithmic variational principles. Benchmark examples for multislip scenarios in fcc single crystals demonstrate the modeling capabilities and the numerical efficiency of the formulation.

In a second step, we consider a reduction of the full multislip scenario to plane double slip with non-convex latent hardening. For this special case, both the plastic deformation as well as the dislocation density tensor can be explicitly derived from the slips and the slip gradients. Together with a particular non-convex latent hardening function and a regularization term for the arising sharp interfaces between different plastic domains and drawing on ideas presented in [3], we show that the presented model is capable of predicting the formation and evolution of plastic laminate microstructure together with size effects caused by GNDs, a combination that is – to our knowledge – lacking in the literature.

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Modeling of quasibrittle damage with an FFT-based algorithm

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We formulate a model for quasi-brittle damage combined with linear elastic behavior, where we use the effective stress concept, e.g. a damage factor

$f(d)$ is introduced that locally reduces elastic stiffness. In order to prevent mesh dependent results typical to continuous strain softening models, an additional field variable φ for regularization is taken into the energy formulation, based on the ideas proposed in [2]. A term including the gradient of φ is added, and variational analysis yields an algebraic and a Helmholtz like equation for φ and $f(d)$, respectively. A numerical algorithm based on fast Fourier transforms was developed, that is time efficient, has a high potential for parallelization, and provides reasonable, mesh-independent results. The algorithm is based on the work of Moulinec and Suquet [1].

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Evolutional contact with Tresca friction on a periodic microstructure in the framework of the energetic formulation

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It can be easily seen, that if we denote the tangential jump in displacements on the contact interface as an internal variable in the incremental variational inequality for the multiscale contact problem, it will be similar to models of the delamination by shear from [1]. Hence, the results of [1] and [2] can be applied. We further look for the spatial homogenization of problems with friction. The limiting energy and the dissipation term in the stability condition (for each fixed time-step) were obtained for the contact with Trescas friction law in [3]. Using these results and the concept of energetic solutions for evolutional quasistatical problems with frictional contact, we show that

irreversible frictional displacements at micro-level lead to the crystal-type evolutional plastic behavior of the homogenized medium at the macro-level.

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Modelling of deformation banding in single crystals undergoing rate-independent multislip

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The phenomenon of spontaneous formation of deformation bands in metal single crystals deformed plastically by crystallographic multislip is investigated theoretically and numerically by using the energy criterion of instability of a uniform deformation path. Previous works on modelling of deformation banding in ductile crystals [1,2,3] were frequently focused on finite increments of single slip in subdomains. A rigorous formulation and theoretical justification of a general incremental criterion of deformation banding in metal single crystals deformed locally by multislip represents still an open problem. In the present work, the second-order energy criterion for incipient deformation banding is derived in a time-continuous setting for a rate-independent elastic-plastic crystal. The need for selective symmetrization of the local interaction matrix for active slip-systems [4] is demonstrated. A computational approach to deformation banding is developed by using non-convex constrained minimization of the incremental work with respect to increments

in crystallographic shears and kinematical degrees of freedom. Calculated examples of deformation banding patterns in fcc single crystals are compared with experimental observations.

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Modelling of phase transformations in magnetostrictive materials like NiMnGa

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Magnetostrictive materials such as Ni_2MnGa exhibit mutually interacting martensitic and ferro/paramagnetic phase transformations. Moreover, such materials are electrically conductive, which leads to other coupling through induced magnetic field and produced Joule heat. After presentation of these phenomena, the thermodynamically-consistent model in terms of small-strain and eddy-current approximations will be formulated. The model uses a general free energy and allows for large velocities which occurs in some experiments on frequency range up to MHz even under small displacements and strains. Existence of weak solutions to such a coupled system of momentum equilibrium, Landau-Lifshitz-Gilbert equation, heat equation, and

the parabolic Maxwell system will be proved by limit passage in a carefully designed semi-implicit regularized scheme. The talk reflects primarily concepts and results from a joint article with Giuseppe Tomassetti [6] which merges and expands a series of previous articles [1-5,7].

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Upscaled phase-field models for interfacial dynamics in strongly heterogeneous domains

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We derive an effective macroscopic Cahn-Hilliard equation in [1] whose homogeneous free energy is represented by 4-th order polynomials, which in-

clude the frequently applied double-well potential. This upscaling is done for perforated/strongly heterogeneous domains. To our best knowledge, this seems to be the first attempt of upscaling the Cahn-Hilliard equation in such domains. The new homogenized equation should have a broad range of applicability due to the well-known versatility of phase-field models. The additionally introduced feature of systematically and reliably accounting for confined geometries by homogenization allows for new modeling and numerical perspectives in both, science and engineering. Our results are applied to wetting dynamics in porous media and to a single channel with strongly heterogeneous walls.

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Material parameters in phase field models for ferroelectrics

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Phase field models have become an important tool for the simulation of ferroelectric domain structures. These models are based on an order parameter (usually the material polarization) which is introduced as an additional independent field variable. As a consequence of thermodynamic considerations, the evolution of the order parameter is governed by a time-dependent Ginzburg-Landau type equation. Traditionally the free energy is expanded at the ferroelectric phase transition from an assumed cubic (high-symmetry) parent phase to a low-symmetry phase exhibiting spontaneous polarization and strain. This results in a free energy satisfying the high-symmetry conditions so that piezoelectricity is not explicitly included in the free energy. We show how this shortcoming can be solved by expanding the free energy at the spontaneously polarized low-symmetry state. Similarly, we include dielectricity directly in the free energy. These extensions of the free energy lead to a slightly different interpretation of the order parameter but solve

the difficult question of how to fit the coefficients of the Landau potential to achieve a desired electromechanical model response. In a next step we show how the key properties interface energy, width, and mobility can be input as material parameters by re-arranging the coefficients of a modified Landau potential and the gradient energy. In the end, all model parameters have a clear physical meaning, thus enhancing the usability of phase field models in the context of ferroelectrics. Finite element simulations are presented to illustrate the main results of the presented theoretical considerations.

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Modelling of incompatible martensitic microstructures

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Martensitic phase transformations in shape memory alloys (SMA) proceed by formation and evolution of microstructures. The crystallographic theory of martensite [1] is the main theoretical tool applied currently to study these

microstructures. This theory is based on energy minimization considerations, but in practical applications it reduces to a purely kinematic analysis of compatibility conditions formulated for stress-free phases. Thus, it rules out the microstructures that are not compatible, even if the respective compatibility conditions are only marginally violated, so that relatively small elastic strains would be sufficient to achieve compatibility.

At the same time, incompatible microstructures are observed experimentally at different scales. Several examples of such microstructures are discussed in this talk, and the important role of elastic strains and of the associated elastic strain energy is demonstrated. The modelling approach relies on energy minimization combined with sharp-interface representation of microstructures. Specifically, microstructure evolution is determined by minimization of the incremental energy supply that includes increments in free energy and rate-independent dissipation. In some cases, it suffices to minimize the free energy alone.

The range of microstructures considered includes martensitic plates, X-microstructures [2], and austenite–twinned martensite interface [3]. The related effects governed by the elastic strain energy associated with the incompatibility of the underlying microstructures include softening and instability in evolving laminates [4], elastic micro-strain energy of microstructured interfaces [3,5], and size effects in pseudoelasticity of SMA polycrystals [5].

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Global solvability via viscosity solutions for a model for solid-solid phase transitions driven by material forces

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Global existence of weak solutions to an initial-boundary value problem is proved for a coupled system of partial differential equations which models phase transitions in solid materials, e.g. shape memory alloys. This model was formulated and studied mathematically in [1,2,3]. In this talk weak solutions are defined by combining two well-known concepts of weak solutions: viscosity solution and the usual weak solution defined by using integration by parts.

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