“On the Membrane Potential in a Coupled Network of Integrate-and-Fire Neurons”

Abstract:

The stochastic integrate-and-fire model describes the membrane potential of a neuron by a stochastic differential equation up until the time when it reaches a certain threshold V_F. At this point the neuron is said to ßpikeör fireänd the voltage is reset to a resting value V_R. In the model we consider, the neuron is coupled to a large network, receiving input from the surrounding neurons. Integrating this influence of the network into the model, we obtain an associated mean-field equation.
The first passage time of the stochastic process modeling the membrane potential through the constant barrier $S(t) \equiv V_R$ plays a key role in finding a unique solution to the mean-field equation. The mathematical difficulties lie in understanding the continuity properties of the first passage time density with respect to the drift of the stochastic differential equation and in finding suitable a priori bounds for this drift.
In the mathematical neuroscience literature, the main tool for analyzing the first passage time density has so far been the Fokker-Planck equation (see, for example, [2],[3] and [4]). In our work, we use a path-space representation to study the continuity of the first passage time density with respect to the drift of the stochastic differential equation. The approach is to use Girsanov's theorem to perform corresponding changes of measures in order to show that this density depends continuously and uniformly on the derivative of the drift. The aim is to further apply this method to set up a fixed-point argument for the solution of the stochastic mean-field equation, in the spirit of [1]. Moreover, it could be adapted to the study of the first passage time density of a wider class of processes, by relaxing the restrictions on the parameters of the equation .

Literatur

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