Courses offered by Professor Alexei Kulik (Kiev) in WS 2016/17

Professor Alexei Kulik is a BMS professor at TU Berlin during the winter semester 2016/17. He will offer two successive courses entitled *Lévy processes and Lévy driven SDEs* and *Diffusions and Lévy-type processes: an analytic approach*. Students should have a good knowledge of basic probability theory (like Wahrscheinlichkeitstheorie 2). The contents are described below. The courses will take place

Mo, 10-12, room MA 541
Tue, 8-10, room MA 645

The first course will take place during the first half of the semester and the second one during the second half of the semester. Students can take an oral exam after the end of the first course (5 ECTS) or an oral exam after the end of the second course on both courses (10 ECTS).

Here are short descriptions of both courses:

Lévy processes and Lévy driven SDEs

The course is focused at the stochastic calculus and the theory of stochastic differential equations (SDEs) with the stochastic bases given by Lévy processes. The list of the topics treated in the course includes the core of the Lévy processes theory, elements of the stochastic calculus for random point measures, the structural theory of Lévy driven SDEs, and the stochastic stability theory for these SDEs.

1. Lévy processes: Brownian motion (BM), compound Poisson process, Lévy-Khinchin theorem.
2. Poisson point measures (PPMs), stochastic integrals of deterministic functions w.r.t. PPMs and compensated PPMs. The Itô-Lévy decomposition.
3. Elements of stochastic calculus with jumps: martingale point measures and their compensators, Itô stochastic integral w.r.t. Brownian motion and martingale measures, Itô's formula.
4. SDEs driven by BM and PPMs: uniqueness and existence, continuity and differentiablity w.r.t. parameters, flows.
5. Solution to a Lévy driven SDE as a Markov process: Markov property, Feller and strong Feller properties, formula for the generator.

6. Stochastic stability of the solutions of Lévy driven SDEs: dissipativity and exponential ergodicity; (sub-) linear Lyapunov conditions and (sub-) exponential recurrence; Harris type theorem.

Diffusions and Lévy-type processes: an analytic approach

The course is focused at the analytical approach to the study of diffusion and Lévy-type processes, based on the characterisation of their transition probabilities and associated semigroups in terms of the Kolmogorov differential equations. The first part of the course is devoted to the parametrix method, which is the classical tool for the construction of the fundamental solutions for 2nd order parabolic PDEs. The method will be introduced in detail and with its probabilistic counterpart. In the second part of the course, the extension of the parametrix method for the crucially important class of the Lévy-type processes will be presented. The parametrix method gives flexible representations and well controlled estimates for the heat kernels of the target processes, which leads to a wide variety of further applications. Some of them will be outlined, and applications to asymptotic statistics will be discussed in more detail.

1. Preliminaries: Kolmogorov's forward and backward differential equations for a diffusion process. Lévy type processes. Weak solutions to SDEs and solutions to Martingale Problems (MPs). Pre-generators.

2. The parametrix method for 2nd order parabolic PDEs: associated integral equation, representations and bounds for the solution, Hölder continuity, derivatives.

3. The Positive Maximum Principle and the semigroup properties of the fundamental solution.

4. Harmonic functions and an alternative proof of uniqueness of the solution to the MP.

5. Extension to a jump-diffusion case.

6. The parametrix method for locally stable Lévy-type processes (principal bounds for stable densities; the choice of the zero approximation; subconvolution property and bounds for the convolution powers; approximate fundamental solution and approximate harmonic functions; uniqueness).

7. Some applications to statistics: LAN (Local Asyptotic Normality) property, LAD (Least Absolute Deviation) estimate for an unknown drift parameter.