Some results on quadratic hedging with insider trading *

(Revised version)

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Abstract

We consider the hedging problem in an arbitrage-free financial market, where there are two kinds of investors with different levels of information about the future price evolution, described by two filtrations $\mathbf{F}$ and $\mathbf{G} = \mathbf{F} \vee \sigma(G)$ where G is a given r.v. representing the additional information. We focus on two types of quadratic approaches to hedge a given square-integrable contingent claim: local risk minimization (LRM) and mean-variance hedging (MVH). By using initial enlargement of filtrations techniques, we solve the hedging problem for both investors and compare their optimal strategies under both approaches.

In particular, for LRM, we show that for a large class of additional non trivial r.v.s G both investors will pursue the same locally risk minimizing portfolio strategy and the cost process of the ordinary agent is just the projection on $\mathbf{F}$ of that of the insider. In the MVH setting, we study also some general stochastic volatility model, including Hull and White, Heston and Stein and Stein models. In this more specific setting and for r.v.s G which are measurable with respect to the filtration generated by the volatility process, we obtain an expression for the insider optimal strategy in terms of the ordinary agent optimal strategy plus a process admitting a simple backward-type representation.

Keywords: insider trading, initial enlargement of filtrations, martingale preserving measure, local risk minimization, mean-variance hedging, stochastic volatility models

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1 Introduction

In this paper we begin the study of an hedging problem for a future stochastic cash flow X (delivered at some instant $t < T$, where T is a given finite horizon) in an arbitrage-free and incomplete financial market characterized by the presence of two kinds of investors, which have different levels of information on the future price evolution.

When the given financial market is complete, every contingent claim can be *perfectly replicated* by a *self-financing* portfolio strategy based on the underlying assets, usually modelled by an $\mathbb{R}^d$-valued semimartingale S. In this case, one can reduce to zero the risk of the claim by a suitable dynamic strategy. In the incomplete case, this is no longer possible for a general claim. Every agent then faces the problem of managing the risk they incur by buying or selling the claim.

In the mathematical finance literature, there are two main quadratic approaches to tackle this difficulty: *local risk minimization (abbr. LRM)* and *mean-variance hedging (abbr. MVH)*. Since one cannot ask simultaneously for the perfect replication of a given general claim by a portfolio strategy and the self-financing property of this strategy, we have to relax one of these two conditions. The LRM keeps then the replicability and relaxes the self-financing condition, by requiring it only on average. On the other hand, the MVH keeps the self-financing condition and relaxes the replicability, by requiring it approximately in L^2-sense.

To be a little more precise, Föllmer and Sondermann (1986) introduced the risk minimization approach, which consists in comparing strategies by means of a risk measure in terms of a conditional mean square error process. When the price process is a (local) martingale under P, it was shown that a unique risk-minimizing strategy exists and it can be computed using the Galtchouck-Kunita-Watanabe (abbr. GKW) decomposition (for a short review on this topic, see Ansel and Stricker (1993)). The case of a semimartingale price process is much more delicate and it induced Schweizer (1988) to introduce the concept of LRM. Existence of a LRM-strategy is now related to the existence of a so-called Föllmer-Schweizer decomposition, which can be viewed as a generalization of the GKW-decomposition and characterized by means of the minimal martingale measure (abbr. MMM) introduced by Föllmer and Schweizer (1991).

On the other hand, in the MVH approach, one looks for self-financing strategies which minimize the residual risk between the contingent claim and the terminal portfolio value. Again, existence and construction of an optimal strategy in the martingale case are stated by means of the GKW-decomposition of the given claim we search to hedge. In the semimartingale case, we have two kinds of characterization of the optimal strategy obtained by Gourieroux et al. (1998) (by means of a suitable change of numéraire) and by Rheinländer and Schweizer (1997), who obtained a representation of it in a feedback form. Anyway, in both papers, the variance-optimal martingale measure (abbr. VOMM), introduced by Schweizer (1996) plays a fundamental rôle.

All these papers deal with financial market models in which all agents have the same information flow, represented by a filtration which in most cases is generated either by the underlying price processes or by the driving brownian motions, as in the classical diffusion models as well as in the stochastic volatility models.

An important and natural development of this study is the introduction, in a general semimartingale model, of an *insider*. While the ordinary agent chooses his trading strategy according to the "public" information flow $\mathbf{F} = (\mathcal{F}_t)_{t\in[0,T]}$, the insider possesses from the beginning additional information about the outcome of some random variable G and therefore has the large filtration $\mathbf{G} = (\mathcal{G}_t)_{t\in[0,T]}$ with $\mathcal{G}_t = \cap_{\epsilon>0}(\mathcal{F}_{t+\epsilon} \vee \sigma(G))$ at his disposal. For instance, the insider may know the price of a stock at time T, or the price range of a stock at time T, or the price of a stock at time T distorted by some noise and so on.

In the past few years, there has been an increasing interest in asymmetry of information, and the enlargement of filtrations techniques, developed by the French School of Probability, revealed a crucial mathematical tool to investigate this topic. The reader could look at the paper by Brémaud and Yor (1978), the Lecture Notes by Jeulin (1980) and the series of papers in the Séminaire de Calcul Stochastique (1982/83) of the University Paris VI published in 1985, containing among others the important paper by Jacod (1985).

On the other hand, the mathematical finance literature focuses mainly on the problem of portfolio optimization of an insider. We refer here to Karatzas and Pikovsky (1996), Amendinger et al. (1998), Grorud and Pontier (1998) and Imkeller et al. (2001). All these works consider the differential of utility between the two agents (as previously described) and one important conclusion is that the differential is the relative entropy of the additional r.v. G with respect to the original probability measure P. We quote also a recent paper by Biagini and Øksendal (2002), which adopts a different approach based on forward integrals with respect to the brownian motion, and a preprint by Baudoin and Nguyen-Ngoc (2002), who study a financial market where the price process may jump and there is an insider possessing some weak anticipation on the future evolution of a stock (i.e. he knows the law of some functional of the price process).

The present paper uses the same probabilistic tools as in these articles, but deals with the hedging problem of a given contingent claim $X \in L^2(P)$ in a general semimartingale financial market admitting the phenomenon of asymmetry of information as formalized above. In particular, we would compare the hedging strategies of the ordinary agent and the insider, when they both adopt the LRM or the MVH approach. We will search to answer the following natural questions: for what kind of additional information will the two agents pursue the same optimal hedging strategies? How are the two optimal strategies and the two intrinsic risks of the claim different?

The remainder of the paper is structured as follows. In Section 2 we collect the main results about initial enlargement of filtrations. In particular, we recall that if the additional r.v. G satisfies $P[G \in \cdot\,|\mathcal{F}_t](\omega) \sim P[G \in \cdot\,]$ for all $t \in [0,T)$, then there exists a version of the conditional density $(p_t^x)_{t\in[0,T)}$ of G possessing good measurability properties (Jacod (1985) and Amendinger (2000)). We quote also a result by Jacod (1985) who states that, under the above assumptions, S is also a semimartingale with respect to the enlarged filtration $\mathbf{G}$ and provides its canonical decomposition. Finally, we recall the representation of p^G and its inverse as a stochastic exponential (Amendinger et al. (1998)).

Section 3 deals with LRM for a claim $X \in L^2(P, \mathcal{F}_t)$ with $t < T$ given. We first review the definitions of cost process and locally risk minimizing strategy (abbr. LRM-strategy) and then its characterization in terms of the Föllmer-Schweizer decomposition and the minimal martingale measure. We then establish a relation between the MMMs of the ordinary agent and the insider and we use it to compare the LRM-strategies for a large class of r.v.s G. More precisely, we show that for such a G the two agents pursue the same optimal strategy and the cost process of the ordinary agent is just the projection on his filtration $\mathbf{F}$ of that of the insider.

In Section 4 we investigate the MVH approach with insider trading. After having recalled the main features of this approach, in particular the Rheinländer-Schweizer feedback representation of the optimal strategy $\vartheta^{MVH,\mathbf{H}}$ for $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$, we compare the MVH-strategies in the martingale case, when the price process S is a (local) P-martingale under both $\mathbf{F}$ and $\mathbf{G}$, and we that their optimal strategies are equal. Then, we show that this equality still hold for the "optimal strategies" of the two agents calculated under their respective VOMMs. Unfortunately, we are not able to compare the MVH-strategies in the general case, but nonetheless we can give a feedback representation of the difference process $\xi^{MVH} = \vartheta^{MVH,\mathbf{G}} - \vartheta^{MVH,\mathbf{F}}$ in a quite general stochastic volatility model (including Hull and White, Stein and Stein and Heston models) for all r.v.s G that are measurable with respect to the filtration generated by the volatility process.

2 Preliminaries on initial enlargement of filtrations

Let a probability space $(\Omega, \mathcal{F}, P)$ be given and equipped with a filtration $\mathbf{F} = (\mathcal{F}_t)_{t\in[0,T]}$ satisfying the usual conditions of completeness and right continuity, where $T \in [0, \infty]$ is a fixed time horizon. We also assume that $\mathcal{F}_0$ is trivial.

Given an $\mathcal{F}$-measurable random variable G taking values in a Polish space $(U, \mathcal{U})$, we denote by $\mathbf{G} = (\mathcal{G}_t)_{t\in[0,T]}$ the filtration $\mathbf{F}$ initially enlarged by G and made right-continuous, i.e.

$$\mathcal{G}_t := \bigcap_{\epsilon>0} (\mathcal{F}_{t+\epsilon} \vee \sigma(G)) \quad t \in [0,T].$$

Furthermore, we set $\mathbf{F}^0 := (\mathcal{F}_t)_{t\in[0,T)}$ and $\mathbf{G}^0 := (\mathcal{G}_t)_{t\in[0,T)}$; note the difference between $[0,T]$ and $[0,T)$. For a given $t \in [0,T)$, we will frequently use also the notations $\mathbf{F}^t := (\mathcal{F}_s)_{s\in[0,t]}$ and $\mathbf{G}^t := (\mathcal{G}_s)_{s\in[0,t]}$.

Now, we make the following *fundamental technical assumption*:

$$P[G \in \cdot\, | \mathcal{F}_t](\omega) \sim P[G \in \cdot\,] \tag{1}$$

for all $t \in [0,T)$ and P-a.e. $\omega \in \Omega$. In other words we are assuming that the regular distributions of G given $\mathcal{F}_t$, $t \in [0,T)$, are equivalent to the law of G for P-almost all $\omega \in \Omega$. It is known that, under this assumption, also the enlarged filtration $\mathbf{G}$ satisfies the usual conditions (Proposition 3.3 in Amendinger (2000)).

We now quote a result by Amendinger (2000), which is based on a previous lemma by Jacod (1985), and which states that there exists "nice" version of the conditional density process resulting from the previous assumption. By $\mathcal{O}(\mathbf{H}^0)$ ($\mathbf{H}^0 \in \{\mathbf{F}^0, \mathbf{G}^0\}$) we will denote the optional σ-field corresponding to the filtration $\mathbf{H}^0$.

Lemma 1 *Under assumption (1), there exists a strictly positive $\mathcal{O}(\mathbf{F}^0) \otimes \mathcal{U}$-measurable process $(\omega, t, x) \mapsto p_t^x(\omega)$, which is right-continuous with left-limits (RCLL) in t and such that*

1. *for all $x \in U$, p^x is a $(P, \mathbf{F}^0)$-martingale, and*
2. *for all $t \in [0, T)$, the measure $p_t^x P[G \in dx]$ on $(U, \mathcal{U})$ is a version of the conditional distributions $P[G \in dx|\mathcal{F}_t]$.*

We now assume that on the stochastic basis $(\Omega, \mathcal{F}, \mathbf{F}, P)$ a continuous, $\mathbf{F}$-adapted, $\mathbb{R}^d$-valued semimartingale $S = (S_t)_{t\in[0,T]}$ is defined, which models the discounted price evolution of d risky assets and with canonical decomposition $S = S_0 + M + A$, where $M \in \mathbb{H}^2_{0,loc}(\mathbf{F})$ and A is an $\mathbf{F}$-predictable process with locally square-integrable variation $|A|$.

For $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$, we will denote by $\mathcal{M}_2(\mathbf{H})$ (resp. $\mathcal{M}_2^e(\mathbf{H})$) the set of all $(P, \mathbf{H})$-absolutely continuous (resp. equivalent) (local) martingale measures with square-integrable Radon-Nikodym densities. More formally

$$\mathcal{M}_2(\mathbf{H}) = \left\{Q \ll P : dQ/dP \in L^2(P), S \text{ is a } (Q, \mathbf{H})\text{-local martingale}\right\}$$

and

$$\mathcal{M}_2^e(\mathbf{H}) = \{Q \in \mathcal{M}_2(\mathbf{H}) : Q \sim P\},$$

where $L^2(P) = L^2(P, \mathcal{F})$. In order to stress the dependence from the underlying probability measure, we will write sometimes $\mathcal{M}_2^e(\mathbf{H}, P)$.

We make the following standing assumption:

$$\mathcal{M}_2^e(\mathbf{H}) \neq \emptyset, \tag{2}$$

for $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$. By Girsanov's theorem, the existence of an element $Q \in \mathcal{M}_2^e(\mathbf{F})$ implies that the predictable process A in the canonical decomposition of S must have the form:

$$A_t = \int_0^t \lambda_s' d\left\langle M\right\rangle_s, \quad t \in [0, T],$$

for some predictable $\mathbb{R}^d$-valued process λ. We denote

$$\widehat{K}_t = \int_0^t \lambda_s' d\left\langle M\right\rangle_s \lambda_s, \quad t \in [0, T],$$

and call this the *mean-variance tradeoff process of S under* $\mathbf{F}$ ($\mathbf{F}$-MVT process).

The following fundamental results by Amendinger (2000), Jacod (1985) and Amendinger et al. (1998), respectively, will be very useful in the sequel of the paper.

Theorem 2 *Let Q be in $\mathcal{M}_2^e(\mathbf{F})$ and let Z denote its density process with respect to P. Moreover, let $p^G = (p^x)_{|x=G}$. Then, under assumptions (1) and (2), the following assertions hold for every $t \in [0, T]$:*

1. *$\widetilde{Z} := Z/p^G$ is a $(P, \mathbf{G}^0)$-martingale, and*

2. *the* $[0,t]$-martingale preserving probability measure (abbr. t-MPM) *(under initial enlargement)*

$$\widetilde{Q}_t(A) := \int_A \frac{Z_t}{p_t^G} dP \quad \textit{for } A \in \mathcal{G}_t \tag{3}$$

has the following properties

(a) *the* σ*-algebra* $\mathcal{F}_t$ *and* $\sigma(G)$ *are independent under* $\widetilde{Q}_t$*,*

(b) $\widetilde{Q}_t = Q$ *on* $(\Omega, \mathcal{F}_t)$*, and* $\widetilde{Q}_t = P$ *on* $(\Omega, \sigma(G))$*, i.e. for all* $A \in \mathcal{F}_t$ *and* $B \in \mathcal{U}$*,*

$$\widetilde{Q}_t[A \cap \{G \in B\}] = Q[A]P[G \in B] = \widetilde{Q}_t[A]\widetilde{Q}_t[B]$$

3. *for every* $p \in [1,\infty]$*,* $\mathbb{H}^p_{(loc)}(Q, \mathbf{F}^t) = \mathbb{H}^p_{(loc)}(\widetilde{Q}_t, \mathbf{F}^t) \subseteq \mathbb{H}^p_{(loc)}(\widetilde{Q}_t, \mathbf{G}^t)$*.*

Proof. See Amendinger (2000), Theorem 3.1 and Theorem 3.2, p. 104. □

Remark 3 *Theorem 2 implies that, under assumption (2) for* $\mathbf{H} = \mathbf{F}$*, there exists an equivalent local martingale measure for* S *also under the enlarged filtration* $\mathbf{G}$*, whose Radon-Nikodym derivative with respect to* P *is not necessarily in* $L^2(P)$*. Assumption (2) is then necessary also for* $\mathbf{H} = \mathbf{G}$*.*

The next theorem (due to J. Jacod) claims that under the fundamental assumption (1), the price process S is also a $\mathbf{G}^0$-semimartingale and it gives its canonical decomposition under the enlarged filtration.

Theorem 4 *For* $i = 1, ..., d$*, there exists a* $\mathcal{P}(\mathbf{F}^0)$*-measurable function* $(\omega, x, t) \mapsto (\mu_t^x(\omega))^i$ *such that*

$$\langle p^x, M^i \rangle = \int (\mu^x)^i p_-^x \, d \langle M^i \rangle .$$

For every such function $(\mu^{\cdot})^i$*, we consider* $(\mu^G)^i = (\mu^x)^i_{|x=G}$ *and we have*

1. $\int_0^t |(\mu_s^G)^i| d\langle M^i \rangle_s < \infty$ $P - a.s.$ *for all* $t \in [0,T)$*, and*
2. M^i *is a* $(P, \mathbf{G}^0)$*-semimartingale, and the continuous local* $(P, \mathbf{G}^0)$*-martingale in its canonical decomposition is*

$$\widetilde{M}_t^i := M_t^i - \int_0^t \left(\mu_s^G\right)^i d \left\langle M^i \right\rangle_s, \quad t \in [0,T). \tag{4}$$

Proof. See Théorème 2.1 of Jacod (1985). □

This theorem with the standing assumption (2) for $\mathbf{H} = \mathbf{G}$ implies that the finite variation process $\widetilde{A}$ in the canonical decomposition of S under $\mathbf{G}$ must satisfy

$$\widetilde{A}_t = \int_0^t \left(\lambda_s + \mu_s^G\right)' d \left\langle \widetilde{M} \right\rangle_s = \int_0^t \left(\lambda_s + \mu_s^G\right)' d \left\langle M \right\rangle_s, \quad t \in [0,T],$$

and then the corresponding **G**-MVT process of S is given by

$$\widehat{K}_t^{\mathbf{G}} = \int_0^t \left(\lambda_s + \mu_s^G\right)' d\left\langle M\right\rangle_s \left(\lambda_s + \mu_s^G\right), \quad t \in [0,T].$$

Finally, the theorem quoted below gives a stochastic exponential representation of the conditional density p^G and its inverse.

Theorem 5 *1. There exists a local $(P,\mathbf{G}^0)$-martingale $\widetilde{N}$ null at 0, which is $(P,\mathbf{G}^0)$-orthogonal to $\widetilde{M}$ (i.e. $\langle \widetilde{M}^i, \widetilde{N}\rangle = 0$ for $i = 1,...,d$) and such that*

$$\frac{1}{p_t^G} = \mathcal{E}\left(-\int \left(\mu^G\right)' d\widetilde{M} + \widetilde{N}\right)_t, \quad t \in [0,T). \tag{5}$$

2. Given $x \in U$, there exists a local $\mathbf{F}^0$-martingale N^x null at 0 which is orthogonal to S and such that

$$p_t^x = \mathcal{E}\left(\int \mu^x dS + N^x\right)_t, \quad t \in [0,T]. \tag{6}$$

Proof. See Proposition 2.9, p. 270, of Amendinger et al. (1998). □

Remark 6 *In the sequel, without further mention, all equalities between strategies or integrands will hold a.s. $d\langle M\rangle dP$.*

3 The LRM approach

3.1 Preliminaries and terminology

We collect in this subsection the main definitions and results of the LRM approach and to do this, we will essentially follow the two survey papers by Pham (2000) and Schweizer (2001). All the objects we will introduce in this section refer to the initially non-trivial filtration $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$.

A portfolio strategy is a pair $\varphi = (V, \vartheta)$ where V is a real-valued adapted process such that $V_T \in L^2(P)$ and ϑ belongs to $\Theta = \Theta^{\mathbf{H}}$, which denotes the set of all **H**-predictable, $\mathbb{R}^d$-valued, S-integrable processes ϑ such that $\int_0^T \vartheta_s dS_s \in L^2(P)$ and $\int \vartheta dS$ is a $(Q,\mathbf{H})$-martingale for all $Q \in \mathcal{M}_2^e(\mathbf{H})$, which is closed in $L^2(P)$.

We now associate to each portfolio strategy $\varphi = (V, \vartheta)$ a process, which will be very useful in the sequel in describing the main features of the LMR approach: the *cost process* $C(\varphi)$.

The cost process of a portfolio strategy $\varphi = (V, \vartheta)$ is defined by

$$C_t(\varphi) = V_t - \int_0^t \vartheta_u dS_u, \quad t \in [0,T].$$

A portfolio strategy φ is called *self-financing* if its cost process $C(\varphi)$ is constant P a.s.. It is called *mean self-financing* if $C(\varphi)$ is a martingale under P.

Fix now a square-integrable, $\mathcal{F}_T$-measurable contingent claim X. We say that a portfolio strategy $\varphi = (V, \vartheta)$ is X-*admissible* if $V_T = X$, P a.s.. Therefore, an X-admissible portfolio strategy φ is called *locally risk minimizing* (abbr. LRM-strategy) if the corresponding cost process $C(\varphi)$ belongs to $\mathbb{H}^2(P, \mathbf{H})$ and is orthogonal to S under $(P, \mathbf{H})$. There exists a LRM-strategy if and only if X admits a decomposition:

$$X = X_0 + \int_0^T \vartheta_t^X dS_t + L_T^X, \quad P \text{ a.s.}, \tag{7}$$

where X_0 is $\mathcal{H}_0$-measurable, $\vartheta^X \in \Theta$ and $L^X \in \mathbb{H}^2(P, \mathbf{H})$ is orthogonal to S. Such a decomposition is called *Föllmer-Schweizer decomposition* of X under $(P, \mathbf{H})$, and the portfolio strategy $\varphi^{LRM} = (V^{LRM}, \vartheta^{LRM})$ with $\vartheta^{LRM} = \vartheta^X$ and

$$V_t^{LRM} = X_0 + \int_0^t \vartheta_s^X dS_s + L_t^X, \quad P \text{ a.s.}, \quad t \in [0, T].$$

is a LRM-strategy for X.

There exists also a very useful characterization of the LRM-strategy by means of the Galtchouk-Kunita-Watanabe decomposition (abbr. GKW-decomposition) of X under a suitable equivalent martingale measure, namely the *minimal martingale measure* (abbr. MMM) introduced by Föllmer and Schweizer (1991). We recall now some basic facts about this measure and its very deep relation with the LRM approach.

We denote by $Z^{min,\mathbf{H}}$, for $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$, the *minimal martingale density under* $\mathbf{H}$, i.e. for the ordinary agent

$$Z_t^{min,\mathbf{F}} = \mathcal{E}\left(-\int \lambda dM\right)_t, \quad t \in [0, T),$$

and for the insider

$$Z_t^{min,\mathbf{G}} = \mathcal{E}\left(-\int \left(\lambda + \mu^G\right) d\widetilde{M}\right)_t, \quad t \in [0, T).$$

Since our goal is comparing the LRM-strategies, we have to assume that, given a contingent claim $X \in L^2(\mathcal{F}_t)$ for some $t < T$, there exists a LRM-strategy (to hedge X) for the ordinary agent as well as for the insider. We make then the following

Assumption 7 *$Z^{min,\mathbf{H}}$ is a uniformly integrable $\mathbf{H}^0$-martingale satisfying $R_2(P)$ for $\mathbf{H}^0 \in \{\mathbf{F}^0, \mathbf{G}^0\}$, i.e. for all $t \in [0, T)$ there exists a constant $C > 0$ such that*

$$\mathbb{E}\left[\left(\frac{Z_t^{min,\mathbf{H}}}{Z_s^{min,\mathbf{H}}}\right)^2 \middle| \mathcal{H}_s\right] \leq C, \quad s \in [0, t].$$

Since Delbaen et al (1997) we know that this assumption is equivalent to assuming the existence of a Föllmer-Schweizer decomposition (and so of a unique LRM-strategy) for every $X \in L^2(P, \mathcal{F}_t)$, for any $t \in [0, T)$, under both $\mathbf{F}$ and $\mathbf{G}$.

Moreover, under Assumption 7, we can define on $\mathcal{F}_t$, for all $t \in [0,T)$, a P-equivalent $\mathbf{H}$-martingale measure $P^{min,\mathbf{H}}$ for S, given by

$$\left.\frac{dP^{min,\mathbf{H}}}{dP}\right|_{\mathcal{H}_t} = Z_t^{min,\mathbf{H}},$$

which is called *minimal martingale measure* for S under $\mathbf{H}$ (abbr. $\mathbf{H}$-MMM).

We now quote without proof (for whom we refer to Föllmer and Schweizer (1991), Theorem 3.14, p. 403) the following fundamental result relating the MMM and the LRM-strategy:

Theorem 8 *(We drop here, for simplicity, the dependence on* $\mathbf{H}$*) Let* X *be a contingent claim in* $L^2(P,\mathcal{F}_t)$ *for some* $t \in [0,T)$*. The LRM-strategy* φ^{LRM}*, hence also the corresponding Föllmer-Schweizer decomposition (7), is uniquely determined. It can be computed in terms of the MMM* P^{min}*: if* $(V_s^{min,X})_{s\in[0,t]}$ *denotes a right-continuous version of the* P^{min}*-martingale* $(\mathbb{E}[X|\mathcal{H}_s])_{s\in[0,t]}$ *with GKW-decomposition*

$$V_s^{min,X} = V_0^{min,X} + \int_0^s \vartheta_u^{min,X} dS_u + L_s^{min,X}, \quad s \in [0,t],$$

then the portfolio strategy $\varphi^{min,X} = (V^{min,X}, \vartheta^{min,X})$ *is the LRM-strategy for* X *and its cost process is given by* $C(\varphi^{LRM}) = \mathbb{E}^{min}[X|\mathcal{H}_0] + L^{min,X}$*.*

3.2 Comparing the LRM-strategies

In this subsection, we want to compare the LRM-strategies of the two differently informed agents. We start with a simple but very useful lemma establishing a relation between the respective MMMs. We recall that if Q is any P-absolutely continuous martingale measure for S and Z its density process under $\mathbf{F}$, then $\widetilde{Q}$ and $\widetilde{Z}$ denote respectively the corresponding MPM and its density process (under $\mathbf{G}$).

Lemma 9 *The minimal martingale densities* $Z^{min,\mathbf{H}}$ *for* $\mathbf{H} \in \{\mathbf{F},\mathbf{G}\}$ *satisfy the following relation:*

$$\mathcal{E}(\widetilde{N})Z^{min,\mathbf{G}} = \widetilde{Z^{min,\mathbf{F}}}, \tag{8}$$

where $\widetilde{N}$ *is the local* $(P,\mathbf{G}^0)$*-martingale, null at* 0 *and* $(P,\mathbf{G}^0)$*-orthogonal to* S *appearing in Theorem 5.*

Proof. By developing the stochastic exponential, we find immediately that

$$\begin{aligned} Z^{min,\mathbf{G}} &= \mathcal{E}\left(-\int \left(\lambda + \mu^G\right) d\widetilde{M}\right) \\ &= \mathcal{E}\left(-\int \lambda dM\right)\mathcal{E}\left(-\int \mu^G d\widetilde{M}\right) \\ &= Z^{min,\mathbf{F}}\mathcal{E}\left(-\int \mu^G d\widetilde{M}\right). \end{aligned}$$

If we multiply both sides of the above equality by $\mathcal{E}(\widetilde{N})$ and apply Yor's formula on stochastic exponentials, we have

$$\mathcal{E}(\widetilde{N})Z^{min,\mathbf{G}} = Z^{min,\mathbf{F}}\mathcal{E}\left(-\int \mu^G d\widetilde{M} + \widetilde{N} + \left[\int \mu^G d\widetilde{M}, \widetilde{N}\right]\right).$$

Since $\widetilde{M}$ is continuous and orthogonal to $\widetilde{N}$, we have

$$\left[\int \mu^G d\widetilde{M}, \widetilde{N}\right] = \left\langle \int \mu^G d\widetilde{M}, \widetilde{N}\right\rangle = 0$$

Then the representation of $1/p^G$ provided by Theorem 5 implies

$$\mathcal{E}(\widetilde{N})Z^{min,\mathbf{G}} = Z^{min,\mathbf{F}}\frac{1}{p^G} = \widetilde{Z^{min,\mathbf{F}}}$$

and the proof is now complete. $\square$

Remark 10 *The previous lemma states in particular that if the orthogonal part $\widetilde{N}$ in the stochastic exponential representation (5) of the conditional density p^G vanishes, then the MMM of the insider is just the MPM corresponding to the MMM of the ordinary agent.*

We now compare the LRM-strategies of both agents when the additional r.v. G is such that $\widetilde{N} = 0$. The next proposition shows that in this case they will adopt the same behaviour and their cost processes satisfy a simple projection relation.

Proposition 11 *Assume $\widetilde{N} = 0$ and let X be a contingent claim in $L^2(P, \mathcal{F}_t)$ for some $t < T$. Then:*

1. $\vartheta_s^{LRM,\mathbf{F}} = \vartheta_s^{LRM,\mathbf{G}}$ *for all* $s \in [0,t]$;
2. $L_t^{min,\mathbf{F}} + (\mathbb{E}^{min,\mathbf{F}}[X] - \mathbb{E}^{min,\mathbf{G}}[X|\mathcal{G}_0]) = L_t^{min,\mathbf{G}}$.

In particular, $C_s(\varphi^{LRM,\mathbf{F}}) = \mathbb{E}[C_s(\varphi^{LRM,\mathbf{G}})|\mathcal{F}_s]$ for all $s \in [0,t]$.

Proof. Associate firstly to X the $(P^{min,\mathbf{G}}, \mathbf{G})$-martingale $X_s^{min,\mathbf{G}} := \mathbb{E}^{min,\mathbf{G}}[X|\mathcal{G}_s]$, $s \leq t$, and consider its GKW-decomposition under $(P^{min,\mathbf{G}}, \mathbf{G})$:

$$X_s^{min,\mathbf{G}} = \mathbb{E}^{min,\mathbf{G}}[X|\mathcal{G}_0] + \int_0^s \vartheta_u^{min,\mathbf{G}} dS_u + L_s^{min,\mathbf{G}}, \quad s \in [0,t], \tag{9}$$

where $\vartheta^{min,\mathbf{G}} \in L^1(S, P^{min,\mathbf{G}})$ and $L^{min,\mathbf{G}}$ is a $(P^{min,\mathbf{G}}, \mathbf{G})$-martingale, orthogonal to S. On the other hand consider the $(P^{min,\mathbf{F}}, \mathbf{F})$-martingale $X_s^{min,\mathbf{F}} := \mathbb{E}^{min,\mathbf{F}}[X|\mathcal{F}_s]$, $s \leq t$. Its GKW-decomposition under $(P^{min,\mathbf{F}}, \mathbf{F})$ is given by

$$X_s^{min,\mathbf{F}} = \mathbb{E}^{min,\mathbf{F}}[X] + \int_0^s \vartheta_u^{min,\mathbf{F}} dS_u + L_s^{min,\mathbf{F}}, \quad s \in [0,t], \tag{10}$$

where $\vartheta^{min,\mathbf{F}} \in L^1(S, P^{min,\mathbf{F}})$ and $L^{min,\mathbf{F}}$ is a $(P^{min,\mathbf{F}}, \mathbf{F})$-martingale, orthogonal to S. Observe now that $\vartheta^{min,\mathbf{F}} \in L^1(S, P^{min,\mathbf{G}})$ and moreover, since $P^{min,\mathbf{G}} = \widetilde{P^{min,\mathbf{F}}}$, item 3 of Theorem 2 implies that $L^{min,\mathbf{F}}$ is also a $(P^{min,\mathbf{G}}, \mathbf{G})$-martingale orthogonal to S and so is $L^{min,\mathbf{F}} + (\mathbb{E}^{min,\mathbf{F}}[X] - \mathbb{E}^{min,\mathbf{G}}[X|\mathcal{G}_0])$. Finally, since the two processes we are considering have the same terminal value X, the uniqueness property of the LRM-strategies implies the first two items of the proposition. The claimed relation between the cost processes is now quite clear. Indeed, since $L^{min,\mathbf{H}}$ is a local $(P, \mathbf{H})$-martingale for $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$ (see Ansel and Stricker (1992) or Schweizer (1995)), the usual localization procedure allows us to assume, without loss of generality, that it is a true $(P, \mathbf{H})$-martingale and then, for all $s \in [0, t]$,

$$\begin{aligned} C_s(\varphi^{LRM,\mathbf{F}}) &= \mathbb{E}^{min,\mathbf{F}}[X] + L_s^{min,\mathbf{F}} = \\ &= \mathbb{E}\left[\mathbb{E}^{min,\mathbf{F}}[X] + L_t^{min,\mathbf{F}}|\mathcal{F}_s\right] = \\ &= \mathbb{E}\left[\mathbb{E}^{min,\mathbf{G}}[X|\mathcal{G}_0] + L_t^{min,\mathbf{G}}|\mathcal{F}_s\right] = \\ &= \mathbb{E}\left[\mathbb{E}^{min,\mathbf{G}}[X|\mathcal{G}_0] + L_s^{min,\mathbf{G}}|\mathcal{F}_s\right] = \\ &= \mathbb{E}\left[C_s(\varphi^{LRM,\mathbf{G}})|\mathcal{F}_s\right]. \end{aligned}$$

The proof is now complete. □

Remark 12 *The conclusion of Proposition 11 is not so surprising. Indeed, under the MPM corresponding to the insider MMM the additional r.v. G is independent to the claim X, which is assumed to be $\mathcal{F}_t$-measurable. Then, in this case the additional knowledge of the insider does not produce any effect on his behaviour.*

Even if it is clearly hard to check the assumption $\widetilde{N} \equiv 0$ on G in a general incomplete market, it is nonetheless not difficult to exhibit several examples of such r.v.s. Indeed, it suffices to consider the stochastic volatility model described in Subsection 4.3 with G equaling the terminal value of the first driving brownian motion W_T^1 or $G = \mathbf{1}_{\{W_T^1 \in (a,b)\}}$ with $a, b \in \mathbb{R} \cup \{-\infty, \infty\}$, or $G = \alpha W_T^1 + (1-\alpha)\varepsilon$ where the random variable ε is independent of $\mathcal{F}_T$ and normally distributed with mean 0 and variance $\sigma^2 > 0$, and α is a real number in $(0, 1)$. To verify this the reader could easily adapt the computations contained in the paper by Amendinger et al. (1998) to the incomplete market setting provided by our stochastic volatility model.

4 The MVH approach

4.1 Preliminaries and terminology

Given a contingent claim $X \in L^2(P)$ and an initial investment $h \in L^2(\mathcal{H}_0)$, we are interested in the following two quadratic optimization problems:

$$\min_{\vartheta^{\mathbf{H}} \in \Theta^{\mathbf{H}}} \mathbb{E}\left[X - h - \int_0^T \vartheta_t^{\mathbf{H}} dS_t\right]^2 \tag{11}$$

for $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$ and where the $\mathbf{H}$-admissible strategies set $\Theta^{\mathbf{H}}$ is as in the previous section.

The financial interpretation is the usual one: two investors search to replicate (approximately, in the L^2-sense) a given future cash-flow X by trading dynamically in the underlying S.

The *ordinary investor* uses only the information contained in the filtration $\mathbf{F}$, e.g. if $\mathbf{F}$ is the natural filtration of S, he observes only the market prices of the underlying assets. On the other hand, the *informed agent* or *insider*, has an additional information which is described by the random variable G, so that the filtration, on which he bases his decisions, is given by $\mathbf{G}$.

From a mathematical viewpoint, this corresponds to project the random variable X onto the following subset of $L^2(P)$

$$\mathcal{G}(h, \Theta^{\mathbf{H}}) := \left\{ h + \int_0^T \vartheta_t^{\mathbf{H}} dS_t : \theta^{\mathbf{H}} \in \Theta^{\mathbf{H}} \right\},$$

that is named *set of investment* $\mathbf{H}$*-opportunities.* Since $\mathcal{G}(h, \Theta^{\mathbf{H}})$ is closed in $L^2(P)$ then problem (11) is meaningful and it admits a unique solution that we will denote by $\vartheta^{MVH,\mathbf{H}}$, for $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$.

We are interested also in the following minimization problem:

$$J^{\mathbf{H}}(X) := \min_{h \in L^2(\mathcal{H}_0)} J^{\mathbf{H}}(h, X) \tag{12}$$

where

$$J^{\mathbf{H}}(h, X) := \min_{\vartheta^{\mathbf{H}} \in \Theta^{\mathbf{H}}} \mathbb{E} \left[X - h - \int_0^T \vartheta_t^{\mathbf{H}} dS_t \right]^2 \qquad h \in L^2(\mathcal{H}_0),$$

is the associated risk function of the investor with information $\mathbf{H}$.

The solution h^{MVH} to this problem is named *approximation price of* X (see Schweizer (1996)).

Assume now that $P \in \mathcal{M}_2^e(\mathbf{H})$. In this case $\Theta^{\mathbf{H}} = L^2(S, P, \mathbf{H})$ (see Remark 5.3 in Pham (2000)). We recall that every contingent claim $X \in L^2(P)$ admits a unique GKW-decomposition

$$X = \mathbb{E}[X|\mathcal{H}_0] + \int_0^T \vartheta_t^{\mathbf{H},X} dS_t + L_T^{\mathbf{H},X}$$

where $\mathcal{H}_0$ is the initial σ-field of $\mathbf{H}$ and $L_T^{\mathbf{H},X}$ is the terminal value of the uniformly integrable $(P, \mathbf{H})$-martingale $(L_t^{\mathbf{H},X})_{t\in[0,T]}$, which orthogonal to S under $(P, \mathbf{H})$ and whose initial value is zero.

Proposition 13 *Assume that* $P \in \mathcal{M}_2^e(\mathbf{H})$.

1. *There exists a unique solution* $\vartheta^{MVH,\mathbf{H}}$ *to problem (11), for all* $h \in L^2(\mathcal{H}_0)$*, given by the process* $\vartheta^{\mathbf{H},X}$ *in the decomposition (4.1), and*

$$J^{\mathbf{H}}(h, X) = \mathbb{E}\left[\mathbb{E}[X|\mathcal{H}_0] - h\right]^2 + \mathbb{E}\left[L_T^{\mathbf{H},X}\right]^2, \tag{13}$$

2. *the approximation price for the agent is given by* $h^{MVH} = \mathbb{E}[X|\mathcal{H}_0]$*, and*

$$J^{\mathbf{H}}(X) = \mathbb{E}\left[L_T^{\mathbf{H},X}\right]^2.$$

Proof.

1. By using GKW-decomposition of X with respect to the filtration $\mathbf{H}$, and conditioning to $\mathcal{H}_0$, which is not necessarily trivial, one obtains

$$\begin{aligned}\mathbb{E}\left[X - h - \int_0^T \vartheta_t^{\mathbf{H}} dS_t\right]^2 &= \mathbb{E}\left[\mathbb{E}[X|\mathcal{H}_0] - h + \int_0^T \left(\vartheta_t^{\mathbf{H},X} - \vartheta_t^{\mathbf{H}}\right) dS_t + L_T^{\mathbf{H},X}\right]^2 \\ &= \mathbb{E}\left[\mathbb{E}[X|\mathcal{H}_0] - h\right]^2 + \mathbb{E}\left[\int_0^T \left(\vartheta_t^{\mathbf{H},X} - \vartheta_t^{\mathbf{H}}\right) dS_t\right]^2 + \\ &\quad + \mathbb{E}\left[L_T^{\mathbf{H},X}\right]^2. \qquad (14)\end{aligned}$$

Then the strategy $\vartheta^{\mathbf{H},X}$ solves problem (11) and we also have the desired formula for the associated value function $J^{\mathbf{H}}(h, X)$, for all $h \in L^2(\mathcal{H}_0)$.

2. By relation (14),

$$J^{\mathbf{H}}(h, X) = \mathbb{E}\left[\mathbb{E}[X|\mathcal{H}_0] - h\right]^2 + \mathbb{E}\left[L_T^{\mathbf{H},X}\right]^2,$$

that implies $h^{MVH} = \mathbb{E}[X|\mathcal{H}_0]$ and concludes the proof of the proposition. □

If P is not an $\mathbf{H}$-martingale measure, Rheinländer and Schweizer (1997) and Gourieroux et al. (1998) (see also Pham (2000)) have nonetheless obtained two characterizations of the solution of problem (11), under the assumption $\mathcal{H}_0$ trivial. But it is very easy to check that all those results still hold even without this assumption. We now recall some basic facts of the first approach.

We know since Delbaen and Schachermayer (1996) that, being the price process S continuous, the *variance optimal martingale measure (abbr. VOMM)* can be defined as the unique martingale probability measure $P^{\mathbf{H},opt}$ solution to the problem

$$\min_{Q \in \mathcal{M}_2(\mathbf{H})} \mathbb{E}\left[\frac{dQ}{dP}\right]^2, \qquad (15)$$

and that this measure is in fact equivalent to P. Moreover, the process

$$Z_t^{\mathbf{H},opt} := \mathbb{E}^{\mathbf{H},opt}\left[\left.\frac{dP^{\mathbf{H},opt}}{dP}\right|\mathcal{H}_t\right], \quad t \in [0,T]$$

can be written as

$$Z_t^{\mathbf{H},opt} = Z_0^{opt} + \int_0^t \zeta_s^{\mathbf{H},opt} dS_s, \quad t \in [0,T] \qquad (16)$$

for some constant Z_0^{opt} (independent from the underlying filtration) and some process $\zeta^{\mathbf{H},opt} \in \Theta^{\mathbf{H}}$. The following theorem contains the characterization of the optimal mean-variance strategy for a given contingent claim $X \in L^2(P)$ in a feedback form.

Theorem 14 *Let $X \in L^2(P)$ be a contingent claim and let $h \in L^2(\mathcal{H}_0)$ be an initial investment. The GKW-decomposition of X under $(P^{\mathbf{H},opt}, \mathbf{H})$ with respect to S is*

$$X = \mathbb{E}^{\mathbf{H},opt}[X|\mathcal{H}_0] + \int_0^T \vartheta_s^{\mathbf{H},opt} dS_s + L_T^{\mathbf{H},opt} = V_T^{\mathbf{H},opt} \tag{17}$$

with

$$V_t^{\mathbf{H},opt} = \mathbb{E}^{\mathbf{H},opt}[X|\mathcal{H}_t] = \mathbb{E}^{\mathbf{H},opt}[X|\mathcal{H}_0] + \int_0^t \vartheta_s^{\mathbf{H},opt} dS_s + L_t^{\mathbf{H},opt}, \quad t \in [0,T].$$

Then, the mean-variance optimal strategy for X is given by

$$\vartheta_t^{MVH,\mathbf{H}} = \vartheta_t^{\mathbf{H},opt} - \frac{\zeta_t^{\mathbf{H},opt}}{Z_t^{\mathbf{H},opt}} \left(V_{t-}^{\mathbf{H},opt} - h - \int_0^t \vartheta_s^{MVH,\mathbf{H}} dS_s \right) \tag{18}$$

$$= \vartheta_t^{\mathbf{H},opt} - \zeta_t^{\mathbf{H},opt} \left(\frac{V_0^{\mathbf{H},opt} - h}{Z_0^{\mathbf{H},opt}} + \int_0^{t-} \frac{1}{Z_s^{\mathbf{H},opt}} dL_s^{\mathbf{H},opt} \right), \tag{19}$$

for all $t \in [0,T]$. Moreover the approximation price for X is given by $h^{MVH} = \mathbb{E}^{\mathbf{H},opt}[X|\mathcal{H}_0]$.

For the proof of this result and many remarks, the reader may look at the survey article by Schweizer (2001).

4.2 Comparing the optimal MVH-strategies

4.2.1 The martingale case under both F and G

Firstly we assume that the price process S is a P-martingale with respect to both $\mathbf{F}$ and $\mathbf{G}$. Given an instant $t \in [0,T)$ and a contingent claim $X \in L^2(P, \mathcal{F}_t)$ we compare the strategies and the risk functions of the informed and the ordinary agent. This means that we are considering a MVH-problem for the ordinary agent and the insider *until time $t < T$*.

For a given $t \in [0,T)$, we will denote by $\vartheta^{MVH,\mathbf{H}}(X)$ the optimal strategy for an $\mathbf{H}$-investor to hedge the claim X. Moreover, we fix two initial investments for the agents, $c \in \mathbb{R}$ for the ordinary one and $g \in L^2(\mathcal{G}_0) = L^2(G)$ for the informed one. It is important to point out that in this case the information drift μ^G vanishes.

The next technical result states a relation between the insider optimal hedging strategies $\vartheta^{MVH,\mathbf{G}}(X)$ under P and the integrand $\widetilde{\vartheta}^{X/\widetilde{Z}_t,\mathbf{G}}$ in the GKW-decomposition of the claim $X/\widetilde{Z}_t$ under the corresponding MPM $\widetilde{P}$.

Lemma 15 *Assume that $P \in \mathcal{M}_2^e(\mathbf{G})$ and let $X \in L^2(P, \mathcal{F}_t)$ for a given $t \in [0,T)$. Then*

$$\vartheta^{MVH,\mathbf{G}}(X) = \widetilde{Z}_- \widetilde{\vartheta}^{X/\widetilde{Z}_t,\mathbf{G}}$$

and

$$J^{\mathbf{G}}(g, X) = \mathbb{E}[\mathbb{E}[X|\mathcal{G}_0] - g]^2 + \mathbb{E}\left[\int_0^t \widetilde{Z}_{s-} dL_s^{\mathbf{G},\widetilde{X}} + \int_0^t V_{s-}^{\mathbf{G}} dN_s \right]^2,$$

where $V_s^{\mathbf{G}} := \mathbb{E}[X|\mathbf{G}_s]$, $\widetilde{\vartheta}^{X/\widetilde{Z}_t,\mathbf{G}}$ *is the integrand with respect to* S *in the GKW-decomposition of* $X/\widetilde{Z}_t$ *under* $(\widetilde{P},\mathbf{G})$, $L^{\mathbf{G},\widetilde{X}}$ *is a* $(P,\mathbf{G})$*-martingale strongly orthogonal to* S, *and* N *as in Theorem 5.*

Proof. We start by considering the $(P,\mathbf{G})$-martingale $V_s^{\mathbf{G}} := \mathbb{E}[X|\mathcal{G}_s]$, $s \in [0,t]$. Since $\widetilde{V}^{\mathbf{G}} := V^{\mathbf{G}}/\widetilde{Z}$ is a local $(\widetilde{P},\mathbf{G}^t)$-martingale, we can write the following GKW-decomposition

$$\widetilde{V}_s^{\mathbf{G}} = V_0^{\mathbf{G}} + \int_0^s \widetilde{\vartheta}_u^{\mathbf{G},\widetilde{X}} dS_u + \widetilde{L}_s^{\mathbf{G},\widetilde{X}}, \quad s \in [0,t], \tag{20}$$

where $\widetilde{\vartheta}^{\mathbf{G},\widetilde{X}} \in L_{loc}(S,\widetilde{P},\mathbf{G}^t)$ and $\widetilde{L}^{\mathbf{G},\widetilde{X}}$ is a $(\widetilde{P},\mathbf{G}^t)$-martingale orthogonal to S.

Integration by parts formula gives

$$dV_s^{\mathbf{G}} = d\left(\widetilde{V}^{\mathbf{G}}\widetilde{Z}\right)_s = \widetilde{Z}_{s-}d\widetilde{V}_s^{\mathbf{G}} + \widetilde{V}_{s-}^{\mathbf{G}}d\widetilde{Z}_s + \left[\widetilde{Z},\widetilde{V}^{\mathbf{G}}\right]_s.$$

By using the decomposition (20) and since, by Theorem 5, $\widetilde{Z}$ satisfies $d\widetilde{Z}_s = \widetilde{Z}_{s-}dN_s$ (in this easy case the process μ of Theorem 5 is null), where N is a local $(P,\mathbf{G}^t)$-martingale orthogonal to S, we also have

$$dV_s^{\mathbf{G}} = \widetilde{Z}_{s-}\widetilde{\vartheta}_s^{\mathbf{G},\widetilde{X}}dS_s + \widetilde{Z}_{s-}d\widetilde{L}_s^{\mathbf{G},\widetilde{X}} + \widetilde{V}_{s-}^{\mathbf{G}}\widetilde{Z}_{s-}dN_s + \widetilde{Z}_{s-}d\left[N,\widetilde{L}^{\mathbf{G},\widetilde{X}}\right]_s.$$

Now, we use Girsanov's Theorem to write

$$\widetilde{L}^{\mathbf{G},\widetilde{X}} = L^{\mathbf{G},\widetilde{X}} + A^{\mathbf{G},\widetilde{X}}$$

where $L^{\mathbf{G},\widetilde{X}} := \widetilde{L}^{\mathbf{G},\widetilde{X}} - \frac{1}{\widetilde{Z}_-}\langle\widetilde{L}^{\mathbf{G},\widetilde{X}},\widetilde{Z}\rangle$ is a local $(P,\mathbf{G}^t)$-martingale, orthogonal to S and $A^{\mathbf{G},\widetilde{X}} = \frac{1}{\widetilde{Z}_-}\langle\widetilde{L}^{\mathbf{G},\widetilde{X}},\widetilde{Z}\rangle$.

But since $V^{\mathbf{G}}$ is a $(P,\mathbf{G}^t)$-martingale, we must have $\widetilde{Z}_s d(A^{\mathbf{G},\widetilde{X}} + [N,\widetilde{L}^{\mathbf{G},\widetilde{X}}])_s = 0$ and so

$$dV_s^{\mathbf{G}} = \widetilde{Z}_{s-}\widetilde{\vartheta}_s^{\mathbf{G},\widetilde{X}}dS_s + \widetilde{Z}_{s-}dL_s^{\mathbf{G},\widetilde{X}} + \widetilde{V}_{s-}^{\mathbf{G}}\widetilde{Z}_{s-}dN_s.$$

This concludes the proof of the lemma. □

Finally, the next proposition gives a complete answer to the comparison problem in the martingale case.

Proposition 16 *Assume that* $P \in \mathcal{M}_2^e(\mathbf{G})$.

1. *If* $X \in L^2(P,\mathcal{F}_t)$, *then*

$$\vartheta_s^{MVH,\mathbf{G}} = \vartheta_s^{MVH,\mathbf{F}}, \quad s \in [0,t].$$

2. *The risk functions of both agents satisfy*

$$J^{\mathbf{F}}(X) - J^{\mathbf{G}}(X) = \mathbb{E}\left[\mathbb{E}[X] - \mathbb{E}[X|\mathcal{G}_0]\right]^2.$$

Proof.

1. To the random variable $X \in L^2(\mathcal{F}_t)$ we associate the $(P, \mathbf{F}^t)$-martingale $V_s := V_s^{\mathbf{F}} := \mathbb{E}[X|\mathcal{F}_s]$, for which the GKW-decomposition holds:

$$V_s = V_0 + \int_0^s \vartheta_u^{\mathbf{F},X} dS_u + L_s^{\mathbf{F},X} \quad s \in [0,t] \tag{21}$$

where $\vartheta^{\mathbf{F},X} \in \Theta^{\mathbf{F}}$ and $L^{\mathbf{F},X}$ is a $(P, \mathbf{F}^t)$-martingale, strongly orthogonal to S for $(P, \mathbf{F}^t)$. Moreover, $Y_s := V_s p_s^G$ is a $(\widetilde{P}, \mathbf{G}^t)$-local martingale and its GKW-decomposition under $(\widetilde{P}, \mathbf{G}^t)$ is given by

$$Y_s = Y_0 + \int_0^s \widetilde{\vartheta}_u^{\mathbf{G},Y} dS_u + \widetilde{L}_s^{\mathbf{G},Y} \quad s \in [0,t]. \tag{22}$$

By (6) the process p_s^G satisfies

$$p_s^G = 1 + \int_0^s p_{u-}^G dN_u^G$$

and by the integration by parts formula applied to Y_s, we obtain

$$\begin{aligned} Y_s &= V_s p_s^G = Y_0 + \int_0^s p_{u-}^G \vartheta_u^{X,\mathbf{F}} dS_u + \int_0^s p_{u-}^G dL_u^{X,\mathbf{F}} \\ &\quad + \int_0^s V_{u-} p_{u-}^G dN_u^G + \left[V, p^G\right]_s . \end{aligned}$$

Since Y is a $(\widetilde{P}, \mathbf{G}^t)$-local martingale, the finite variation part in the above decomposition vanishes and then

$$\begin{aligned} Y_s &= V_s p_s^G = Y_0 + \int_0^s p_{u-}^G \vartheta_u^{X,\mathbf{F}} dS_u + \int_0^s p_{u-}^G dL_u^{X,\mathbf{F}} \\ &\quad + \int_0^s V_{u-} p_{u-}^G dN_u^G. \end{aligned} \tag{23}$$

If we compare this orthogonal decomposition with (22), we obtain that

$$\widetilde{\vartheta}_s^{Y,\mathbf{G}} = p_{s-}^G \vartheta_s^{X,\mathbf{F}}.$$

We finally apply Lemma (15) and we have

$$\begin{aligned} \vartheta_s^{MVH,\mathbf{G}}(X) &= \widetilde{Z}_{s-} \widetilde{\vartheta}_s^{X/\widetilde{Z}_t,\mathbf{G}} \\ &= \widetilde{Z}_{s-} \widetilde{\vartheta}_s^{Y_t,\mathbf{G}} \\ &= \widetilde{Z}_{s-} p_{s-}^G \vartheta_s^{X,\mathbf{F}} \\ &= \vartheta_s^{X,\mathbf{F}}. \end{aligned}$$

2. From the GKW-decompositions of X under $\mathbf{F}$ and $\mathbf{G}$, one can deduce

$$\begin{aligned} L_t^{\mathbf{F},X} &= X - \mathbb{E}[X] - \int_0^t \vartheta_s^{\mathbf{F},X} dS_s \\ &= (\mathbb{E}[X|\mathcal{G}_0] - \mathbb{E}[X]) + X - \mathbb{E}[X] - \int_0^t \vartheta_s^{\mathbf{G},X} dS_s \\ &\quad + \int_0^t \left(\vartheta_s^{\mathbf{G},X} - \vartheta_s^{\mathbf{F},X}\right) dS_s \\ &= (\mathbb{E}[X|\mathcal{G}_0] - \mathbb{E}[X]) + \int_0^t \left(\vartheta_s^{\mathbf{G},X} - \vartheta_s^{\mathbf{F},X}\right) dS_s + L_t^{\mathbf{G},X}. \end{aligned}$$

By item 1 of this proposition, we have

$$\mathbb{E}\left[L_t^{\mathbf{F},X}\right]^2 = \mathbb{E}[\mathbb{E}[X|\mathcal{G}_0] - \mathbb{E}[X]]^2 + \mathbb{E}\left[L_t^{\mathbf{G},X}\right]^2,$$

that is

$$J^{\mathbf{F}}(X) = J^{\mathbf{G}}(X) + \mathbb{E}[\mathbb{E}[X|\mathcal{G}_0] - \mathbb{E}[X]]^2.$$

The proof is now complete. □

Remark 17 *If* both *investors are allowed to minimize only over all pairs* $(c, \vartheta) \in \mathbb{R} \times \Theta^{\mathbf{H}}$ *(*$\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$*), then the risk functions are equal, i.e.* $J^{\mathbf{F}}(X) = J^{\mathbf{G}}(X)$*. Indeed, by (13) and since*

$$L_t^{\mathbf{H},X} = X - \mathbb{E}[X|\mathcal{H}_0] - \int_0^t \vartheta_s^{\mathbf{H},X} dS_s,$$

we have

$$\begin{aligned} J^{\mathbf{F}}(c, X) &= \mathbb{E}\left[\mathbb{E}[X] - c\right]^2 + \mathbb{E}\left[\mathbb{E}[X|\mathcal{G}_0] - X\right]^2 + \mathbb{E}\left[\int_0^t \left(\vartheta_s^{\mathbf{G},X} - \vartheta_s^{\mathbf{F},X}\right)^2 d\langle S\rangle_s\right] \\ &= J^{\mathbf{G}}(c', X) + \mathbb{E}\left[\mathbb{E}[X] - c\right]^2 + \mathbb{E}\left[\mathbb{E}[X|\mathcal{G}_0] - X\right]^2 - \mathbb{E}\left[\mathbb{E}[X|\mathcal{G}_0] - c'\right]^2 \\ &= J^{\mathbf{G}}(c', X) + \mathbb{E}[X - c]^2 - \mathbb{E}[X - c']^2, \end{aligned}$$

where c *and* c' *are two given initial real investment for, respectively, the ordinary and the informed agent. By setting* $c = c' = \mathbb{E}[X]$*, which is in this case the approximation price for both investors, we have the claimed equality* $J^{\mathbf{F}}(X) = J^{\mathbf{G}}(X)$*.*

4.2.2 The semimartingale case

For the general case, that is S is a continuous $(P, \mathbf{F})$-semimartingale, the Rheinländer-Schweizer feedback representation (18) of the optimal MVH-strategies suggests to compare

- the "optimal strategies" $\vartheta^{opt,\mathbf{F}} := \vartheta^{X,P^{opt,\mathbf{F}}}$ and $\vartheta^{opt,\mathbf{G}} := \vartheta^{X,P^{opt,\mathbf{G}}}$ of the ordinary agent and the insider under their own VOMMs $P^{opt,\mathbf{F}}$ and $P^{opt,\mathbf{G}}$, and

- the ratios $\zeta^{opt,\mathbf{F}}/Z^{opt,\mathbf{F}}$ and $\zeta^{opt,\mathbf{G}}/Z^{opt,\mathbf{G}}$ in the Rheinländer-Schweizer backward representation (18).

We assume that both agents start with the same initial investment $c \in \mathbb{R}$. We begin by the first item and, to do this, we will use the results of the previous subsection. Before this, we need some more results on the VOMM $P^{opt,\mathbf{H}}$ ($\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$), for which our main reference remains the paper by Delbaen and Schachermayer (1997).

Let $K_0^{\mathbf{H}}$ denote the subspace of $L^\infty(P)$ spanned by the "simple" stochastic integrals of the form

$$f = \phi' \left(S_{\tau_2} - S_{\tau_1}\right)$$

where $\tau_1 \leqslant \tau_2$ are stopping-times (with respect to the filtration $\mathbf{H}$) such that the stopped process S^{τ_2} is bounded and ϕ is a bounded $\mathbb{R}^d$-valued $\mathcal{H}_{\tau_1}$-measurable function. In this paper, S is assumed to be a continuous semimartingale under both $\mathbf{F}$ and $\mathbf{G}$ and so a probability measure Q on $\mathcal{F}$ is a local $\mathbf{H}$-martingale measure for S iff Q vanishes on K_0.

Moreover, by $\widehat{K}^{\mathbf{H}}$ we denote the closure of the span of $K_0^{\mathbf{H}}$ and the constants in $L^2(P)$:

$$\widehat{K}^{\mathbf{H}} := \overline{\text{span}\left(K_0^{\mathbf{H}}, 1\right)}.$$

By Delbaen-Schachermayer (1997) (Lemma 2.1) and our standing assumption (2), we know that $P^{opt,\mathbf{H}}$ is the unique element of $\widehat{K}^{\mathbf{H}}$ vanishing on $\widehat{K}_0^{\mathbf{H}}$ and equaling 1 on the constant function 1. (Here we have identified any measure Q with the linear functional $\mathbb{E}_Q[\cdot]$ and linear functionals on $L^2(P)$ with elements of $L^2(P)$)

Now, since $\widehat{K}^{\mathbf{F}} \subseteq \widehat{K}^{\mathbf{G}}$, it is easy to see, by a standard Hilbert space argument, that $P^{opt,\mathbf{F}}$ is just the projection of $P^{opt,\mathbf{G}}$ into $\widehat{K}^{\mathbf{F}}$.

Indeed, denote by f this projection, i.e. $f := \pi(P^{opt,\mathbf{G}}, \widehat{K}^{\mathbf{F}})$. Then, we have $\mathbb{E}[fg] = \mathbb{E}^{opt,\mathbf{G}}[g] = 0$ for all $g \in \widehat{K}_0^{\mathbf{G}}$ and, since $1 \in \widehat{K}^{\mathbf{G}}$, $\mathbb{E}[f] = \mathbb{E}[f1] = \mathbb{E}^{opt,\mathbf{G}}[1] = 1$. By the previously mentioned Lemma 2.1 in Delbaen-Schachermayer (1997), we conclude that $f = P^{opt,\mathbf{F}}$. Furthermore this property of the VOMM does not depend on the structure of the filtration $\mathbf{G}$.

A first consequence of this remark is that, for the ordinary agent, solving the MVH-problem under either $P^{opt,\mathbf{F}}$ or $P^{opt,\mathbf{G}}$ leads to the same optimal strategy, i.e. $\vartheta^{\mathbf{F},P^{opt,\mathbf{F}}} = \vartheta^{\mathbf{F},P^{opt,\mathbf{G}}}$.

Finally, since $P^{opt,\mathbf{G}}$ is a local martingale measure for S under both $\mathbf{F}$ and $\mathbf{G}$, item 2. of Proposition 16 applies and provides the equality between $\vartheta^{opt,\mathbf{G}}$ and $\vartheta^{opt,\mathbf{F}}$. We have so proved the following:

Proposition 18 *If $X \in L^2(P, \mathcal{F}_t)$ for some $t \in [0, T)$, then for all $s \leq t$*

$$\vartheta_s^{opt,\mathbf{G}} = \vartheta_s^{opt,\mathbf{F}}. \tag{24}$$

Comparing now the VOMM ratios in our general framework is a quite difficult problem. We are able to give an answer by considering some particular insider's information in some particular incomplete model. In fact, in the next subsection, we will see that in a given stochastic volatility model (including Hull and White, Heston and Stein and Stein models) if the additional r.v. G is measurable with respect to the filtration generated by the volatility process, then the two VOMM ratios coincide. This result will allow us to obtain a feedback representation for the difference process between the two optimal strategies $\vartheta^{MVH,\mathbf{F}}$ and $\vartheta^{MVH,\mathbf{G}}$.

4.3 Stochastic volatility models

We consider the following stochastic volatility model for a discounted price process S:

$$dS_t = \sigma(t, S_t, Y_t)S_t[\lambda(t, S_t, Y_t)dt + dW_t^1] \tag{25}$$

where W^1 is a brownian motion and Y is assumed to satisfy the following SDE

$$dY_t = \alpha(t, S_t, Y_t)dt + \gamma(t, S_t, Y_t)dW_t^2 \tag{26}$$

with W^2 another brownian motion independent from the first one. The coefficients are assumed to satisfy the usual hypotheses ensuring the existence of a unique strong solution and of an equivalent local martingale measure with square integrable Radon-Nikodym density. Furthermore, we assume that the underlying filtration $\mathbf{F} = (\mathcal{F}_t)$ is that generated by the two driving brownian motions, i.e. $\mathcal{F}_t = \sigma(W_s^1, W_s^2 : s \leqslant t)$ for all $t \in [0, T]$, and that λ does not depend on the process S, that is $\lambda(t, S_t, Y_t) = \lambda(t, Y_t)$. We point out that this assumption is satisfied by the Hull and White, Heston and Stein and Stein models (e.g. see Hobson (1998b)).

We will denote by $\mathbf{F}^1 = (\mathcal{F}_t^1)$ (resp. $\mathbf{F}^2 = (\mathcal{F}_t^2)$) the filtration generated by W^1 (resp. W^2).

We assume that the additional random variable G is $\mathcal{F}_T^2$-measurable, e.g. $G = W_T^2$, $G = \mathbf{1}_{(W_T^2 \in [a,b])}$ with $a < b < \infty$ or $G = Y_T$ when Y and W^2 generate the same filtration (for example, in the Hull and White model).

In this case, the VOMM is the same for the ordinary and the informed agent. Indeed, by Biagini et al. (2000) (Theorem 1.16), we have for $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$,

$$\frac{dP^{\mathbf{H},opt}}{dP} = \frac{\mathcal{E}\left(-\int_0^{\cdot} \beta_t^{\mathbf{H}} dS_t\right)_T}{\mathbb{E}\left[\mathcal{E}\left(-\int_0^{\cdot} \beta_t^{\mathbf{H}} dS_t\right)_T\right]} \tag{27}$$

with $\beta_t^{\mathbf{H}} = \frac{\lambda(t,Y_t) - h_t^{\mathbf{H}}}{\sigma(t,S_t,Y_t)S_t}$. So, we focus on the process $h^{\mathbf{H}}$. Now, by assumption the process λ does not depend on S and then, again by Biagini et al. (2002) (Section 2), $h^{\mathbf{F}} = 0$.

Moreover, being G $\mathcal{F}_T^2$-measurable and since W^1 and W^2 are independent, the dynamics of S does not change if we pass from $\mathbf{F}$ to $\mathbf{G}$. Indeed, since in this case assumption (1) is equivalent to assume $P(G \in \cdot\,|\mathcal{F}_t^2) \sim P(G \in \cdot\,)$ for all $t \in [0, T)$, it is easy to see that the conditional density process $(p_s^G)_{s \in [0,T)}$ can be chosen $\mathbf{F}_2^0$-optional, where $\mathbf{F}_2^0 := (\mathcal{F}_t^2)_{t \in [0,T)}$. The equality

$$d\left\langle p^G, \int \sigma(u, S_u, Y_u)S_u dW_u^1 \right\rangle_t = \sigma(t, S_t, Y_t)S_t d\left\langle p^G, W^1\right\rangle_t = 0, \quad t \in [0, T),$$

implies, thanks to Theorem 4, $\mu^G \equiv 0$.

So, always by Biagini et al. (2002) (Section 2), $h^{\mathbf{G}} = 0$. This implies $\beta^{\mathbf{F}} = \beta^{\mathbf{G}}$ and then $P^{\mathbf{F},opt} = P^{\mathbf{G},opt} =: P^{opt}$.

Proposition 19 *Let G be $\mathcal{F}_T^2$-measurable, $X \in L^2(P, \mathcal{F}_t)$ with $t < T$, $c \in \mathbb{R}$ and $g \in L^2(\mathcal{G}_0)$ two given initial investments for, respectively, the ordinary agent and the insider. Then*

$$\vartheta_s^{MVH,\mathbf{G}} = \vartheta_s^{MVH,\mathbf{F}} + \xi_s^{MVH}, \quad s \in [0,t], \tag{28}$$

where the process ξ^{MVH} has the following backward representation:

$$\begin{aligned} \xi_s^{MVH} &= \rho_s^{opt} \left(V_{s-}^{opt,\mathbf{G}} - V_{s-}^{opt,\mathbf{F}} + \int_0^s \xi_u^{MVH} dS_u \right) & (29) \\ &= \zeta_t^{\mathbf{F},opt} \left(\frac{V_0^{\mathbf{G},opt} - g - \left(V_0^{\mathbf{F},opt} - c \right)}{Z_0^{\mathbf{F},opt}} + \int_0^{t-} \frac{1}{Z_s^{\mathbf{F},opt}} dV_s^{\mathbf{G},opt} \right), & (30) \end{aligned}$$

for all $s \in [0,t]$, where $V_s^{opt,\mathbf{H}} := \mathbb{E}^{opt}[X|\mathcal{H}_s]$ for $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$ and $\rho_s^{opt} := \zeta_s^{opt,\mathbf{F}} / Z_s^{opt,\mathbf{F}} = \zeta_s^{opt,\mathbf{G}} / Z_s^{opt,\mathbf{G}}$, $s \in [0,t]$.

Proof. Since $P^{opt,\mathbf{F}} = P^{opt,\mathbf{G}} = P^{opt}$, it is easy to remark that by isometry $\zeta_s^{opt,\mathbf{F}} = \zeta_s^{opt,\mathbf{G}}$ and so $\zeta_s^{opt,\mathbf{F}} / Z_s^{opt,\mathbf{F}} = \zeta_s^{opt,\mathbf{G}} / Z_s^{opt,\mathbf{G}} =: \rho_s^{opt}$ for $s \leqslant t$. Indeed, since by localization we can assume that S is a true martingale under P^{opt}, it suffices to note that $Z_T^{opt,\mathbf{F}} = Z_T^{opt,\mathbf{G}}$ implies $\int_0^T \zeta_s^{opt,\mathbf{F}} dS_s = \int_0^T \zeta_s^{opt,\mathbf{G}} dS_s$ and so, by isometry, we have

$$\mathbb{E}^{opt} \left[\int_0^T \left(\zeta_s^{opt,\mathbf{F}} - \zeta_s^{opt,\mathbf{G}} \right)^2 d \langle S \rangle_s \right] = 0.$$

Then, by Proposition 16, the optimal strategies of the two agents under the VOMM are equal, i.e. $\vartheta^{\mathbf{F},opt} = \vartheta^{\mathbf{G},opt}$. Finally, by comparing the backward representations (18) of the two optimal hedging strategies $\vartheta^{MVH,\mathbf{F}}$ and $\vartheta^{MVH,\mathbf{G}}$, we have the representation (29) of the difference process ξ^{MVH}.

For the representation (30), we have to compare the characterizations provided by (19) for $\mathbf{H} \in \{\mathbf{F}, \mathbf{G}\}$. By doing this, we obtain for all $s \leqslant t$

$$\xi_s^{MVH} = \zeta_t^{\mathbf{F},opt} \left(\frac{V_0^{\mathbf{G},opt} - g - \left(V_0^{\mathbf{G},opt} - c \right)}{Z_0^{\mathbf{F},opt}} + \int_0^{t-} \frac{1}{Z_s^{\mathbf{F},opt}} \left(dL_s^{\mathbf{G},opt} - dL_s^{\mathbf{F},opt} \right) \right). \tag{31}$$

It remains to study the last stochastic differential appearing in (31). From the GKW-decomposition of X under $(P^{\mathbf{F},opt}, \mathbf{F})$ and $(P^{\mathbf{G},opt}, \mathbf{G})$ and by Proposition 18 we deduce that

$$L_t^{\mathbf{F},opt} = L_t^{\mathbf{G},opt} + \left(\mathbb{E}^{opt}[X|\mathcal{G}_0] - \mathbb{E}^{opt}[X] \right).$$

Thus, for every $s \leqslant t$,

$$L_s^{\mathbf{G},opt} = \mathbb{E} \left[L_t^{\mathbf{F},opt} | \mathcal{G}_s \right] - \left(\mathbb{E}^{opt}[X|\mathcal{G}_0] - \mathbb{E}^{opt}[X] \right).$$

On the other hand we have that, for every $s \leqslant t$,

$$\begin{aligned}\mathbb{E}\left[L_t^{\mathbf{F},opt}|\mathcal{G}_s\right] &= \mathbb{E}^{opt}\left[X - \mathbb{E}^{opt}[X] - \int_0^t \vartheta_s^{\mathbf{F},opt} dS_s|\mathcal{G}_s\right] \\ &= V_s^{\mathbf{G},opt} - \mathbb{E}^{opt}[X] - \int_0^s \vartheta_u^{\mathbf{F},opt} dS_u,\end{aligned}$$

since, being $\vartheta^{\mathbf{F},opt} \in \Theta^{\mathbf{F}} \subseteq \Theta^{\mathbf{G}}$, $\int \vartheta^{\mathbf{F},opt} dS$ is a $(Q, \mathbf{G})$-martingale for all $Q \in \mathcal{M}_2^e(\mathbf{G})$ and so even for P^{opt}. Thus, for all $s \leqslant t$,

$$\mathbb{E}\left[L_t^{\mathbf{F},opt}|\mathcal{G}_s\right] = V_s^{\mathbf{G},opt} - \left(X - L_s^{\mathbf{F},opt}\right),$$

and so

$$L_s^{\mathbf{G},opt} = \left(V_s^{\mathbf{G},opt} - X\right) + \left(\mathbb{E}^{opt}[X] - V_0^{\mathbf{G},opt}\right) + L_s^{\mathbf{F},opt},$$

which implies $dL_s^{\mathbf{G},opt} = dL_s^{\mathbf{F},opt} + dV_s^{\mathbf{G},opt}$, $s \leqslant t$. We now combine this equality with formula (31) and obtain the representation (30). □

Remark 20 *The two characterizations of the difference optimal process ξ^{MVH} provided by the previous proposition imply that, in this particular setting, if the ordinary agent can observe the dynamics of the insider approximation price, he could fill his informational gap and reconstruct the optimal hedging strategy of the insider.*

5 Conclusions

This paper represents a first attempt to analyze the sensitiveness of the hedging strategies with respect to a change of the information flow. We have studied this problem for the locally risk minimization and the mean-variance hedging separately. We have shown in particular that if both agents use the first approach and the additional information of the insider satisfies a certain property, namely the orthogonal part in the stochastic exponential representation of its conditional density process vanishes, their hedging strategies coincide and the cost processes of the ordinary investor is just the projection on his filtration $\mathbf{F}$ of the insider cost process.

On the other hand, the asymmetry of information in the MVH approach is much more delicate to investigate. Motivated by the feedback characterization of the optimal strategies yielded by Rheinländer and Schweizer (1997), we have shown that the integrands in the GKW-decomposition of a claim X under the respective VOMMs of the two agents are equal. Finally, we have obtained a feedback representation for the difference between the hedging strategies in a rather general stochastic volatility model where the additional r.v. G is measurable with respect to the filtration generated by the volatility process.

The problem of comparing the hedging strategies of the two investors in the semimartingale case and for all r.v. G satisfying assumption (1) remains open in the LRM as well as in the MVH approach.

Moreover, a natural development of this study would be to investigate the hedging problem in a financial market with an insider possessing either a weak anticipation on the future evolution of the stock price (Baudoin (2003) and Baudoin and Nguyen-Ngoc (2002)) or an additional dynamical information (as in Corcuera et al. (2002)).

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