Analysis of Asset Prices in an Equilibrium Market Model

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Extended Abstract

The aim of this paper is to introduce and analyze an equilibrium financial market model in continuous time. The market model consists of N financial assets modelled by general stochastic processes (not necessarily semimartingales) on a probability base $(\Omega, F_\infty, (F_t)_{t\geq 0}, \mathbb{P})$ and of K economic agents having time dependent wealth preferences given by the set of utility functions $\{U^k_{t,T}\}_{0\leq t\leq T<\infty, k=1,..,K}$ and some initial endowments. We discuss the choice of the numéraire in the portfolio selection problem for the economic agents and the importance of the capital market numéraire. We introduce the concept of non-degenerate financial market and show that a non-degenerate market is in equilibrium (i.e. there exists a set of optimal portfolios for the K agents satisfying the market clearing condition for each of the N assets) if and only if the asset prices expressed in the capital market numéraire are regular càdlàg F_t-martingales with respect to the historical probability measure $\mathbb{P}$ taking values in the (n-1)-dimensional simplex $\{x \in [0,1]^N : \sum_{i=1}^N x_i = 1\}$ of $\mathbb{R}^N$. The change of numéraire and of probability measures technique is used to show in an easy way that prices of assets and self-financing portfolios expressed in any numeraire/currency are càdlàg semimartingales. The market price of risk process, self-financing portfolios, the existence of a risk neutral measure $\mathbb{P}^*$ as well as the completeness of the financial market are analyzed. Pricing and hedging of contingent claims under the historical measure $\mathbb{P}$ as well as under the risk neutral measure $\mathbb{P}^*$ are discussed. A connection to the CAPM and APT models in continuous time is established. We also give a complete characterization of equilibrium markets models with continuous default-free asset prices as multidimensional Black-Scholes models with stochastic volatilities and stochastic correlations.

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