Good Deal Bounds for Derivatives when the Underlying Asset Prices are Driven by a Marked Point Process

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Abstract:
The paper addresses the problem of pricing financial derivatives in continuous time incomplete markets, when underlying asset prices are allowed to be driven by a general marked point process as well as by a multidimensional Wiener process. Following Cochrane and Saa-Requejo (1998) (henceforth CSR) we impose the economic restriction of ruling out "good deals", i.e. derivative prices with high Sharpe ratios, thus allowing us to obtain reasonably tight bounds for derivative prices. Our analysis of the problem is slightly diferent from that in Cochrane and Saa-Requejo, since we formulate the problem as a stochastic optimal control problem where the contol variables is choosen as the kernel processes in the Girsanov transformation from the objective measure to the class of risk neutral martingale measures.

Using this framework, our contributions are as follows.

- The basic pricing formula is proved to be a solution of an Hamilton-Jacobi-Bellman equation. Using our formulation of the problem as an optimal control problem involving the martingale measure, we can give a more systematic and complete derivation of the HJB equation than the one previously given by CSR. Even for the purely Wiener driven case, we can also treat a more general model class than that discussed in CSR.
- The main contribution of the paper is that we extend the CSR framework to the case of a driving marked point process. For this case the HJB equation takes the form of a non linear integro-differtial equation with an embedded variational optimization problemen.
- For the case where the point process is compound Poisson, we provide an explicit solution to the variational problem embedded in the HJB equation. We also give qualitative results on the structure of the solution.
- Several concrete examples, including the Merton jump diffusion stock price model, are studied in some detail. For these examples we provide numerical solutions of the HJB equation, and are thus able to provide explicit good deal bounds.