Shape Optimization using Grid Free Solver and Evolutionary Algorithm

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MESHLESS METHOD

- USEFUL FOR PROBLEMS INVOLVING MULTIBODIES SUCH AS CORE BODY, WINGS, FINS, WIRE TUNNELS
- USEFUL FOR PROBLEMS INVOLVING EXTREMELY LARGE MESH DEFORMATION CASTINGS, EXTRUSIONS AND MOULDINGS, PROPAGATION OF CRACKS, PROPAGATION OF INTERFACES BETWEEN SOLIDS AND LIQUIDS
- USEFUL IN ADAPTATION VIA POINT ENRICHMENT
- USEFUL WHENEVER GEOMETRIC CONFIGURATION HAS PARTS IN RELATIVE MOTION
- VERY PROMISING IN SHAPE OPTIMISATION IN GRID FREE ENVIRONMENT

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VARIOUS OPTIMIZATION AND DESIGN PROBLEMS OF INTEREST

- MAXIMIZATION OF SEPARATIVE POWER

  CONTROL VARIABLES: GAP, SHAPE OF HOLLOW TUBE, NUMBER AND PITCH OF HOLES IN THE BAFFLE

- CHANGE IN COUNTERCURRENT BY CHANGING SHAPE OF HOLLOW TUBE

- FIXING OPTIMAL LOCATION OF CONTROL SURFACES ON A FLIGHT VEHICLE

  CONTROL SURFACES CAN BE DEFLECTED (δ1, δ2, δ3, δ4) OR MOVED LONGITUDINALLY ALONG THE VEHICLE TO GET CHANGES IN PITCHING MOMENT

  IN MANY CASES

  MOMENT AVAILABLE FOR DEFLECTION $<$ AERODYNAMIC MOMENT ON FINS

  SOLUTION LIES IN SHIFTING THEM.

  WHICH IS THE OPTIMAL LOCATION?

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VARIOUS OPTIMIZATION AND DESIGN PROBLEMS OF INTEREST

- AIR FRAME - SCRAMJET ENGINE INTEGRATION FOR A HYPERSONIC CRUISE VEHICLE

- MACH NUMBER AND VELOCITY AND PRESSURE DISTRIBUTION AT INLET TO ENGINE DEPEND ON AIRFRAME AHEAD OF IT

- CAN WE CHANGE THE SHAPE OF FOREBODY TO GET DESIRED MACH NUMBER AT INLET ?

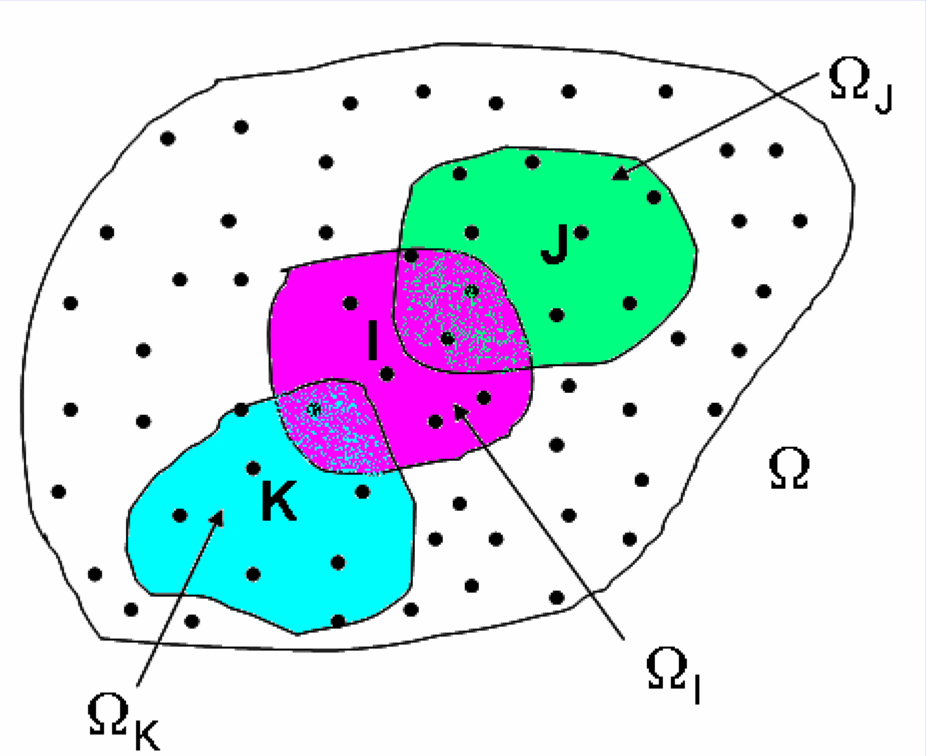
- DESIGN OF COMPRESSOR BLADE OF A TURBO MACHINERY WITH MINIMUM LOSS

MESHLESS METHOD

- SMOOTH PARTICLE HYDRODYNAMICS (SPH, LUCY 1977, MONAGHAN 1982)
- LSKUM (GHOSH & DESHPANDE 1988, RAMESH, DAUHOO, ANANDHA, PRAVEEN)
- MOVING LEAST SQUARES NAYROLES et al. (1992), BELYTSCHKO, LU AND GU (1994)
- PARTITION OF UNITY (PU) METHODS

  DUARTE & ODEN (1996), BABUSKA & MELENK (1995)

MESHLESS METHOD



Ω = DOMAIN

Ω_I , Ω_J, Ω_K ARE THREE TYPICAL SUBDOMAINS (CALLED CONNECTIVITY BY US) OF 3 NODES OR POINTS I, J, K

I = NODE INDEX , NODES ARE $\vec{X}_1, \vec{X}_2, \ldots\ldots \vec{X}_N$

THE SUBDOMAINS Ω_1, Ω_2 , ... Ω_N PROVIDE OVERLAPPING COVER FOR Ω.

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MESHLESS METHOD

THE CENTRAL PROBLEM IS TO FIND A SUITABLE APPROXIMATION TO $u(x, y)$ ON Ω GIVEN $u_1, u_2, \ldots u_N$ WHICH ARE VALUES OF u AT NODES IN Ω.

SOMETIMES INSTEAD OF u, THE INTEREST IS TO FIND GRADIENT OF u AT NODES FROM THE INPUT DATA $u_1, u_2, \ldots u_N$

WINDOW FUNCTION OR WEIGHT FUNCTION $w(x, y) \neq 0$ OVER A COMPACT SUPPORT, OUTSIDE IT $w(x,y) = 0$.

THE SUPPORT IS CALLED DOMAIN OF INFLUENCE OF NODE. IN OUR NOTATION CONNECTIVITY OF NODE I IS ITS SUPPORT.

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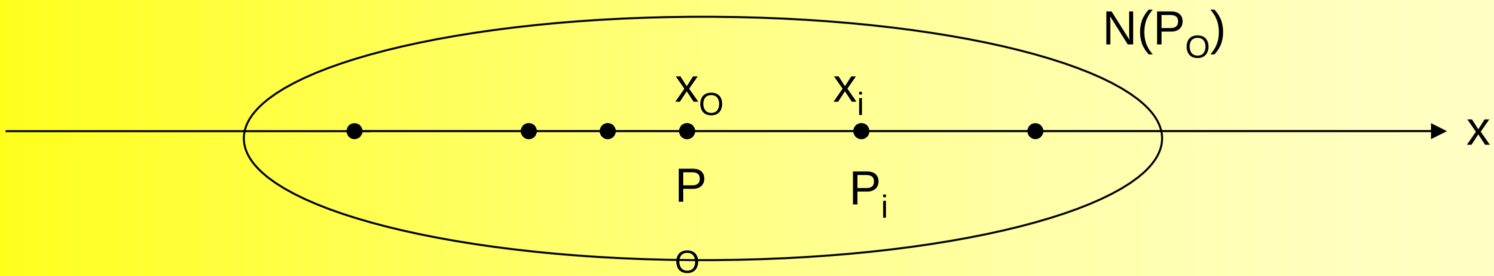
FIRST ORDER ACCURATE LS FORMULA

CONSIDER 1D CASE



POINT OF INTEREST IS P_O

$N(P_O)$ = CONNECTIVITY OF P_O OR NEIGHBOURHOOD OF P_O

THE FIRST ORDER LEAST SQUARES (LS) FORMULA FOR

DERIVATIVE u_{xo} IS OBTAINED BY MINIMISING :

$$E = \sum_i \left(\Delta u_i - u_{xo}\Delta x_i\right)^2$$

$$u_{xo}^{(1)} = \left(\frac{\partial u}{\partial x}\right)_{P_o} = \frac{\sum \Delta u_i \Delta x_i}{\sum \Delta x_i^2}$$

STANDARD NOTATION : $\Delta u_i = u_i - u_o$, $\Delta x_i = x_i - x_o$

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FIRST ORDER ACCURATE LS FORMULA

TAYLOR SERIES

$$\Delta u_i = u_{xo}\Delta x_i + \frac{\Delta x_i^2}{2} u_{xxo} + O(\Delta x_i^3)$$

$$\frac{\sum \Delta u_i \Delta x_i}{\sum \Delta x_i^2} = \frac{\sum \{u_{xo}\Delta x_i + \frac{\Delta x_i^2}{2} u_{xxo} + O(\Delta x_i^3)\}\Delta x_i}{\sum \Delta x_i^2}$$

$$= u_{xo} + \left(\frac{\sum \frac{\Delta x_i^3}{2}}{\sum \Delta x_i^2} \right) u_{xxo} + O(\Delta x_m^2)$$

$$= u_{xo} + O(\Delta x_m)$$

$$\Delta x_m = \underset{i}{Max}\{|\Delta x_i|\}$$

EVIDENTLY $u_{xo}^{(1)}$ IS FIRST ORDER ACCURATE

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DEFECT CORRECTION & HIGHER ORDER ACCURACY

START WITH

$$\frac{\sum \Delta u_i \Delta x_i}{\sum \Delta x_i^2} = u_{xo} + \underbrace{\left(\frac{\sum \frac{\Delta x_i^3}{2}}{\sum \Delta x_i^2} \right)} u_{xxo} + O(\Delta x_m^2)$$

TO INCREASE ORDER OF ACCURACY WE MUST CANCEL

SECOND TERM ON RHS

$$u_{xo} = \frac{\sum \Delta u_i \Delta x_i}{\sum \Delta x_i^2} - \frac{\sum \frac{\Delta x_i^3}{2}}{\sum \Delta x_i^2} u_{xxo}$$

$$= \frac{\sum \Delta x_i \left\{ \Delta u_i - \frac{\Delta x_i^2}{2} u_{xxo} \right\}}{\sum \Delta x_i^2}$$

REPLACE

$$\tfrac{1}{2} \Delta x_i^2 u_{xxo} = \tfrac{1}{2} \Delta x_i (u_{xi} - u_{xo}) = \tfrac{1}{2} \Delta x_i \Delta u_{xi}$$

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DEFECT CORRECTION & HIGHER ORDER ACCURACY

$$u_{xo} = \frac{\sum \Delta x_i \left\{ \Delta u_i - \frac{\Delta x_i}{2} (u_{xi} - u_{xo} \right\}}{\sum \Delta x_i^2}$$

WE GET LS FORMULA FOR u_{xo} WHICH HAS IMPLICT DEPENDENCE, i. e. TO DETERMINE u_x AT P_O WE REQUIRE VALUES OF DERIVATIVES u_{xi} AT OTHER POINTS P_i

$$u_{xo}^{(1)} = \frac{\sum \Delta x_i \Delta u_i}{\sum \Delta x_i^2}$$

$$u_{xo}^{(2)} = \frac{\sum \Delta x_i \left\{ \Delta u_i - \frac{\Delta x_i}{2} (u_{xi}^{(2)} - u_{xo}^{(2)} \right\}}{\sum \Delta x_i^2}$$

OBVIOUSLY ITERATIONS ARE REQUIRED, THESE ARE CALLED INNER ITERATIONS.

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INNER ITERATIONS FOR IMPLEMENTING DEFECT CORRECTION

GET

$$u_{xo}^{(1)} = \frac{\sum \Delta x_i \Delta u_i}{\sum \Delta x_i^2} \qquad \text{AT ALL } P_O$$

USE THESE VALUES TO UPDATE

$$\tilde{u}_{xo} = u_{xo}^{(1)} - \frac{\sum \frac{\Delta x_i^2}{2}\left(u_{xi}^{(1)} - u_{xo}^{(1)}\right)}{\sum \Delta x_i^2}$$

AGAIN OBTAIN BETTER ESTIMATE THROUGH ONE MORE ITERATION

$$\tilde{\tilde{u}}_{xo} = u_{xo}^{(1)} - \frac{\sum \frac{\Delta x_i^2}{2}\left(\tilde{u}_{xi} - \tilde{u}_{xo}\right)}{\sum \Delta x_i^2}$$

AND SO ON.

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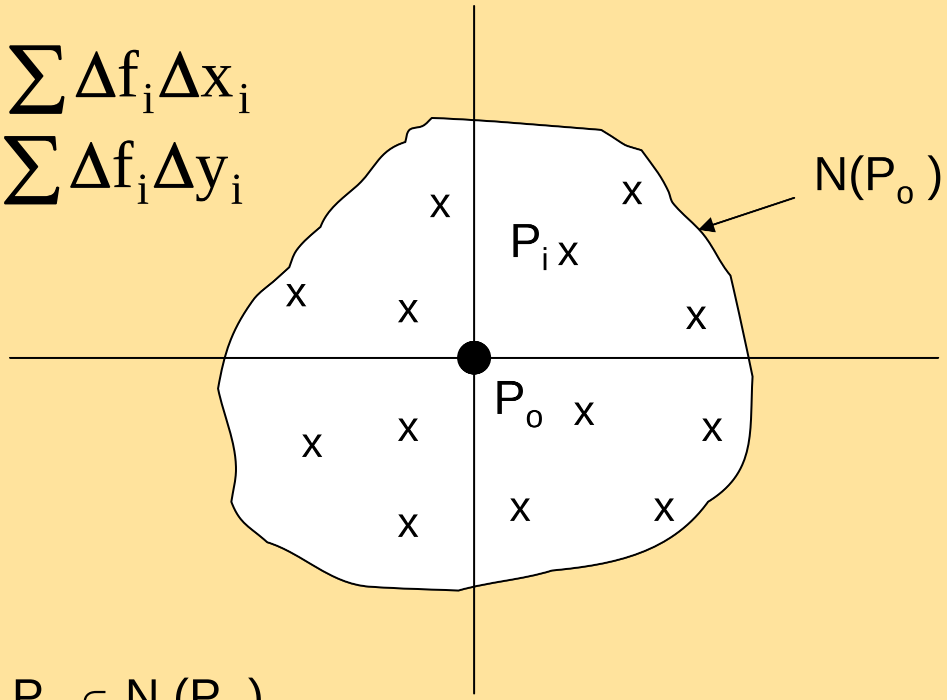
2D SECOND ORDER ACCURATE LS FORMULA FOR f_{xo} and f_{yo} IS OBTAINED BY MINIMISING:

$$E = \sum_i \left(\Delta f_i - f_{xo}\Delta x_i - f_{yo}\Delta y_i\right)^2 \quad \text{w. r. t.} \quad f_{xo} \text{ and } f_{yo}$$

$$f_{xo}\left(\sum \Delta x_i^2\right) + f_{yo}\left(\sum \Delta x_i \Delta y_i\right) = \sum \Delta f_i \Delta x_i$$
$$f_{xo}\left(\sum \Delta x_i \Delta y_i\right) + f_{yo}\left(\sum \Delta y_i^2\right) = \sum \Delta f_i \Delta y_i$$



SUMMATION IN Σ IS OVER ALL $P_i \in N(P_o)$

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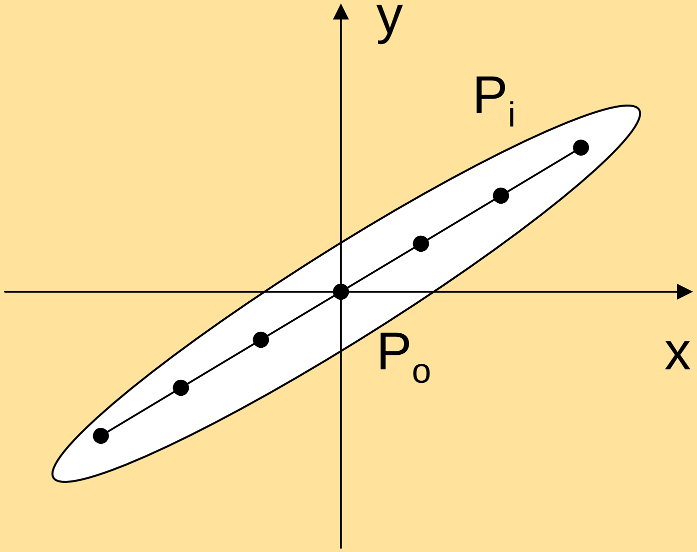
2D SECOND ORDER ACCURATE LS FORMULA

SOLVING FOR f_{xo} , f_{yo} WE GET FIRST ORDER FORMULAE FOR DERIVATIVES

$$f_{xo}^{(1)} = \frac{\left(\sum \Delta f_i \Delta x_i\right)\left(\sum \Delta y_i^2\right) - \left(\sum \Delta f_i \Delta y_i\right)\left(\sum \Delta x_i \Delta y_i\right)}{\det}$$

$$f_{yo}^{(1)} = \frac{\left(\sum \Delta f_i \Delta y_i\right)\left(\sum \Delta x_i^2\right) - \left(\sum \Delta f_i \Delta x_i\right)\left(\sum \Delta x_i \Delta y_i\right)}{\det}$$

$$\det = \left(\sum \Delta x_i^2\right)\left(\sum \Delta y_i^2\right) - \left(\sum \Delta x_i \Delta y_i\right)^2$$



det = 0 IFF ALL x_i , y_i FALL ON A STRAIGHT LINE.

$N(P_o)$ IS A THIN PENCIL WHEN ALL P_i NEARLY FALL ON A LINE.

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2D SECOND ORDER ACCURATE LS FORMULA

TO GET SECOND ORDER ACCURATE LS FORMULAE DEFINE

$$E_2 = \sum_i \{\Delta f_i - \Delta x_i f_{xo} - \Delta y_i f_{yo} - \frac{\Delta x_i^2}{2} f_{xxo} - \Delta x_i \Delta y_i f_{xyo} - \frac{\Delta y_i^2}{2} f_{yyo}\}^2$$

MINIMISE E_2 w. r. t. f_{xo} , f_{yo}

$$f_{xo}\left(\sum \Delta x_i^2\right) + f_{yo}\left(\sum \Delta x_i \Delta y_i\right) = \sum \Delta f_i \Delta x_i - f_{xxo}\left(\sum \frac{\Delta x_i^3}{2}\right) - f_{xyo}\sum \Delta x_i^2 \Delta y_i - f_{yyo}\left(\sum \frac{\Delta x_i \Delta y_i^2}{2}\right)$$

$$f_{xo}\left(\sum \Delta x_i \Delta y_i\right) + f_{yo}\left(\sum \Delta y_i^2\right) = \sum \Delta f_i \Delta y_i - f_{xxo}\left(\sum \frac{\Delta x_i^2 \Delta y_i}{2}\right) - f_{xyo}\left(\sum \Delta x_i \Delta y_i^2\right) - f_{yyo}\left(\sum \frac{\Delta y_i^3}{2}\right)$$

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2D SECOND ORDER ACCURATE LS FORMULA

LET US STUDY RHS OF ABOVE EQUATIONS

RHS OF FIRST EQUATION

$$= \sum \{\Delta f_i \Delta x_i - f_{xxo} \frac{\Delta x_i^3}{2} - f_{xyo} \Delta x_i^2 \Delta y_i - f_{yyo} \frac{\Delta x_i \Delta y_i^2}{2}\}$$

$$= \sum \Delta x_i \{\Delta f_i - \frac{\Delta x_i^2}{2} f_{xxo} - \Delta x_i \Delta y_i f_{xyo} - \frac{\Delta y_i^2}{2} f_{yyo}\}$$

$$= \sum \Delta x_i \{\Delta f_i - \frac{\Delta x_i}{2} (f_{xxo} \Delta x_i + f_{yyo} \Delta y_i)$$

$$- \frac{\Delta y_i}{2} (f_{xyo} \Delta x_i + f_{yyo} \Delta y_i)\}$$

EASY TO VERIFY THAT

$$f_{xi} - f_{xo} = \Delta x_i f_{xxo} + \Delta y_i f_{xyo} + \text{H.O.T.}$$

$$f_{yi} - f_{yo} = \Delta x_i f_{xyo} + \Delta y_i f_{yyo} + \text{H.O.T.}$$

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2D SECOND ORDER ACCURATE LS FORMULA

RHS OF FIRST EQUATION

$$= \sum \Delta x_i \{\Delta f_i - \frac{\Delta x_i}{2}(f_{xi} - f_{xo}) - \frac{\Delta y_i}{2}(f_{yi} - f_{yo})\}$$

$$= \sum \Delta x_i \Delta \tilde{f}_i$$

WHERE

$$\Delta \tilde{f}_i = \Delta f_i - \frac{\Delta x_i}{2}(f_{xi} - f_{xo}) - \frac{\Delta y_i}{2}(f_{yi} - f_{yo})$$

$$= \{f_i - \frac{\Delta x_i}{2} f_{xi} - \frac{\Delta y_i}{2} f_{yi}\} - \{f_o - \frac{\Delta x_i}{2} f_{xo} - \frac{\Delta y_i}{2} f_{yo}\}$$

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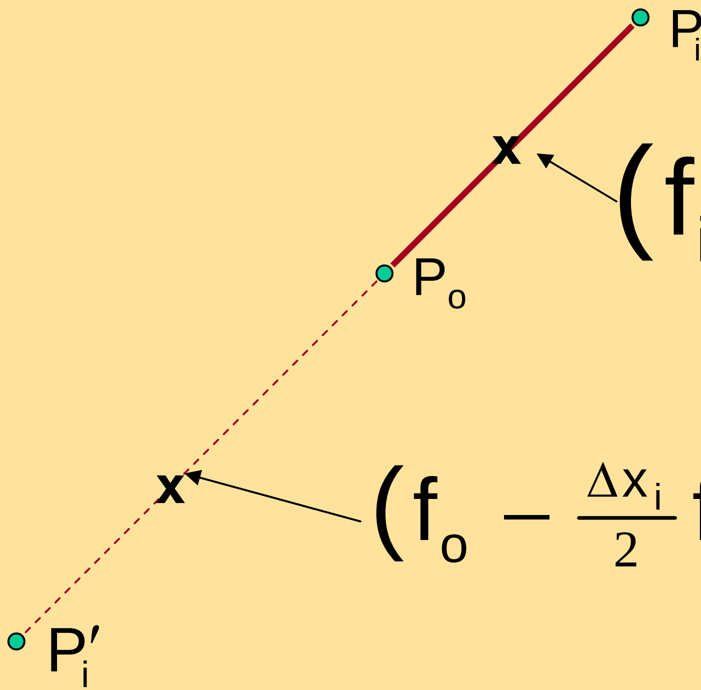
2D SECOND ORDER ACCURATE LS FORMULA



$$\left(f_i - \frac{\Delta x_i}{2} f_{xi} - \frac{\Delta y_i}{2} f_{yi}\right)$$

$$\left(f_o - \frac{\Delta x_i}{2} f_{xo} - \frac{\Delta y_i}{2} f_{yo}\right)$$

2D SECOND ORDER ACCURATE LS FORMULA

RHS OF SECOND EQUATION

$$= \sum \Delta f_i \Delta y_i - f_{xxo}\left(\sum \frac{\Delta x_i^2 \Delta y_i}{2}\right) - f_{xyo}\left(\sum \Delta x_i \Delta y_i^2\right) - f_{yyo}\left(\sum \frac{\Delta y_i^3}{2}\right)$$

$$= \sum \Delta y_i \underbrace{\left\{\Delta f_i - \frac{\Delta x_i^2}{2} f_{xxo} - f_{xyo}\Delta x_i \Delta y_i - \frac{\Delta y_i^2}{2} f_{yyo}\right\}}_{\text{SAME AS BEFORE}}$$

$$= \sum \Delta y_i \Delta \tilde{f}_i$$

THUS 2 LINEAR EQUATIONS ARE

$$\left(\sum \Delta x_i^2\right) f_{xo} + \left(\sum \Delta x_i \Delta y_i\right) f_{yo} = \sum \Delta \tilde{f}_i \Delta x_i$$

$$\left(\sum \Delta x_i \Delta y_i\right) f_{xo} + \left(\sum \Delta y_i^2\right) f_{yo} = \sum \Delta \tilde{f}_i \Delta y_i$$

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2D SECOND ORDER ACCURATE LS FORMULA

WITH DEFECT CORRECTION WE GET SIMILAR FORMULAE FOR 2nd ORDER ACCURATE DERIVATIVES

$$f_{xo}^{(2)} = \frac{\left(\sum \Delta\tilde{f}_i \Delta x_i\right)\left(\sum \Delta y_i^2\right) - \left(\sum \Delta\tilde{f}_i \Delta y_i\right)\left(\sum \Delta x_i \Delta y_i\right)}{\det}$$

$$f_{yo}^{(2)} = \frac{\left(\sum \Delta\tilde{f}_i \Delta y_i\right)\left(\sum \Delta x_i^2\right) - \left(\sum \Delta\tilde{f}_i \Delta x_i\right)\left(\sum \Delta x_i \Delta y_i\right)}{\det}$$

Δf_i IN FIRST ORDER LS FORMULAE ARE REPLACED BY $\Delta\tilde{f}_i$ IN SECOND ORDER FORMULAE

L S K U M – N S

LSKUM - CHARACTERISTIC FEATURES

- **OPERATES ON AN ARBITRARY DISTRIBUTION OR A CLOUD OF POINTS. HENCE IT CAN OPERATE ON STACKED, STRUCTURED, UNSTRUCTURED, PRISMATIC, CARTESIAN, BACKGROUND CARTESIAN OVERLAPPING WITH BODY FITTED MESH, CHIMERA MESHES, FAME MESH, ETC.**

- **REQUIRES CONNECTIVITY $N(P_0)$ FOR EACH NODE P_0**
  $N(P_0)$ = { P_j SUCH THAT P_j IS A NEIGHBOUR OF P_0 }

- **LEAST SQUARES FORMULA OPERATES ON $N(P_0)$ TO GET DISCRETE APPROXIMATION TO SPATIAL DERIVATIVES AT P_0**

LSKUM - CHARACTERISTIC FEATURES

- TIME MARCHING SCHEME COUPLED WITH LEAST SQUARES SPATIAL DISCRETISATION AND MOMENT METHOD CONCEPT GIVE LSKUM BASED STATE UPDATE FORMULA AT EACH NODE P_0

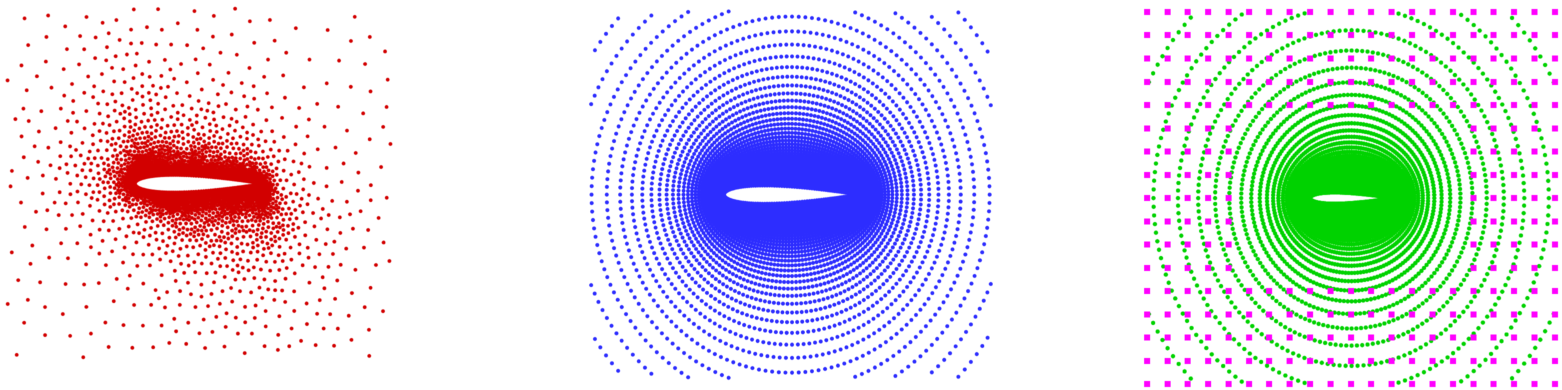
- TWO STEP DEFECT CORRECTION GIVES SECOND ORDER ACCURACY

- BOUNDARY CONDITION TREATMENT IN THE KINETIC THEORY FRAMEWORK : KCBC, KOBC, KPBC

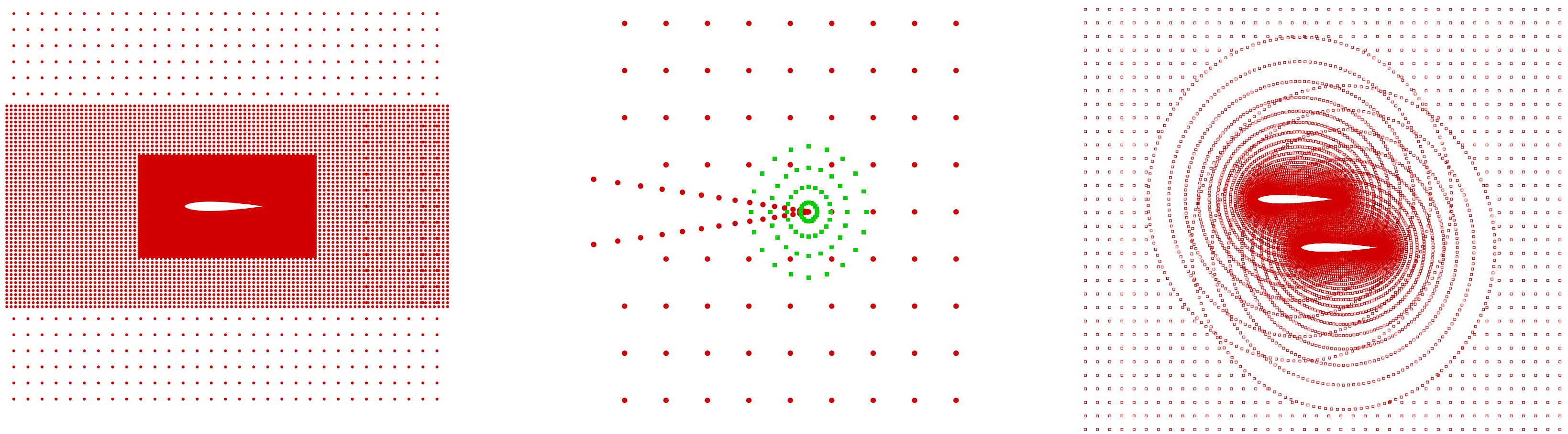
LSKUM - CHARACTERISTIC FEATURES

- WITH SMALL MODIFICATION LSKUM ON MOVING GRID CAN BE DEVELOPED, LEADS TO LSKUM-MG USEFUL IN UNSTEADY AERODYNAMICS

- USE OF ENTROPY VARIABLES (CALLED q-VARIABLES) LEADS TO q-LSKUM, ROTATED q-LSKUM, WITH ROTATION ALONG STREAMLINE CO-ORDINATE SYSTEM LEADS TO LESS DISSIPATIVE LSKUM, ROTATIONALLY INVARIANT LSKUM (KUMARI)



POINT DISTRIBUTIONS FROM DIFFERENT METHODS



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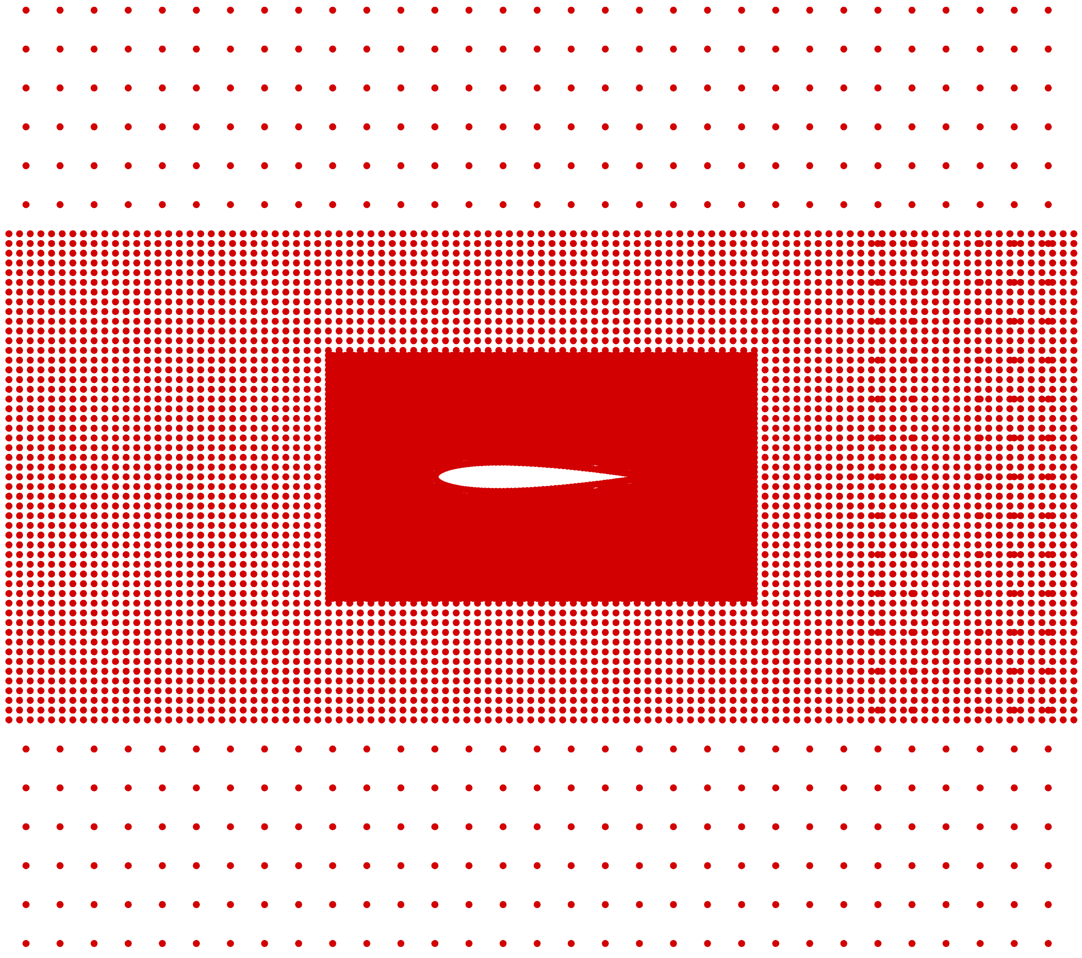

POINT DISTRIBUTION FROM EMBEDDED CARTESIAN GRIDS

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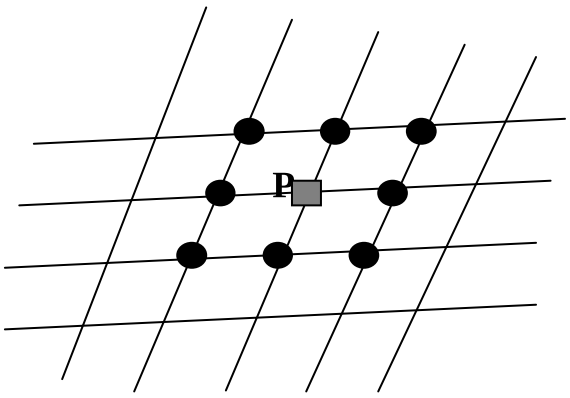
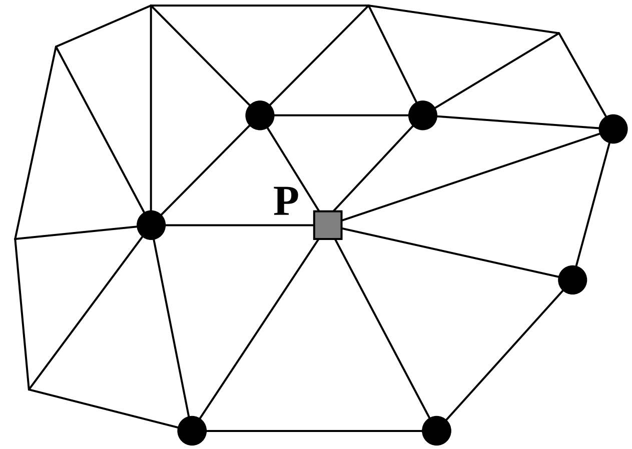
CONNECTIVITY GENERATION

SIMPLE CLOUDS

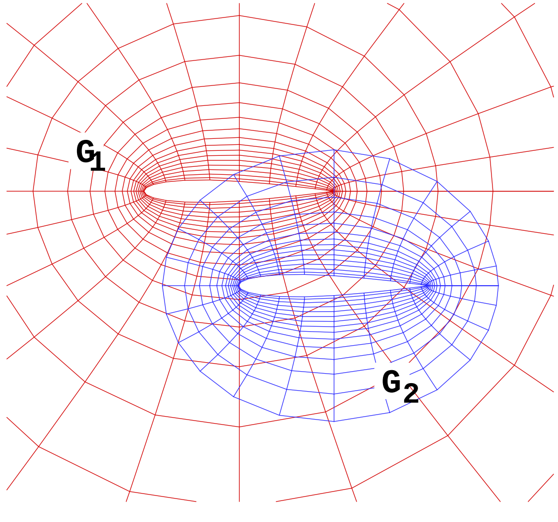
STRUCTURED MESH



UNSTRUCTURED MESH



CHIMERA CLOUDS



OVERLAPPED MESH

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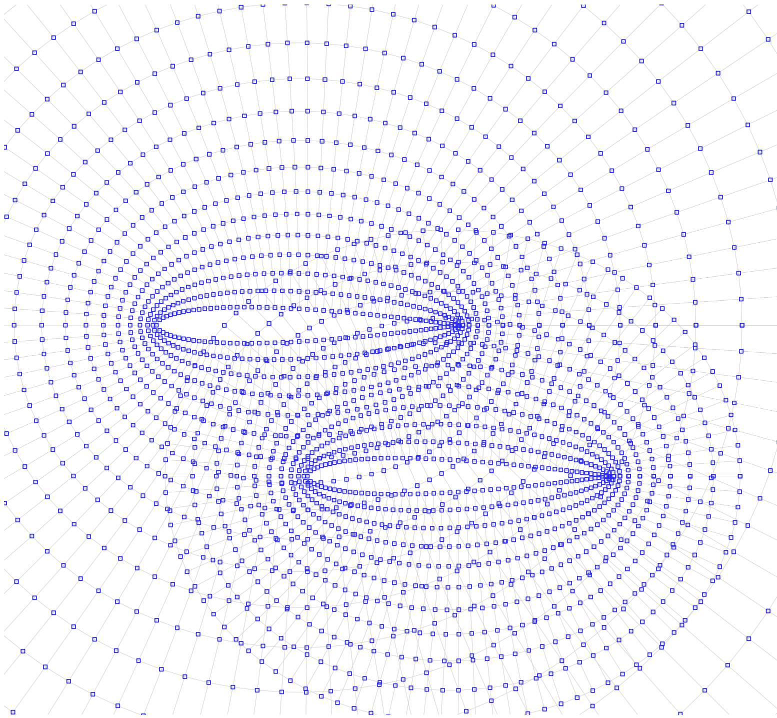
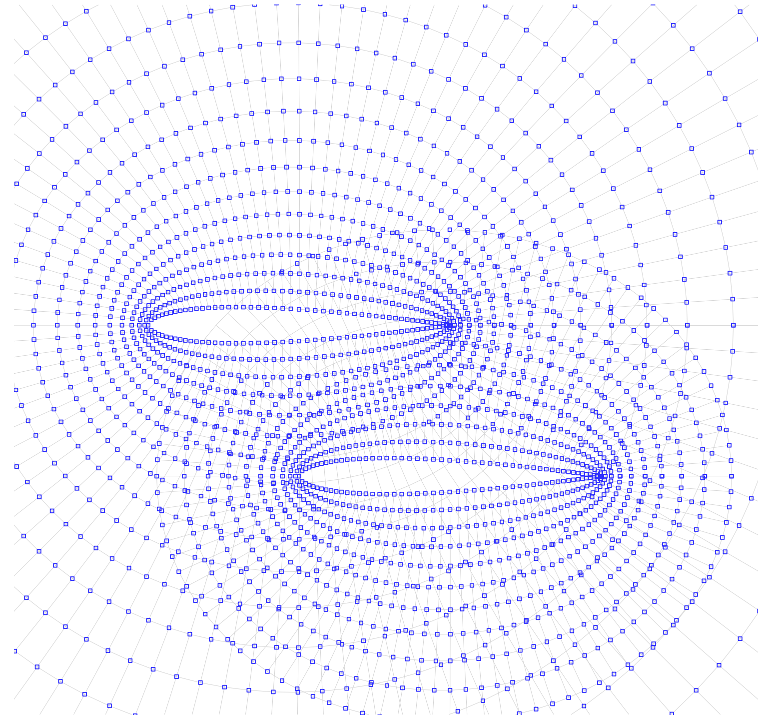
CHIMERA CLOUDS

OVELAPPED SIMPLE CLOUDS



OVELAPPED SIMPLE CLOUDS AFTER BLANKING SOLID POINTS



- **BLANKING BY SURFACE NORMAL TEST**

- **CONNECTVITY GENERATION : (i) QUADTREE METHOD , (ii) GRADIENT SEARCH METHOD**

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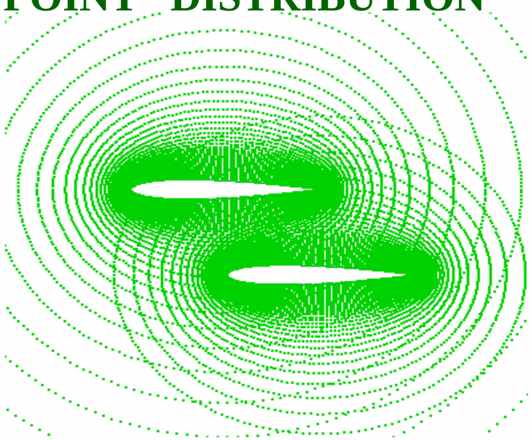
APPLICATIONS OF GRID FREE LSKUM SOLVER

APPLICATIONS OF GRID FREE LSKUM SOLVER

- FLOW PAST 2D MULTI ELEMENT GEOMETRIES WITH MULTIPLE CHIMERA CLOUDS
- 3D MULTI-BODY CONFIGURATION OF ASLV TYPE
- CONTROL SURFACE DEFLECTION
- FLOW PAST M165 CONFIGURATION WITH FAME CLOUD
- VISCOUS SEPARATED FLOW ON AIRFOIL
- CAPTURING SECONDARY VORTICITY IN STRONGLY ROTATING VISCOUS FLOW AROUND A 2D INTAKE

FLOW PAST 2D MULTI ELEMENT GEOMETRIES WITH MULTIPLE CHIMERA CLOUDS

POINT DISTRIBUTION



Transonic flow past Naca0012 staggered biplane

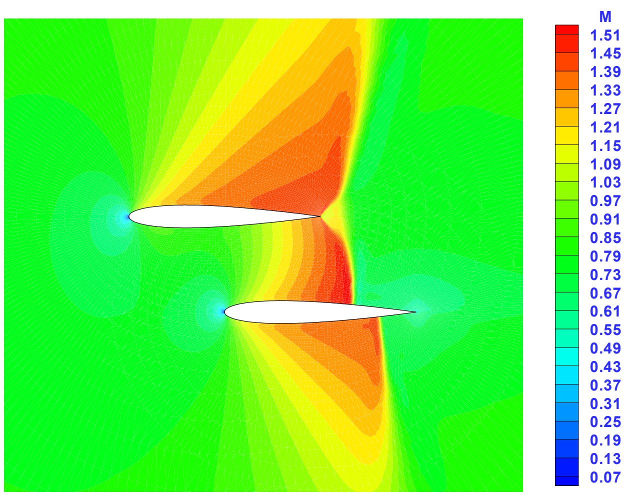
- **FLOW CONDITONS**

  M_∞ : 0.85 AND α : 0^o

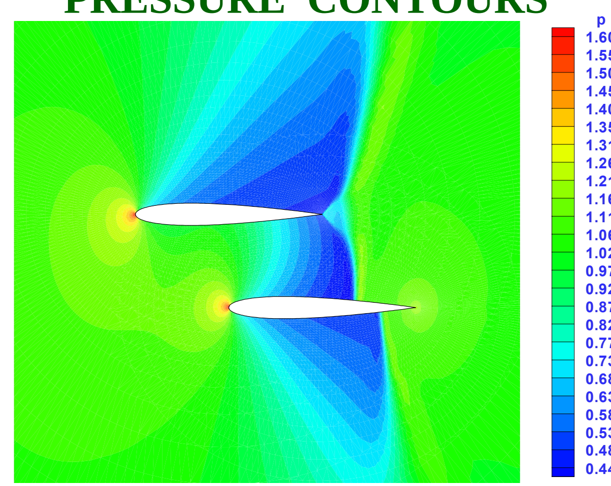
- **POINTS : 241 x 51 : UPPER AIRFOIL**
  241 x 31 : LOWER AIRFOIL

- **CHIMERA CLOUDS**

MACH CONTOURS



PRESSURE CONTOURS



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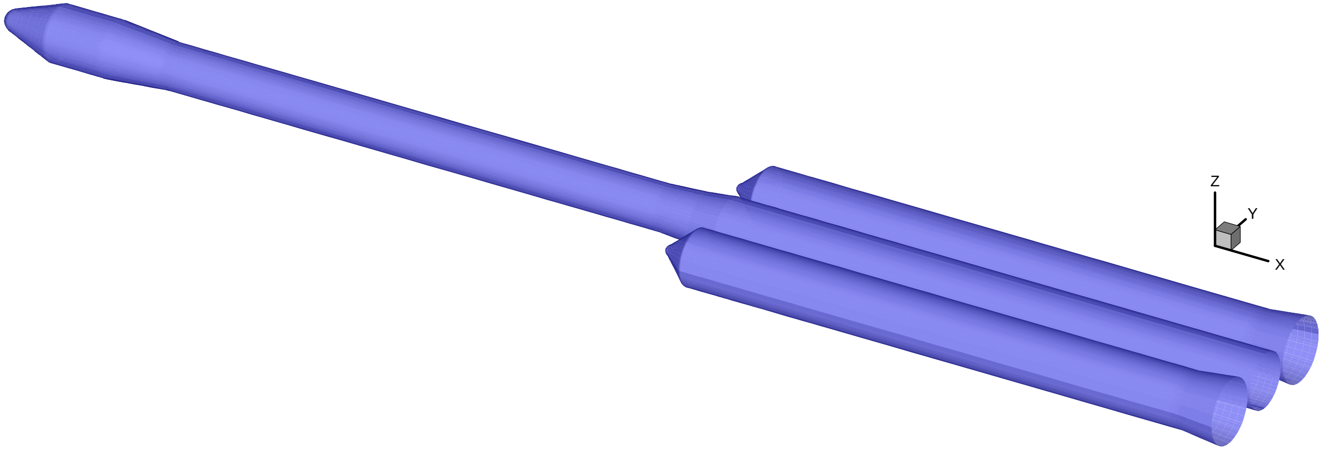
3D MULTI-BODY CONFIGURATION OF ASLV TYPE

DEVELOPMENT OF PREPROCESSOR FOR 3-D OVERLAPPED MESHES

- BLANKING USING STANDARD ANALYTICAL SHAPES
- NEWTON RAPHSON METHOD TO FIND CLOSEST POINT
- MERGING NEIGHBOURS FROM VARIOUS BLOCKS

APPLICATION TO MULTI-BODY LAUNCH VEHICLE

$M_\infty = 2.09$ $\alpha = 0^O$ & 4^O



REF. : NAL PROJECT DOCUMENT CF 9427

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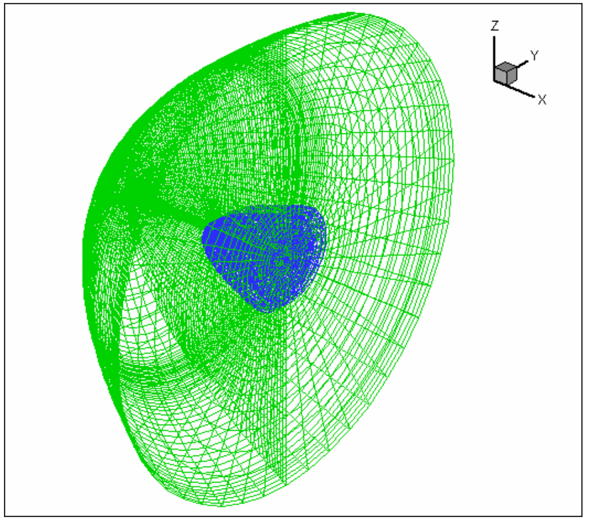
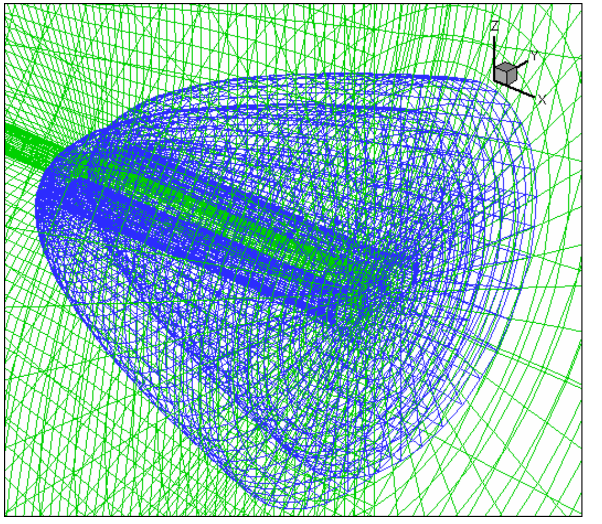
POINT DISTRIBUTION FOR LAUNCH VEHICLE

OVERLAPPED MESHES



ZOOMED VIEW NEAR STRAP-ON

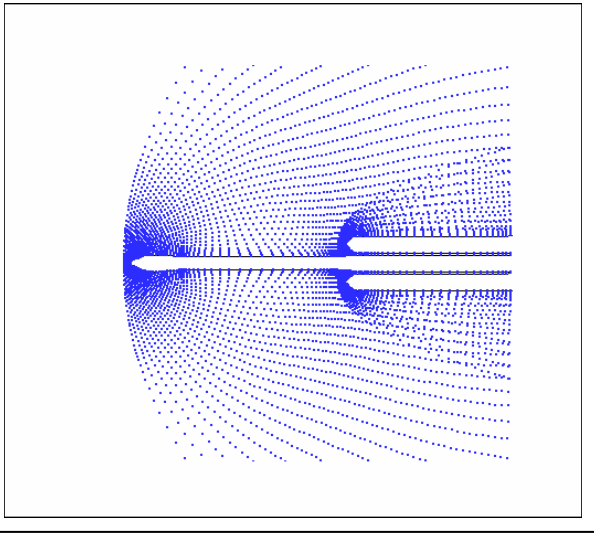


GRID SIZE :

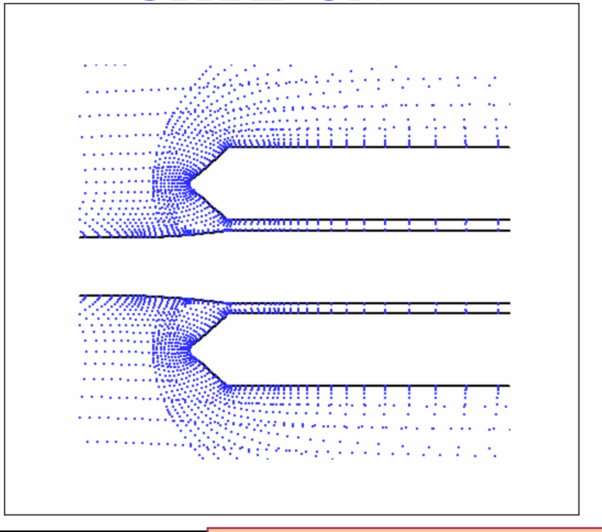
0.09 MILLION

0.14 MILLION

POINT DISTRIBUTION IN MEDIAN PLANE



ZOOMED VIEW NEAR STRAP-ON



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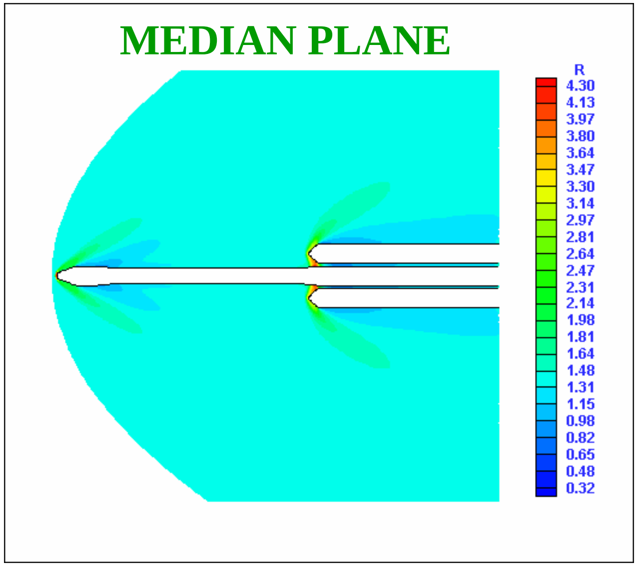
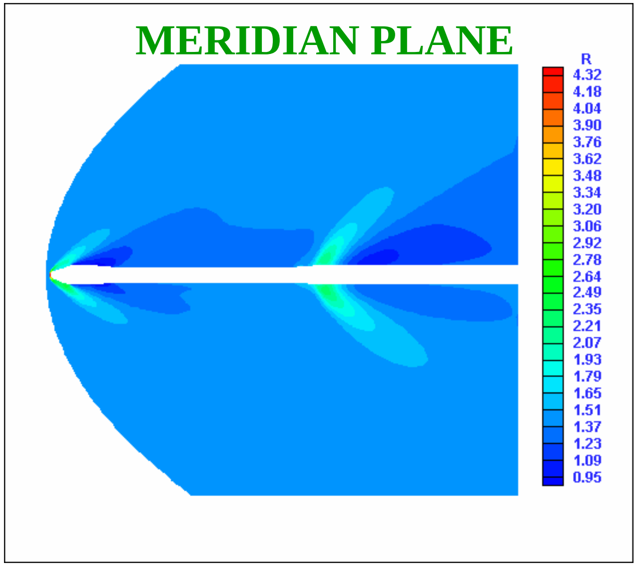
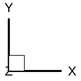
DENSITY CONTOURS

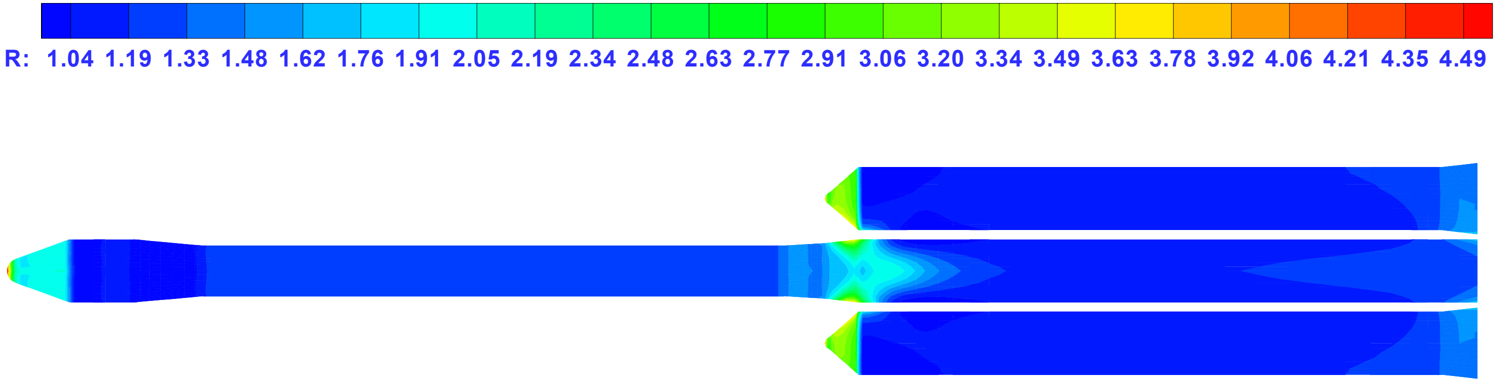
$M_\infty = 2.09$ $\alpha = 4^o$







SURFACE CONTOURS



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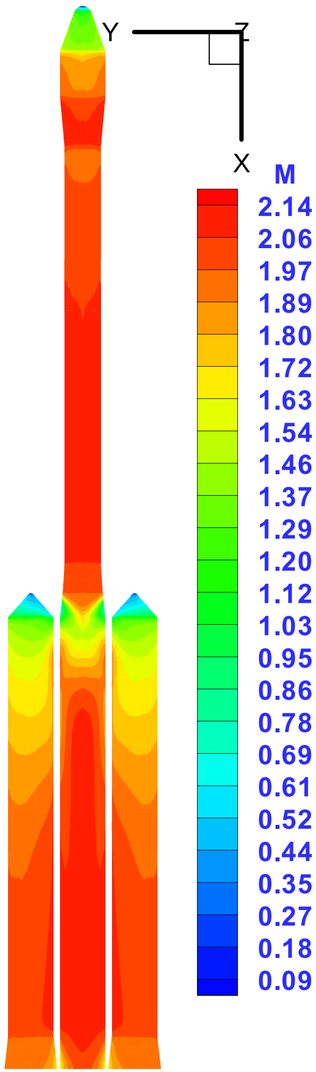
MACH CONTOURS

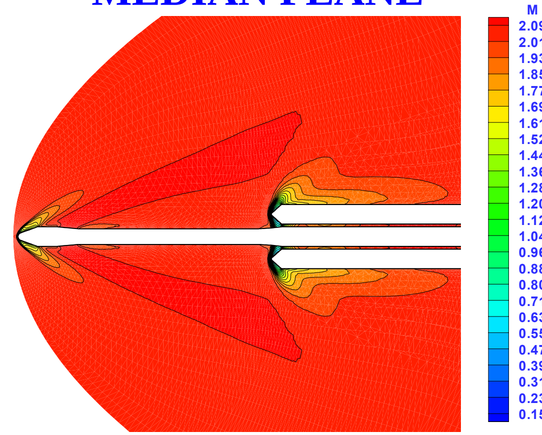
SURFACE CONTOURS



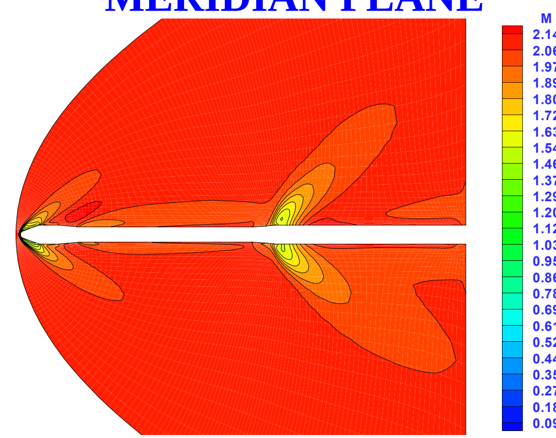
$M_{\infty} = 2.09$

$\alpha = 4^{O}$

MEDIAN PLANE



MERIDIAN PLANE



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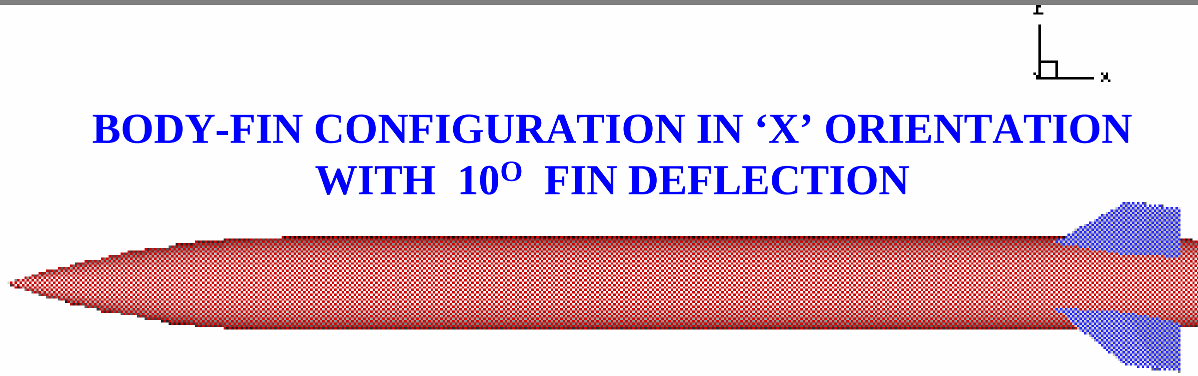
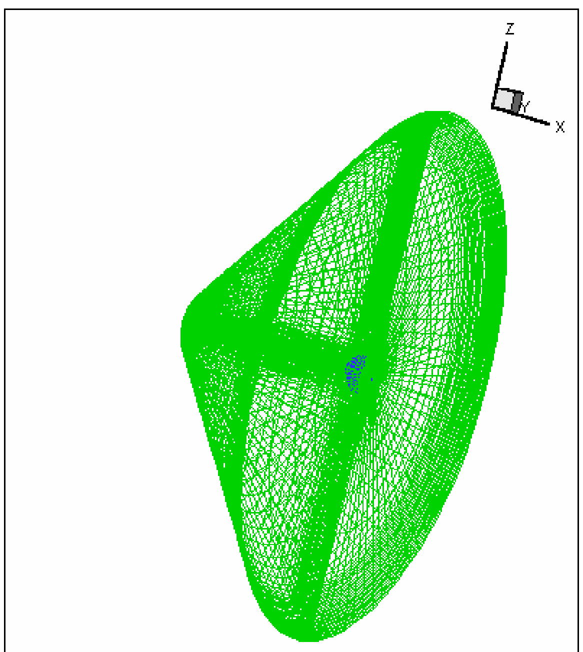
APPLICATION TO CONTROL SURFACE DEFLECTION

BODY-FIN CONFIGURATION IN 'X' ORIENTATION
WITH 10^{O} FIN DEFLECTION

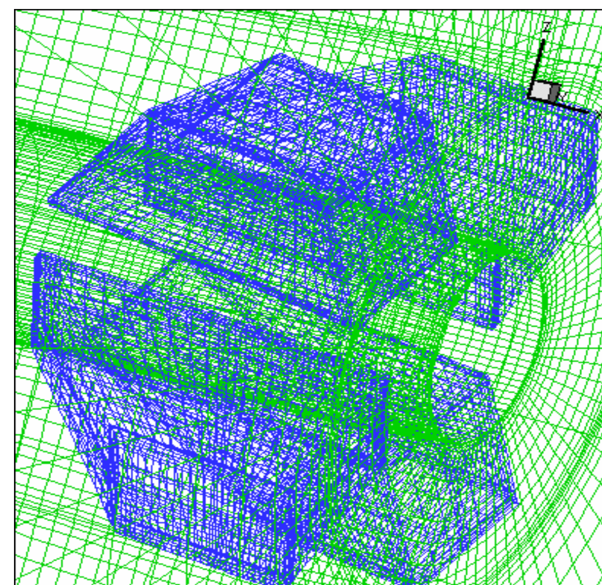


REF. : NASA TM X-2774, 1973 AND
AIAA 2000-4590

OVERLAPPED MESH



ZOOMED VIEW OF OVERLAPPED MESH



GRID SIZE :

69 x 45 x 31 (BODY)

29 x 21 x 10 x 2 (FIN)

AFTER BLANKING

0.1 MILLION POINTS

(5 THOUSNDS POINTS ARE BLANKED)

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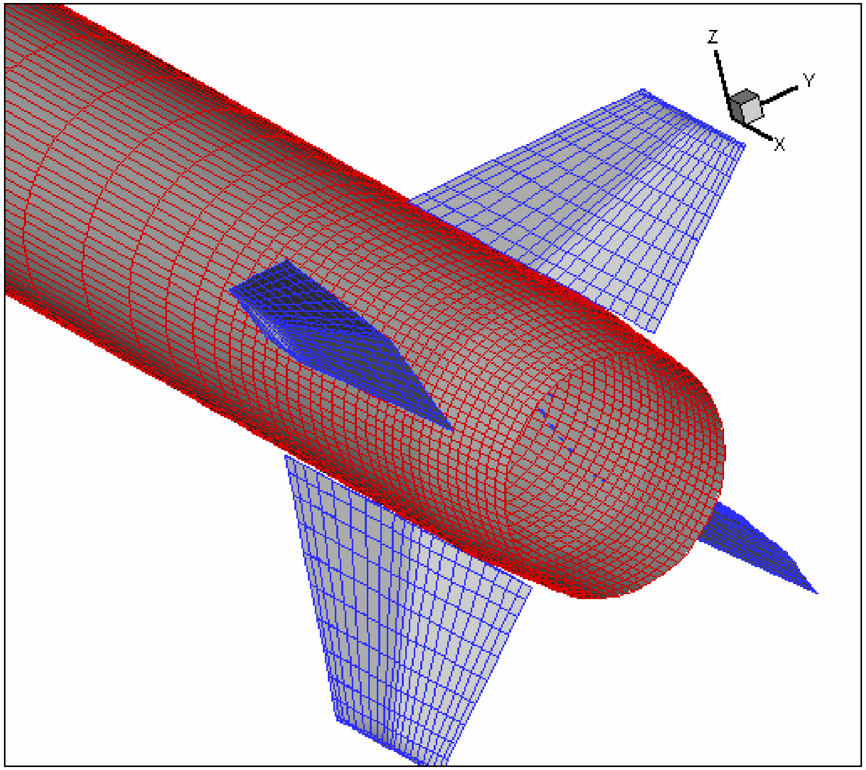
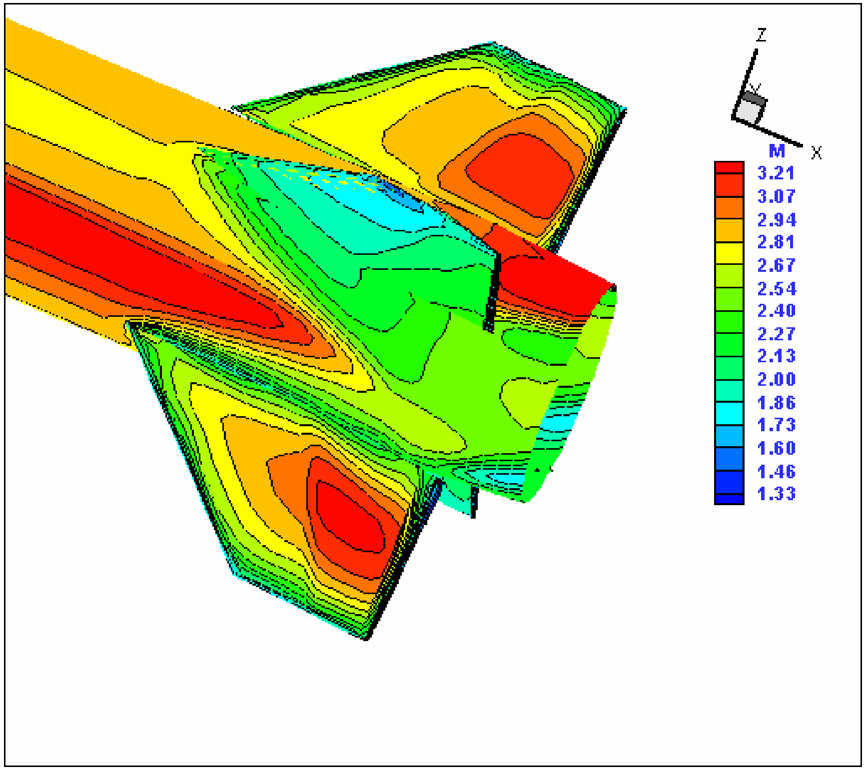
CONTROL SURFACE DEFLECTION EFFECTIVENESS

$M_\infty = 2.86 \quad \alpha = 10^O \quad \delta = 10^O$

SURFACE MESH SHOWING FIN DEFLECTION



MACH CONTOURS



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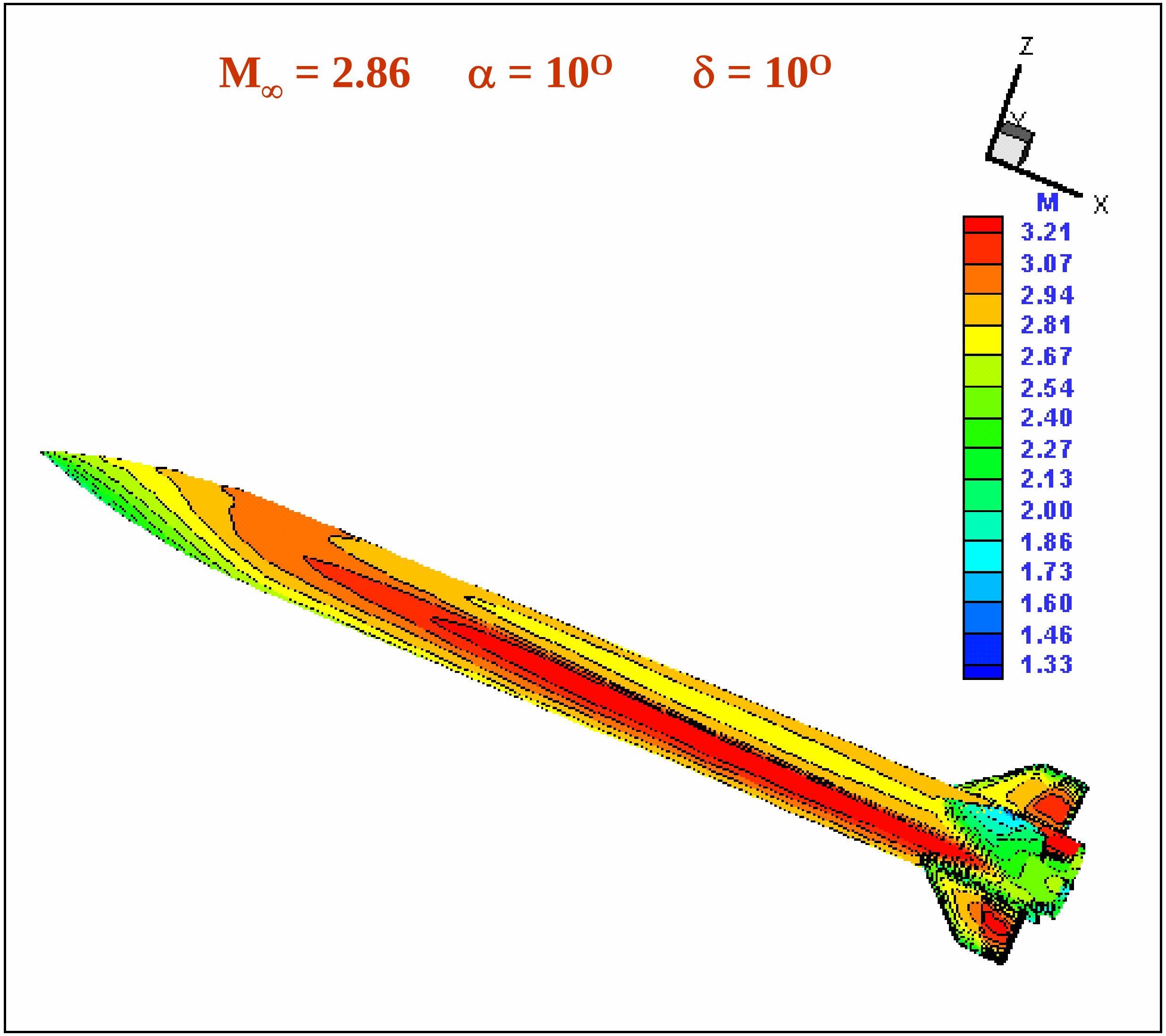
MACH CONTOURS



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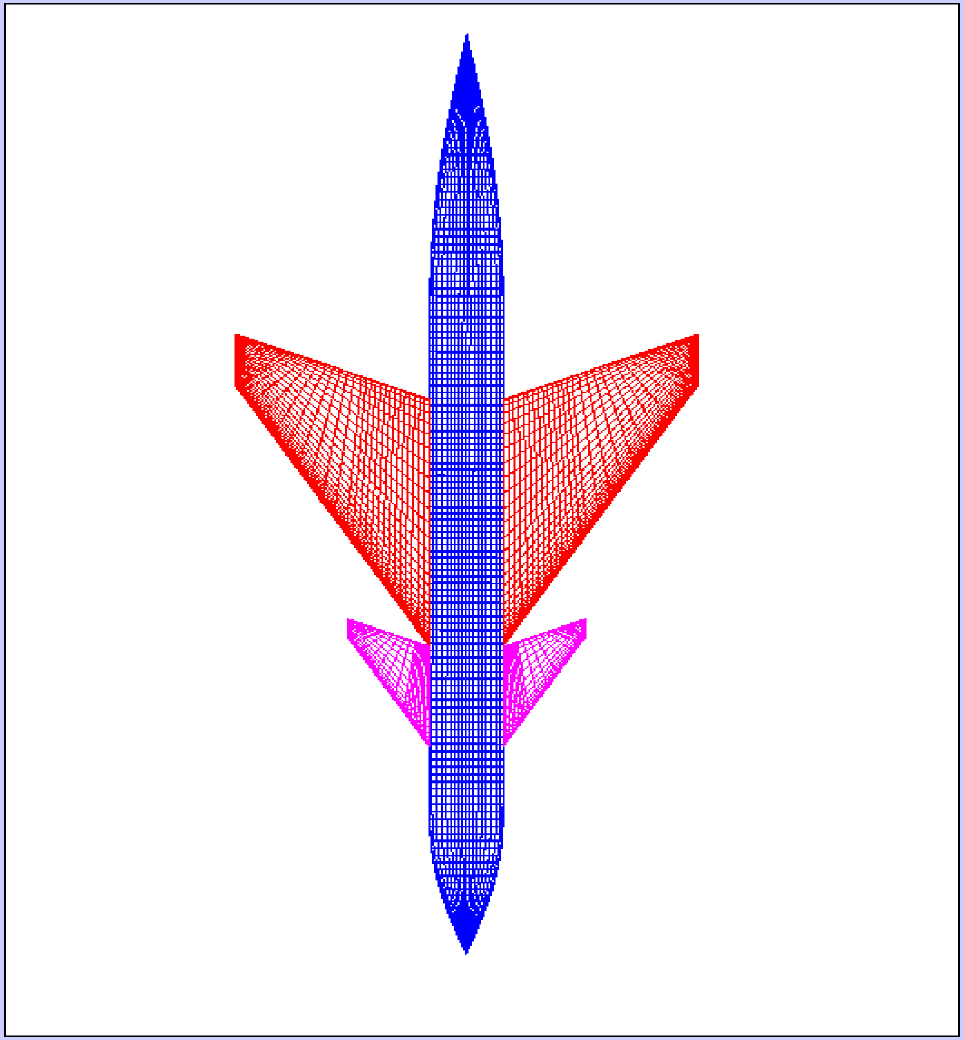
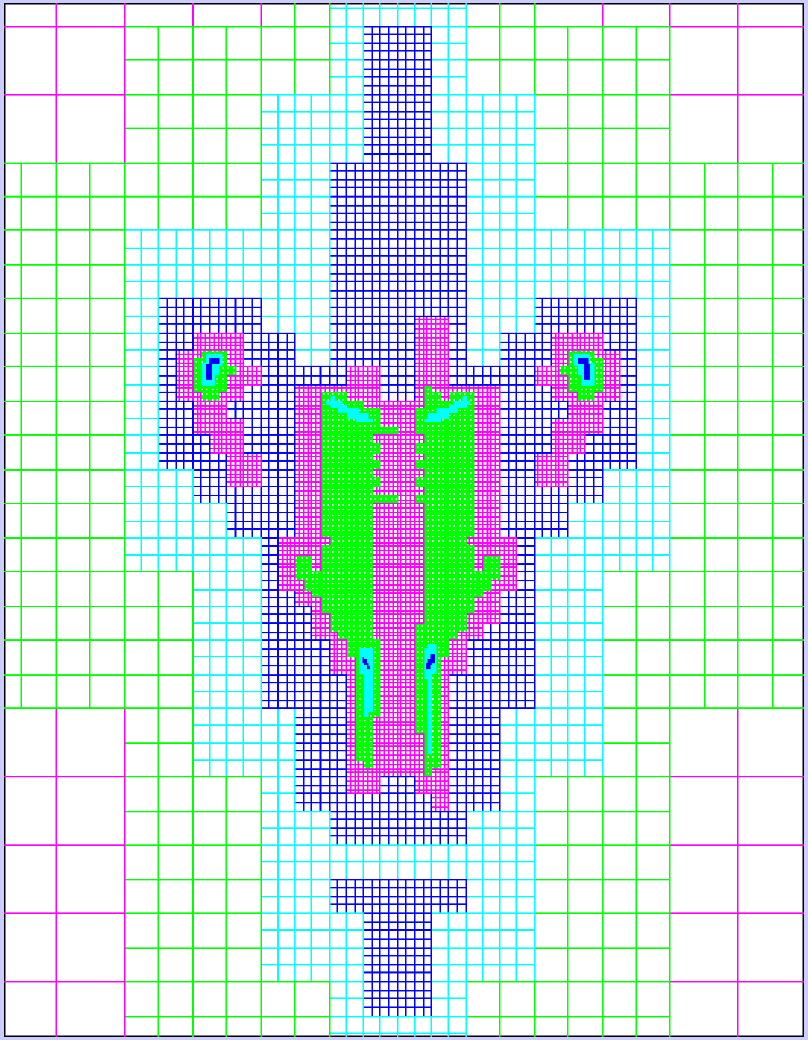
FLOW PAST M165 CONFIGURATION WITH FAME CLOUD

- <u>F</u>EATURE <u>A</u>SSOCIATED <u>M</u>ESH <u>E</u>MBEDDING
- A HYBRID OVERSET GRID CONSISTING OF MULTIPLE BODY FITTED GRIDS AROUND EACH GEOMETRICALLY SIMPLE PART AND A BACK-GROUND CARTESIAN GRID
- CARTESIAN GRID IS PROGRESSIVELY REFINED TO BLEND WITH BODY FITTED GRID IN TERMS OF GRID SPACING
- AN OPTIMAL STRATEGY – BODY FITTED MESH RESOLVES THE GEOMETRY PROPERLY AND CARTESIAN MESH FILLS THE FIELD WHERE THERE ARE NO FEATURES TO BE RESOLVED
- CONVENTIONAL GRID-BASED SOLVERS REQUIRE INTERPOLATION TO TRANSFER DATA BETWEEN GRIDS – INTERPOLATION DOES NOT RESPECT THE GOVERNING EQUATIONS OF FLUID FLOW
- LSKUM TAKES POINTS FROM ALL THE GRIDS AND <u>*UPDATES CONSISTENTLY AT ALL POINTS – NO INTERPOLATION IS REQUIRED*</u>

M165 configuration body fitted and Cartesian Grid





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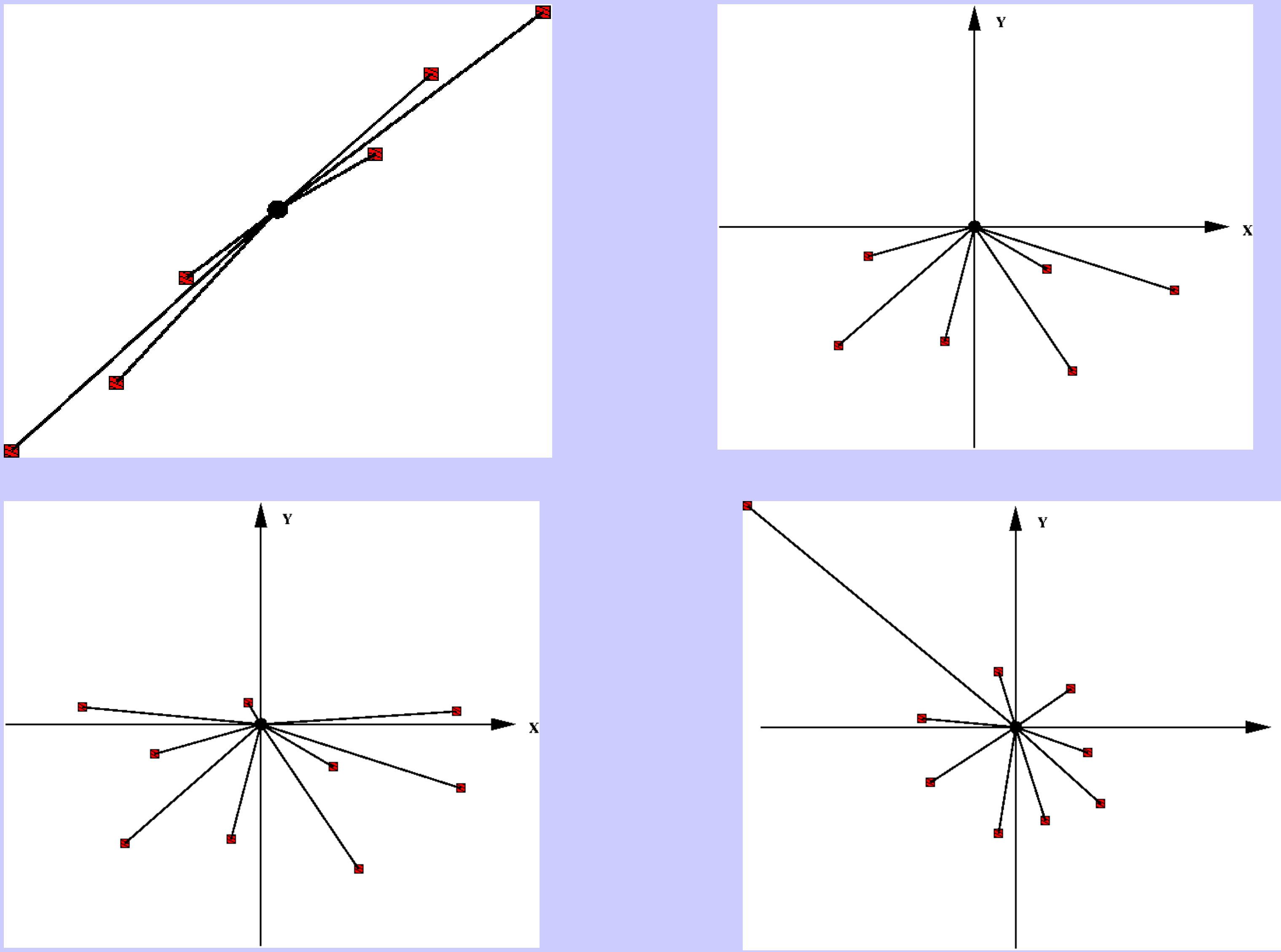
Examples of bad connectivity



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Singular Value Decomposition (SVD) test

$$\mathbf{A} = \mathbf{X}^{T}\mathbf{X},$$

Least squares matrix for connectivity with "n" points

$$\mathbf{X} = \begin{bmatrix} \Delta \mathbf{x}_1 & \Delta \mathbf{y}_1 & \Delta \mathbf{z}_1 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \Delta \mathbf{x}_n & \Delta \mathbf{y}_n & \Delta \mathbf{z}_n \end{bmatrix}$$

Condition number of X, $\kappa = \dfrac{\lambda_{\mathbf{max}}}{\lambda_{\mathbf{min}}}$

$\lambda_{\mathbf{min}}, \lambda_{\mathbf{max}}$ are the smallest and largest singular values of X

$\kappa >> \mathbf{1}$ means problem is ill-conditioned; geometrically, it tells us that connectivity is flat. In this case, use full stencil.

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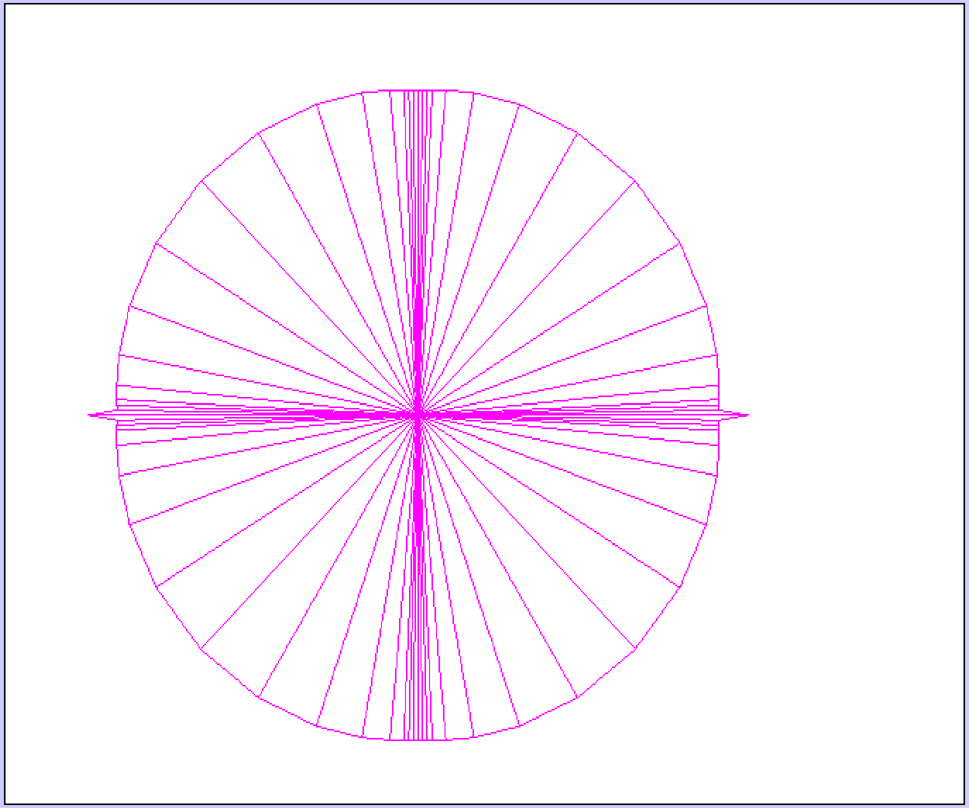
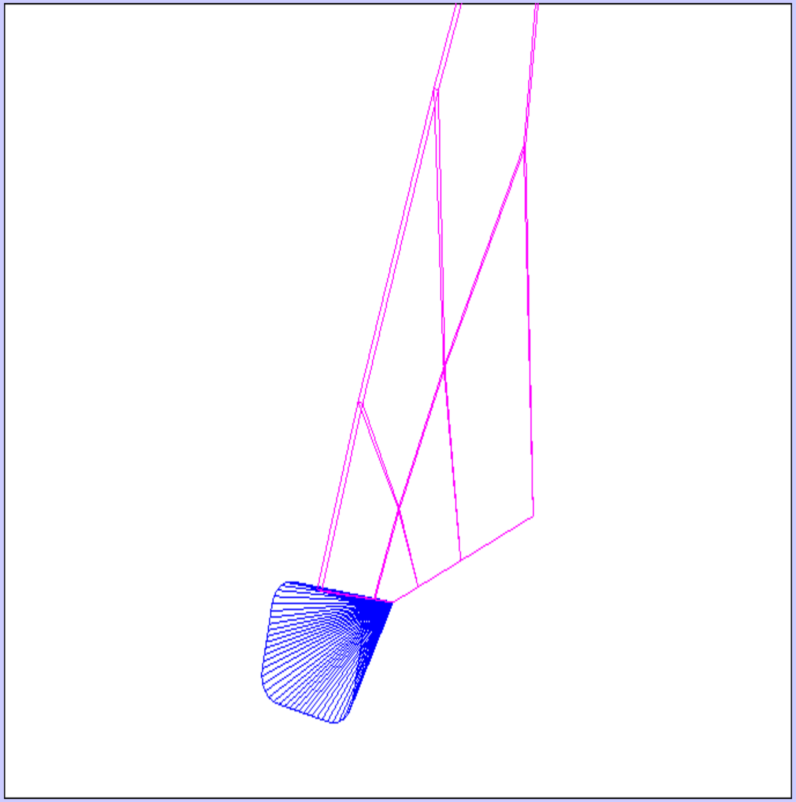
Grid near fuselage tip

Grid singularity leads to flat connectivity – identified by SVD test and use full stencil for derivatives normal to disc.





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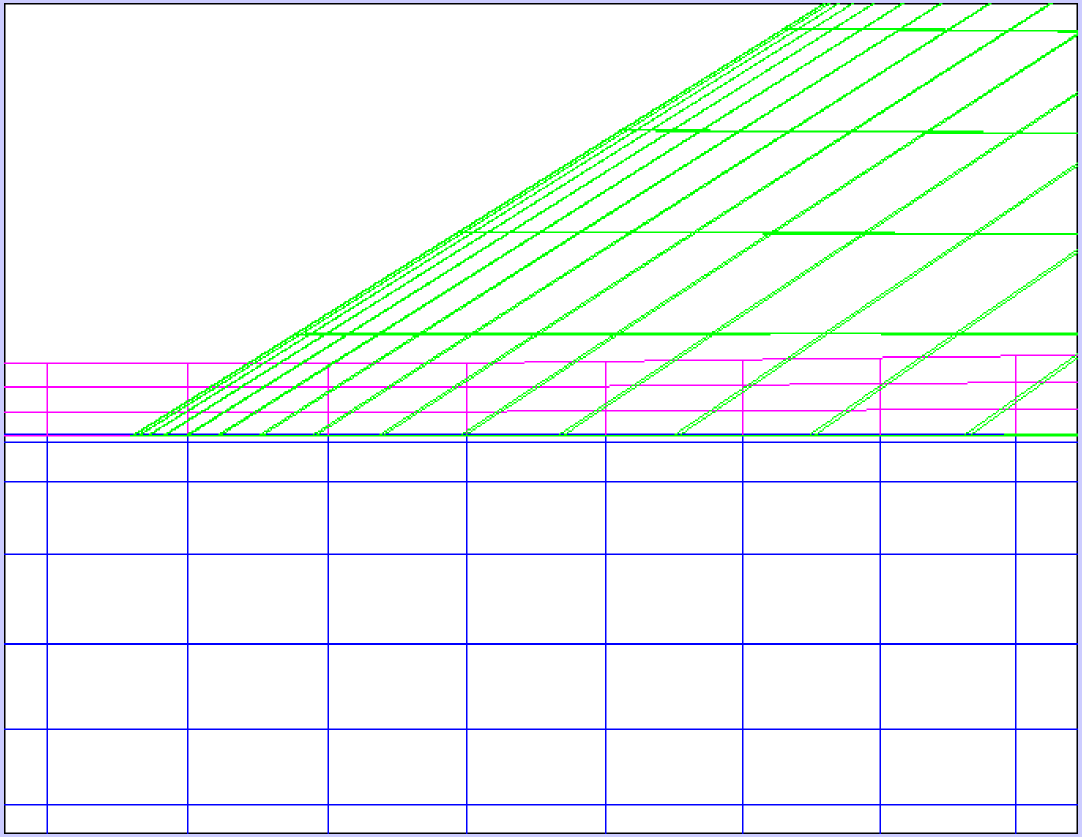
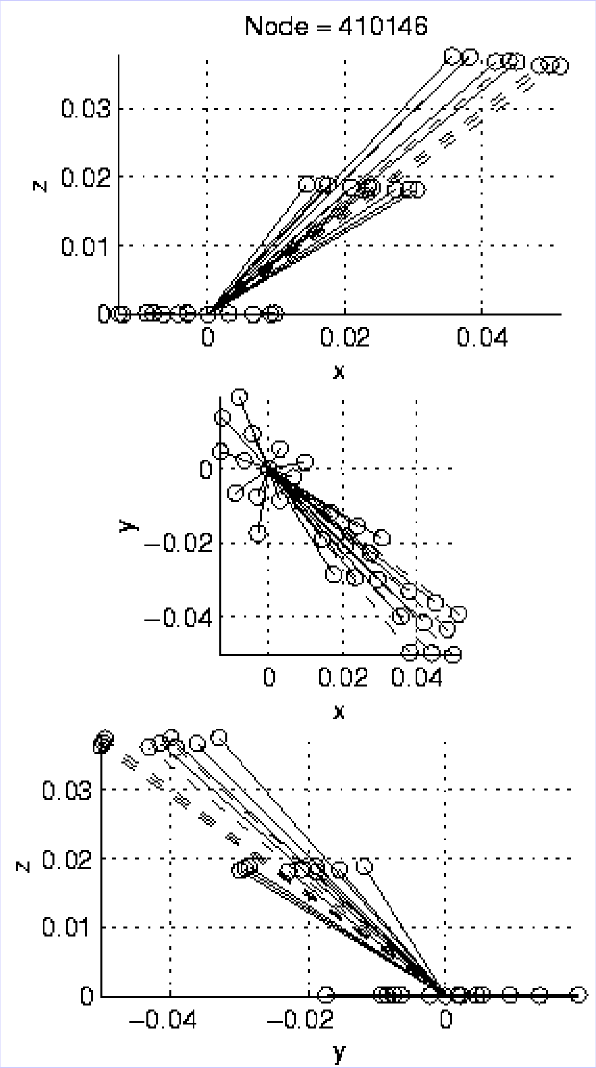
Wing-fuselage overlap region

Sweepback leads to one-sided connectivity – identified by SVD test, use full stencil





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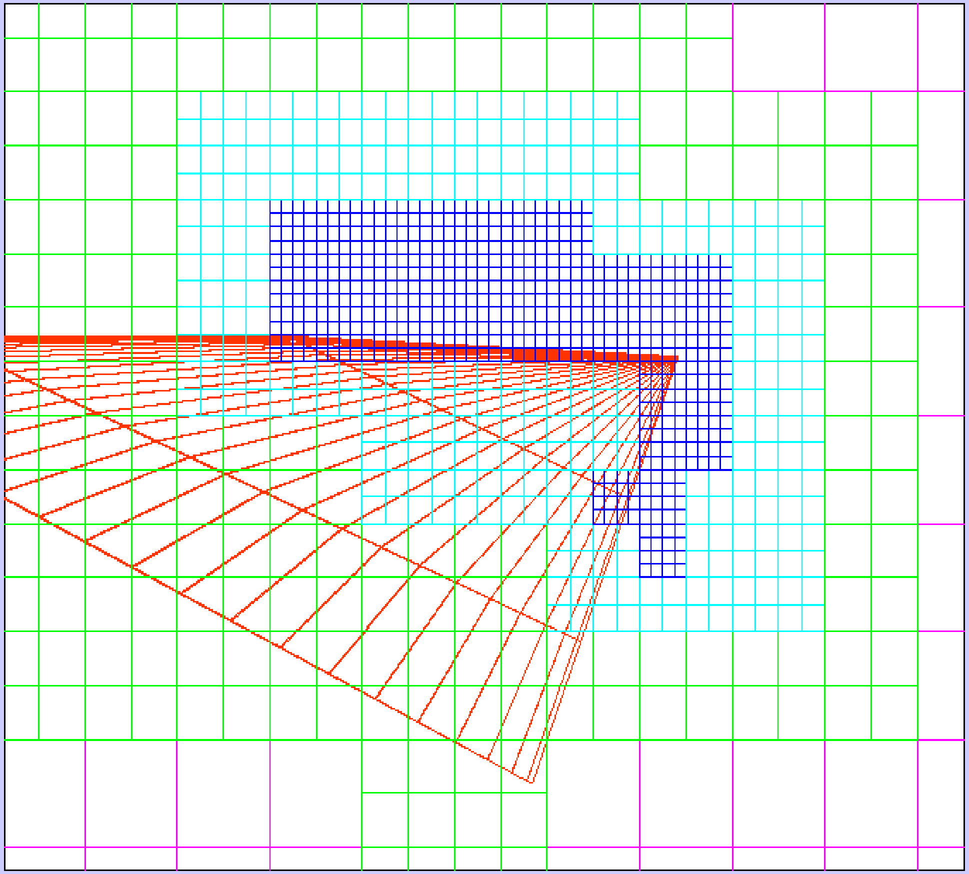
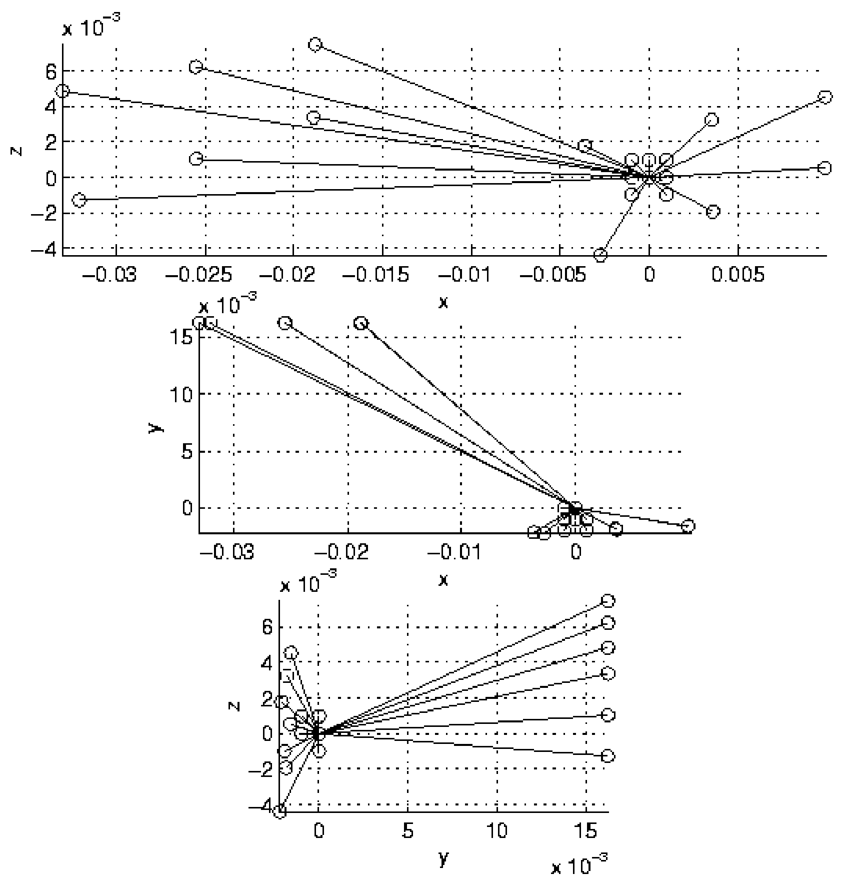
Grid and connectivity near wing tip

Anisotropic connectivity – but we cannot remove points since information transfer will be affected. *LSKUM still works on this crazy stencil.*





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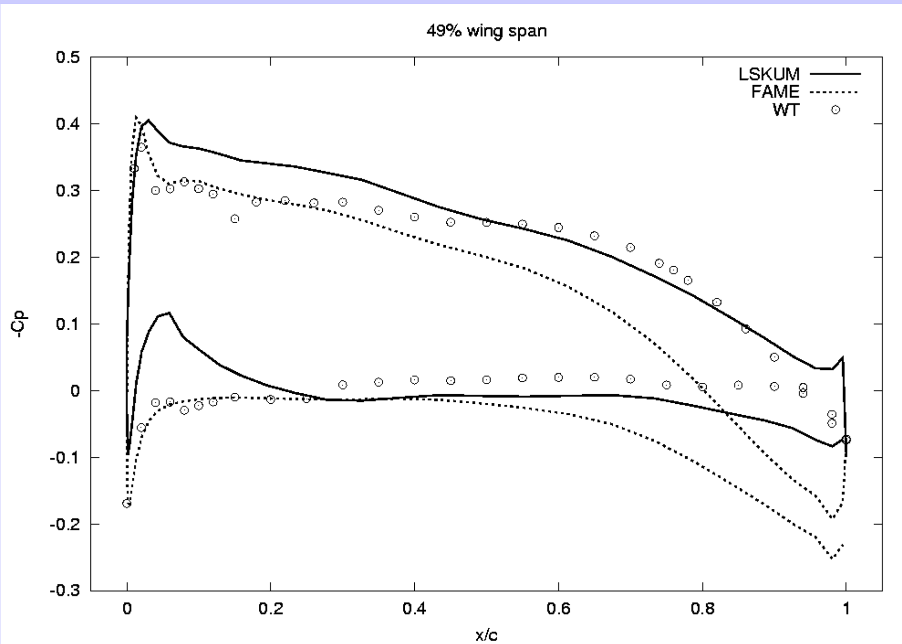
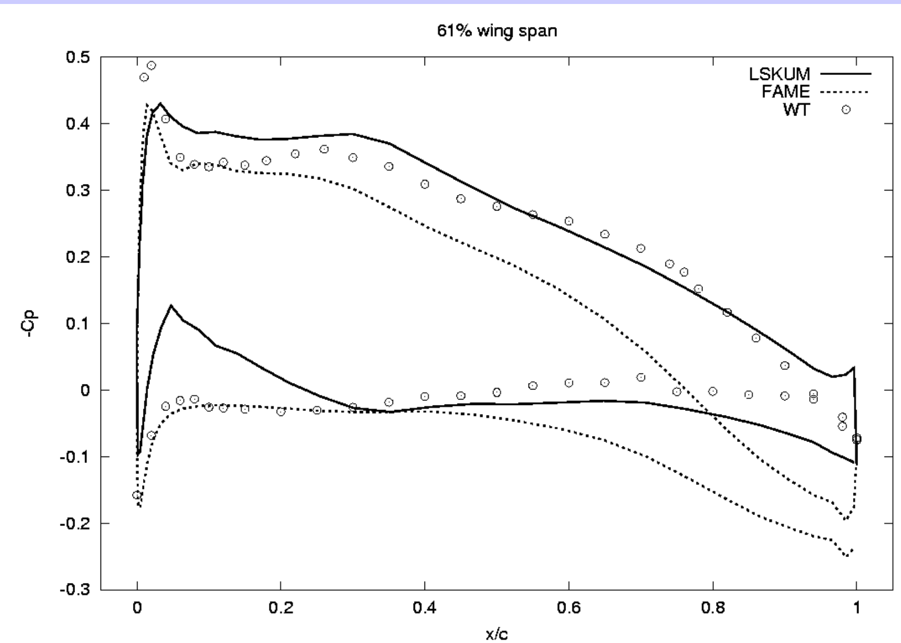
Cp on the wing

49% span



61% span



Comparison between LSKUM (solid line), FAME (dotted line) and wind tunnel (symbols) results

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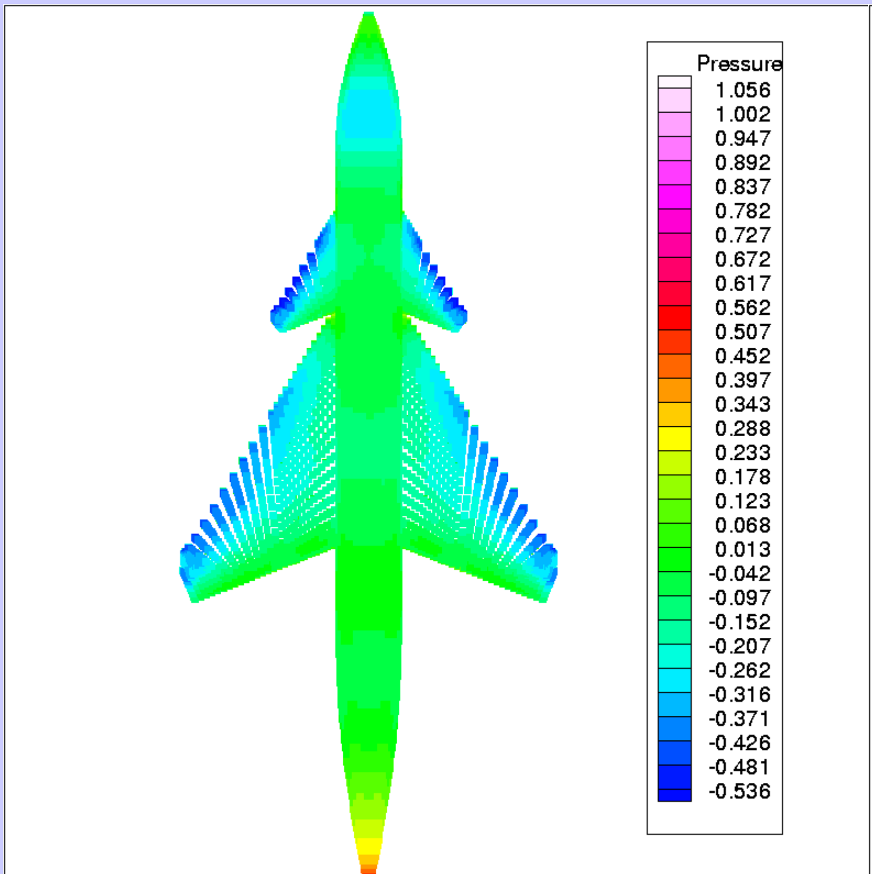
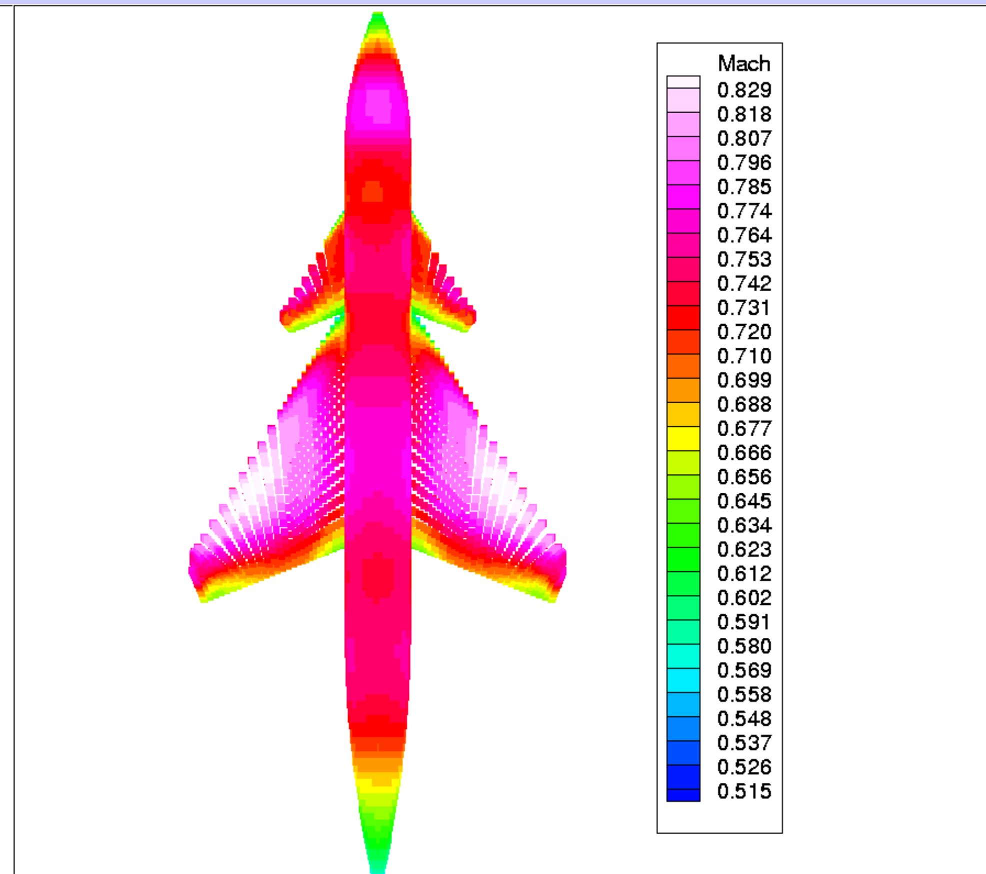
Cp and Mach number on M165

Cp



Mach number



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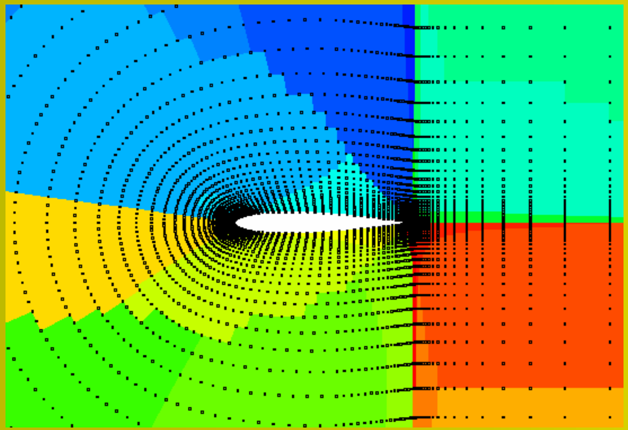
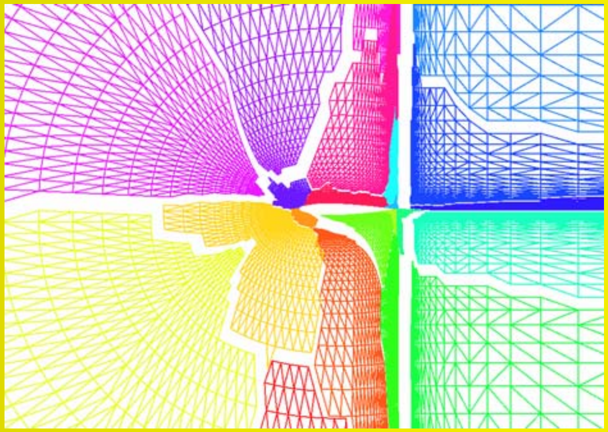
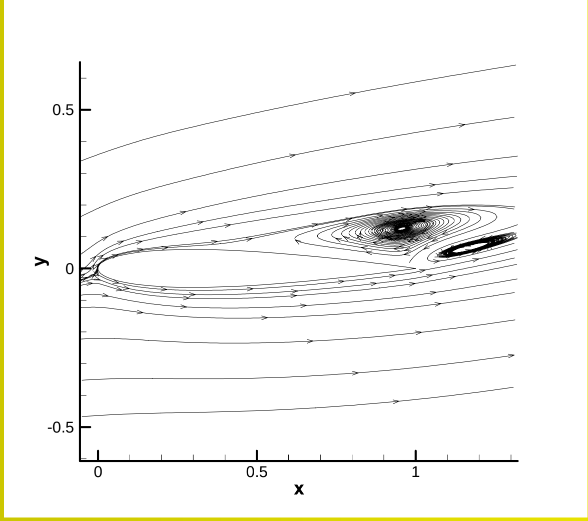
VISCOUS SEPARATED FLOW ON AIRFOIL



Cloud of points around NACA0012 Airfoil decomposed in 22 parts



Mesh around NACA0012 airfoil decomposed in 22 parts



Boundary layer separation for transonic flow past NACA0012 Airfoil

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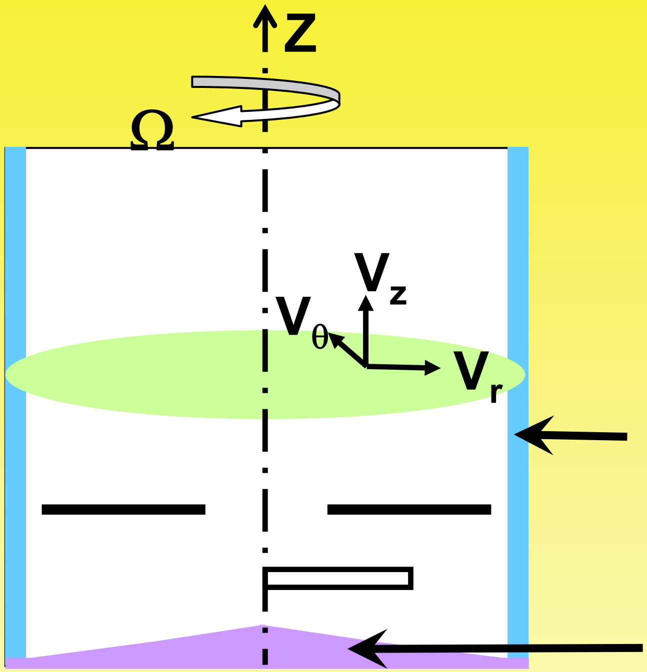
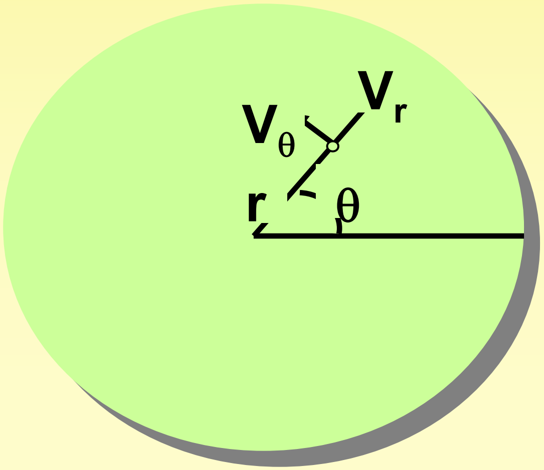
CAPTURING SECONDARY VORTICITY IN STRONGLY ROTATING VISCOUS FLOW AROUND A 2D INTAKE



Stewartson 1/3 layer $\left(\frac{\mu\ H}{\rho\ \Omega}\right)^{\frac{1}{3}}$

Ekman 1/2 Layer $\left(\frac{\mu}{\rho\ \Omega}\right)^{\frac{1}{2}}$



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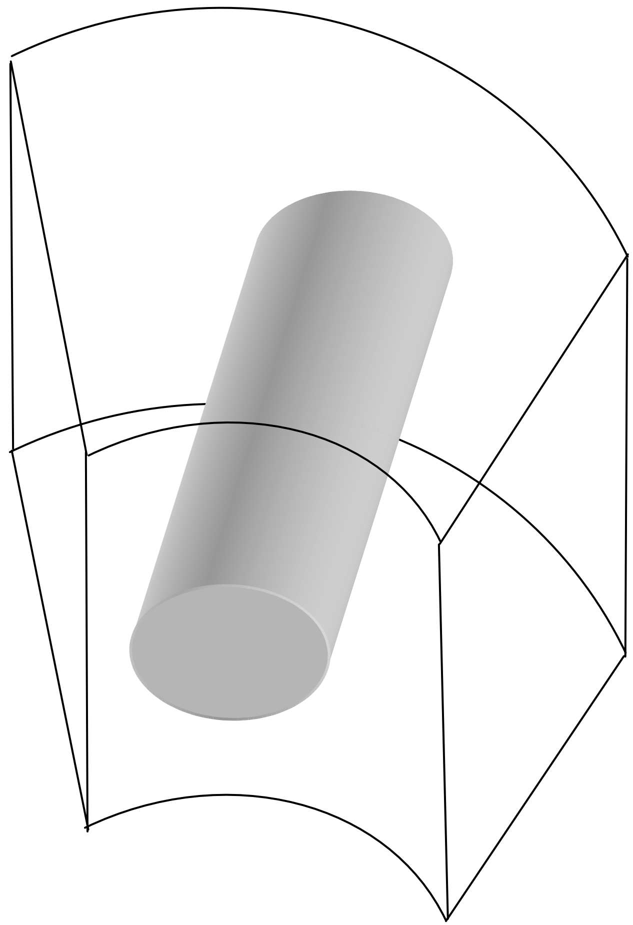

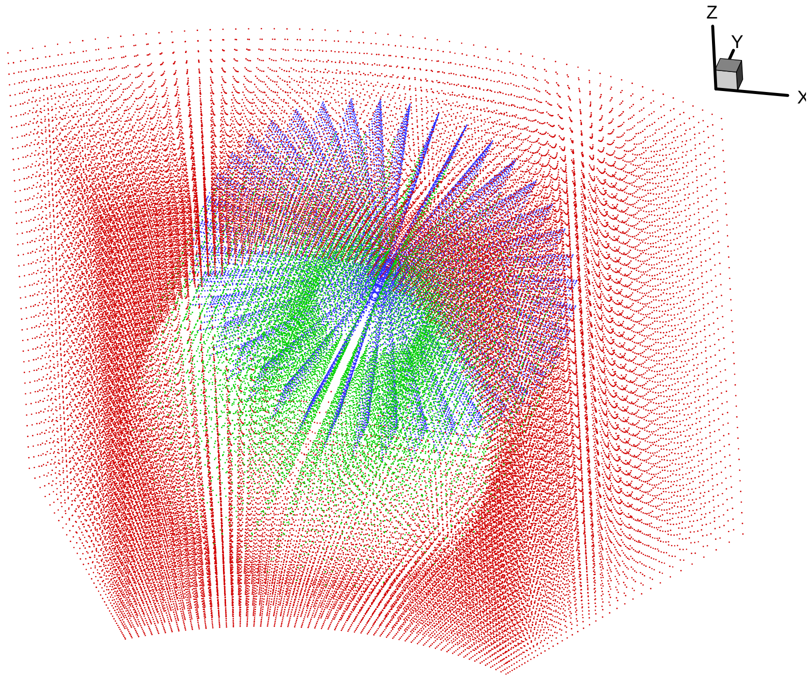

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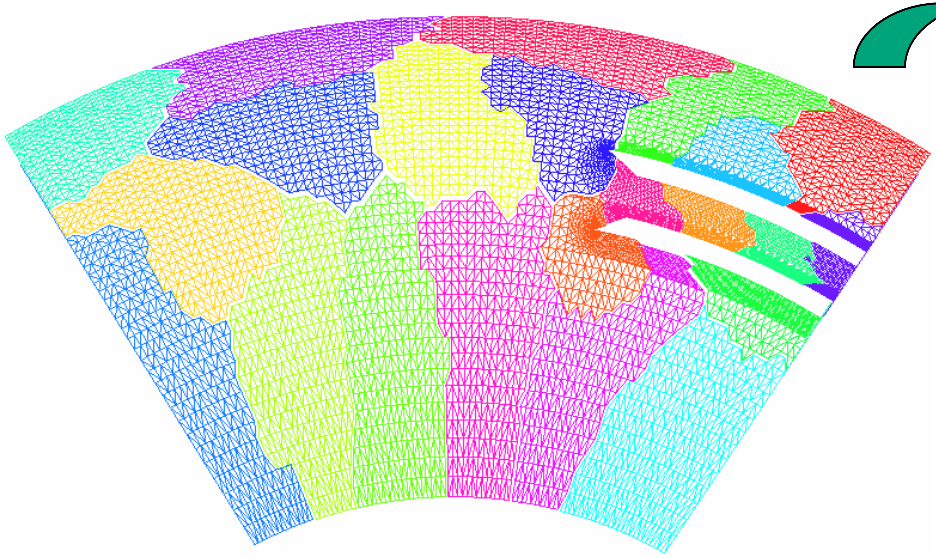
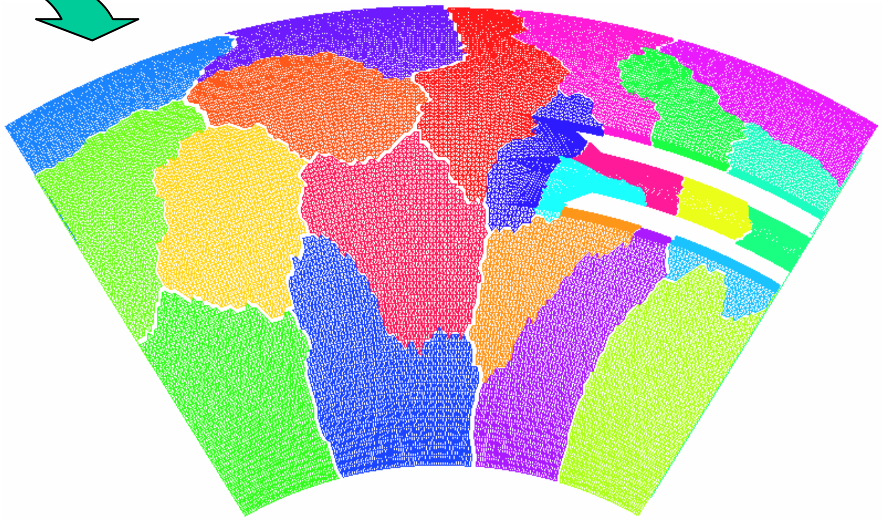
COMPUTATION IN r-θ PLANE



COARSE GRID (6268 NODES)



MEDIUM GRID (26110 NODES)

HELPING TO SHIFT FROM TEST BASED TO SIMULATION BASED CONFIDENCE

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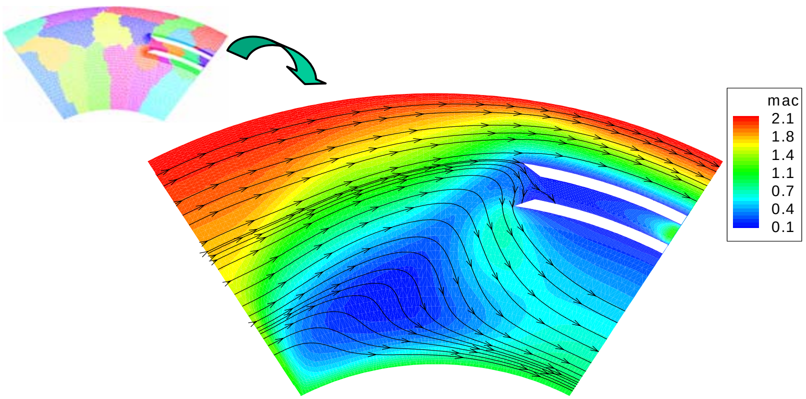
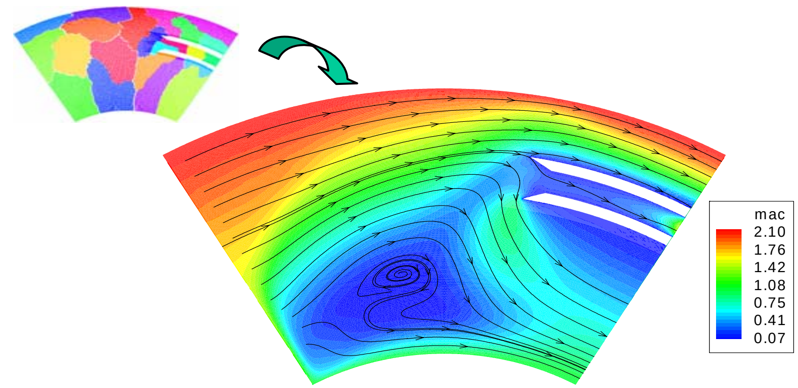
COMPUTATION IN r-θ PLANE

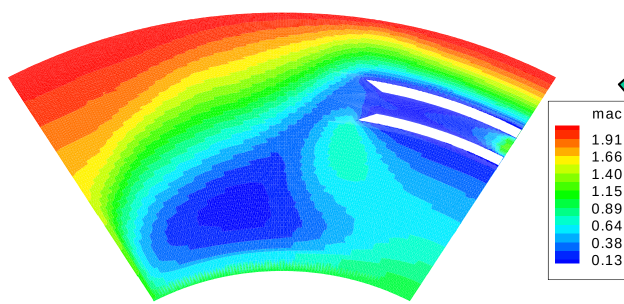
Results of fine grid after mapping from Coarse to Medium and then to Fine grid



Coarse Grid (6268 nodes)



Fine Grid (102002 nodes)



Mapped contours on Medium Grid (26110 nodes)

HELPING TO SHIFT FROM TEST BASED TO SIMULATION BASED CONFIDENCE

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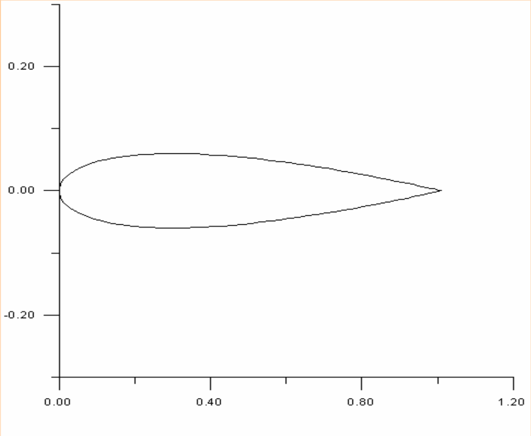
OPTIMIZATION PROBLEM CHOSEN

Initial configuration : NACA0012 Airfoil



Objective Function (OF) $= \dfrac{0.4}{C_d^{\,2}}.\exp(-10 \times Max\{|0.4 - C_l|, 10^{-4}\})$

Solver : 2D LSKUM-NS

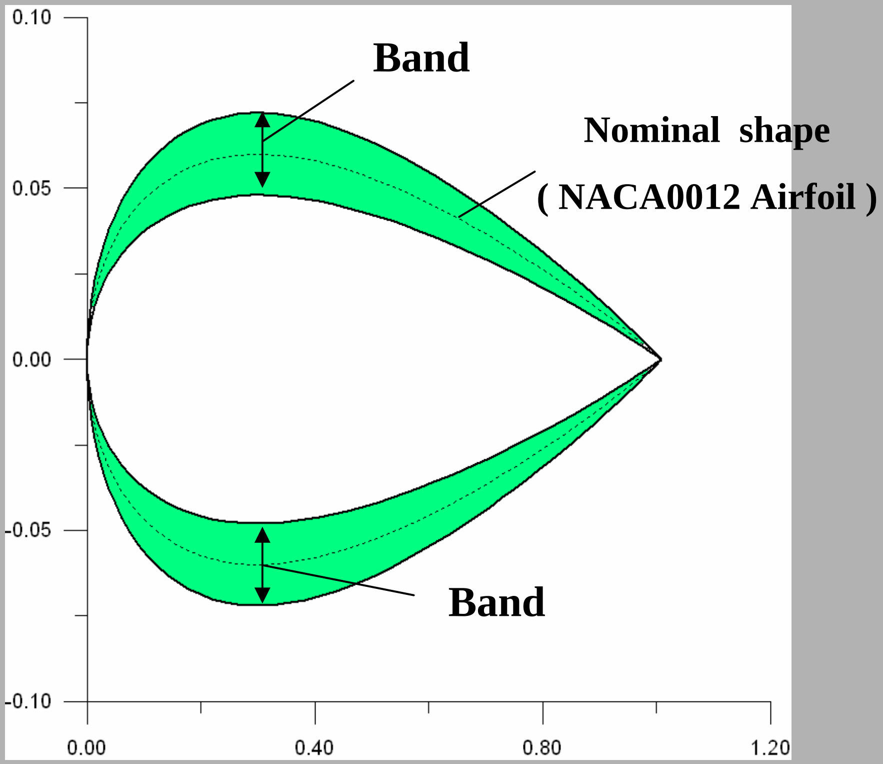
Optimization method : EVOLUTIONARY ALGORITHM (EA)

2D LSKUM-NS + EA

- REGRIDDING AFTER EVERY SHAPE CHANGE IS NOT REQUIRED
- SB POINTS ON OLD SHAPE DELETED AND SB POINTS ON NEW SHAPE ADDED.
- CLOUD OF POINTS CHANGES ONLY LOCALLY NEAR SB
- FLAGGING FOR INTERIOR POINT, SOLID BODY (SB) POINT AND BLANKED POINT
- SHAPE GENERATION USING CUBIC SPLINE
- SELECTION, CROSSOVER, MUTATION AND ELITISM

NOMINAL SHAPE AND ALLOWED Y-BAND

DEFINE A PERTURBATION REGION AROUND NOMINAL SHAPE WITHIN A BAND. EACH MEMBER (CHROMOSOME) OF THE POPULATION IS A PERTURBATION OVER THE NOMINAL SHAPE (NACA0012 AIRFOIL).



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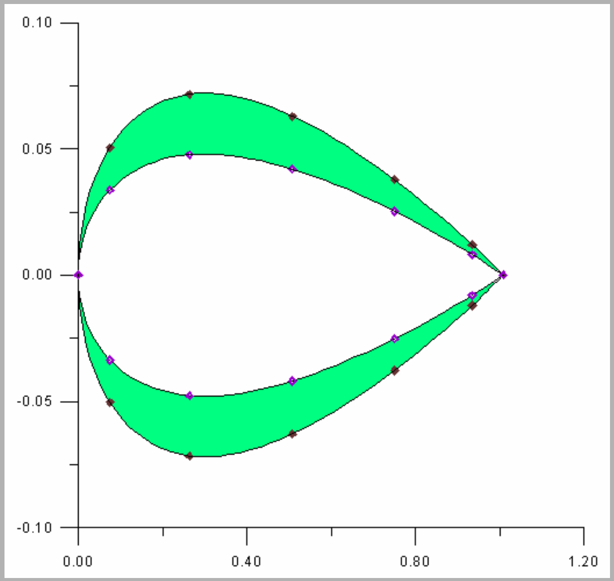
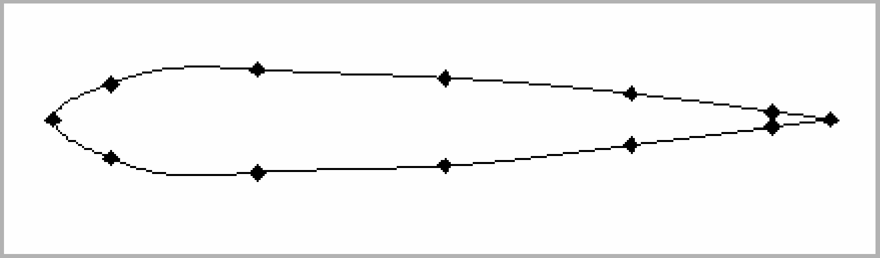
CONTROL POINTS AND CUBIC SPLINE

USING 7 CONTROL POINTS EACH FOR UPPER AND LOWER SPLINES.



CUBIC SPLINES ARE CHOSEN TO REPRESENT THE SHAPE AND FITTING PARABOLA AT L.E.

A TYPICAL SHAPE



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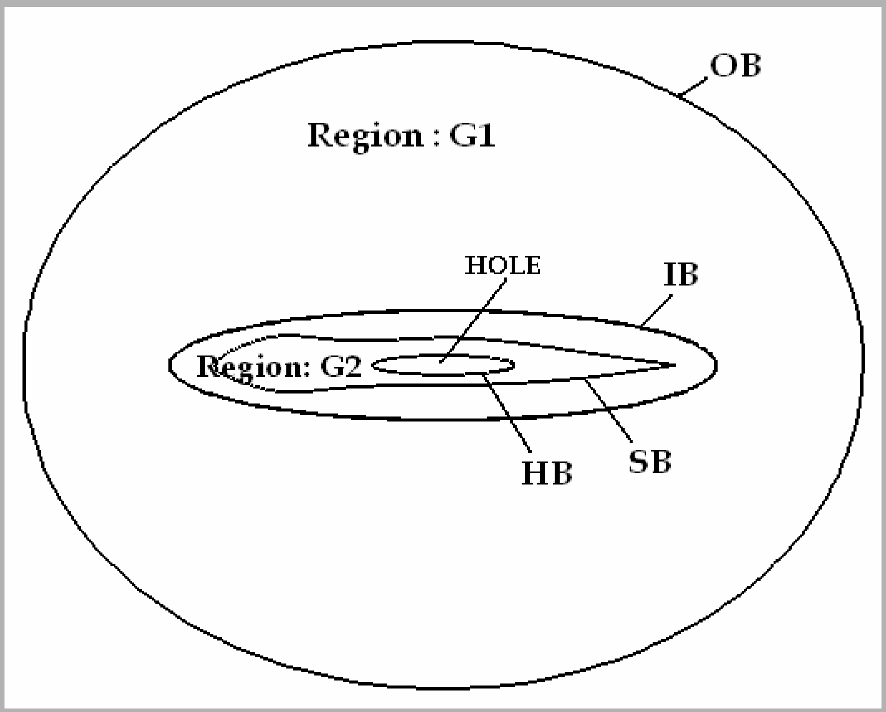
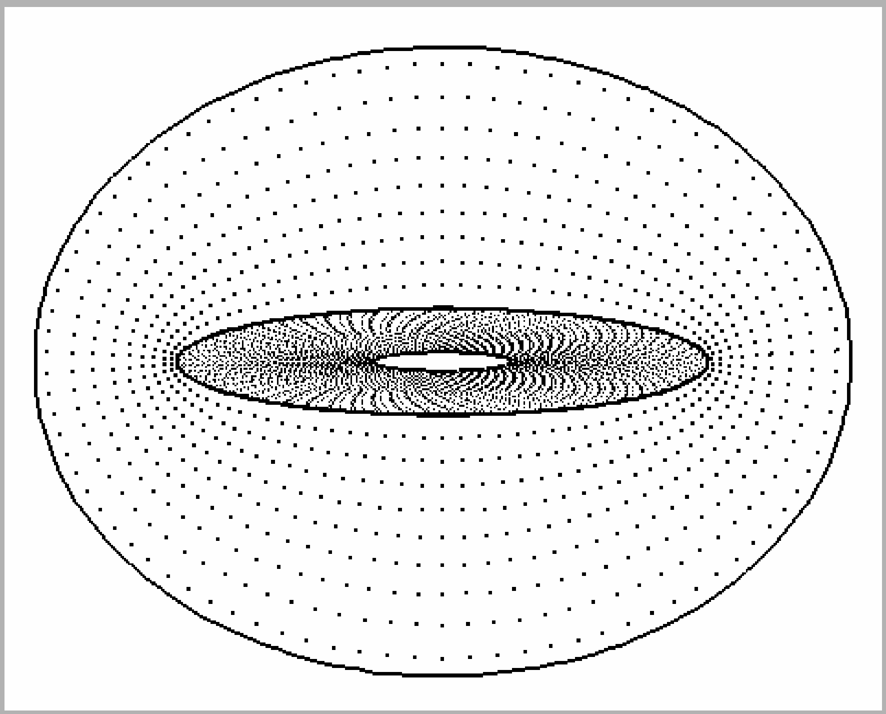
POINT GENERATION





OB = OUTER BOUNDARY

IB = INTERMEDIATE BOUNDARY

SB = SOLID BOUNDARY

HB = HOLE BOUNDARY

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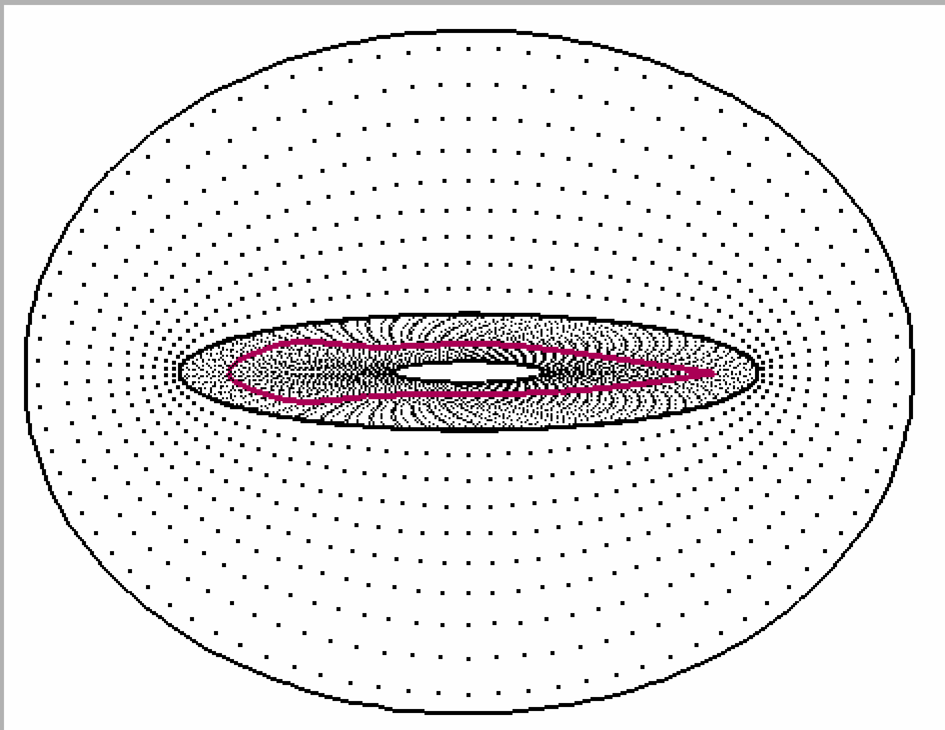
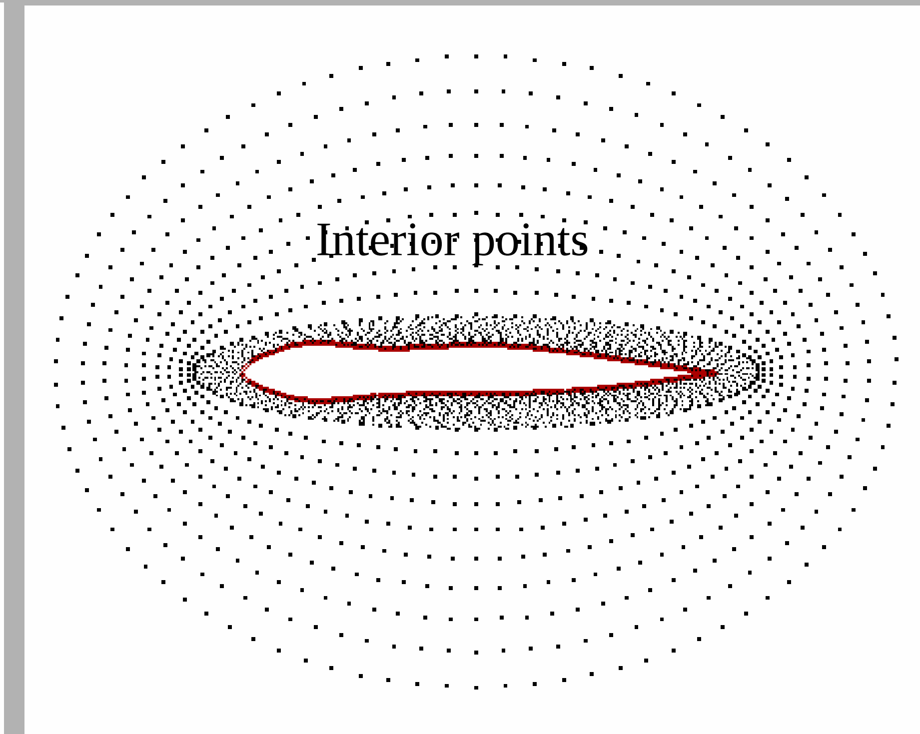
POINT ADDITION



- The points of the airfoil are added into the previous cloud of points.
- These are the points of intersection between gridlines and SB.

POINT DELETION

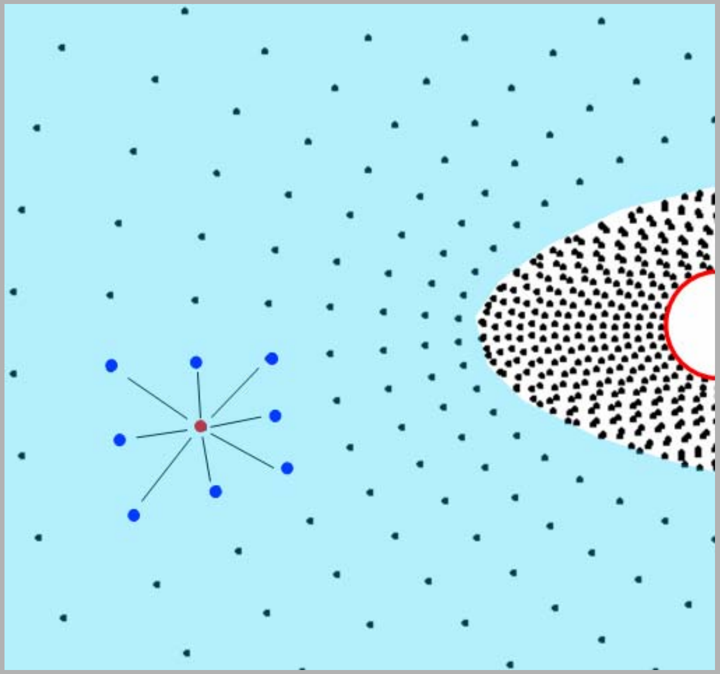


- Flagging using the ray tracing algorithm.
- All points except the SB points are here after referred as interior points.
- Points deleted will be referred as blanked points

CONNECTIVITY GENERATION

Two levels of connectivity generation used.

1. Connectivity of points in Region G1. Its connectivity is not changed during computation.



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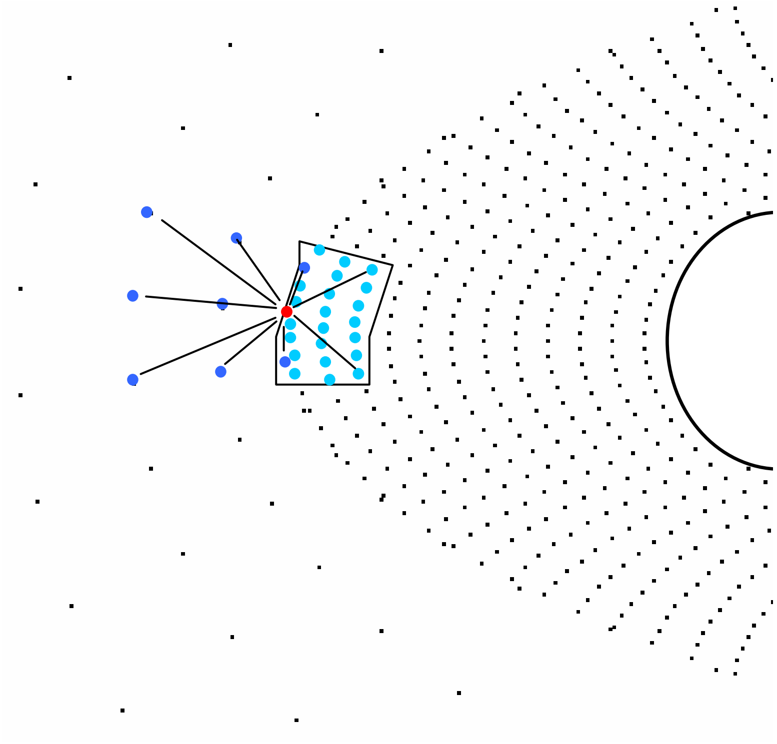
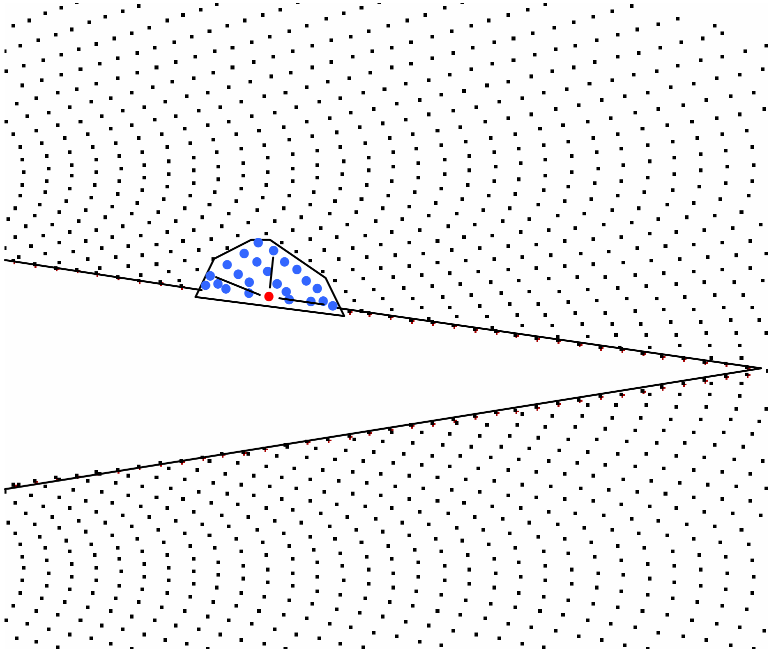
2. Connectivity for the points in Region G2.



CONNECTIVITY GENERATION AT IB



CONNECTIVITY GENERATION AT SB

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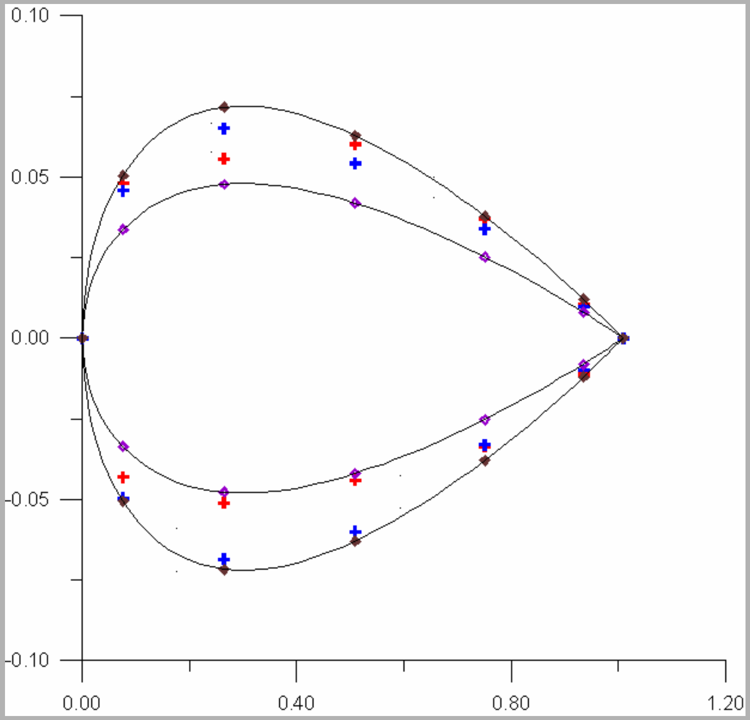
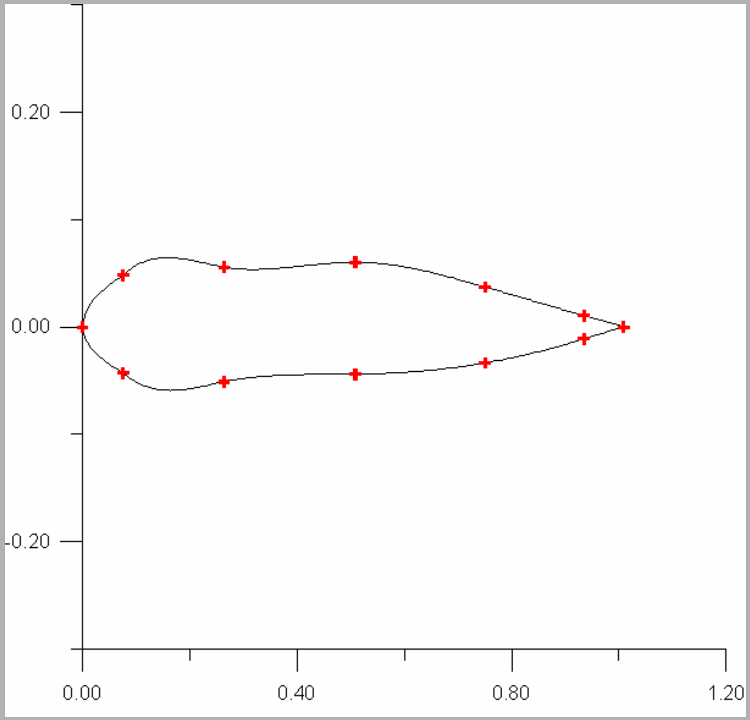
COORDINATES OF CONTROL POINTS CHANGE WITH SHAPE



Two shapes + and +



A typical shape in initial generation

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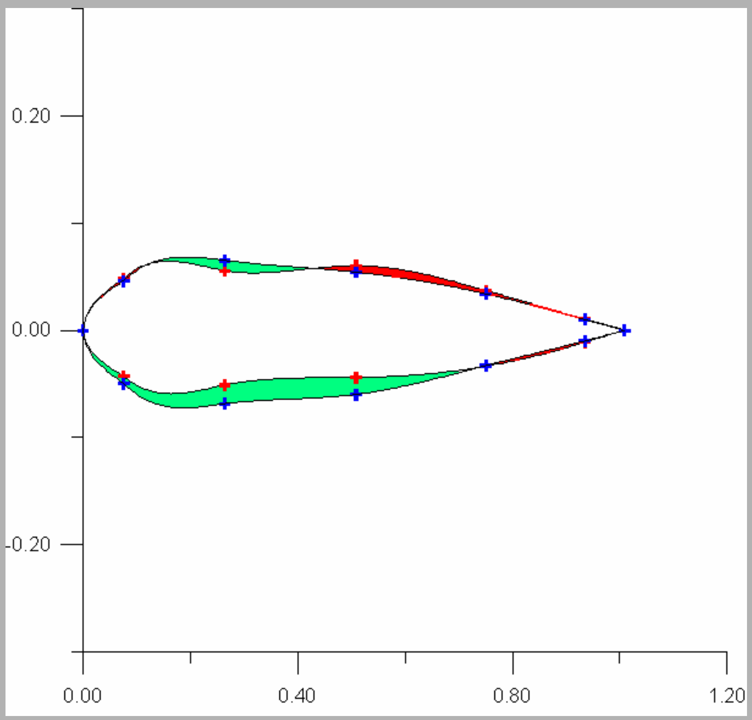
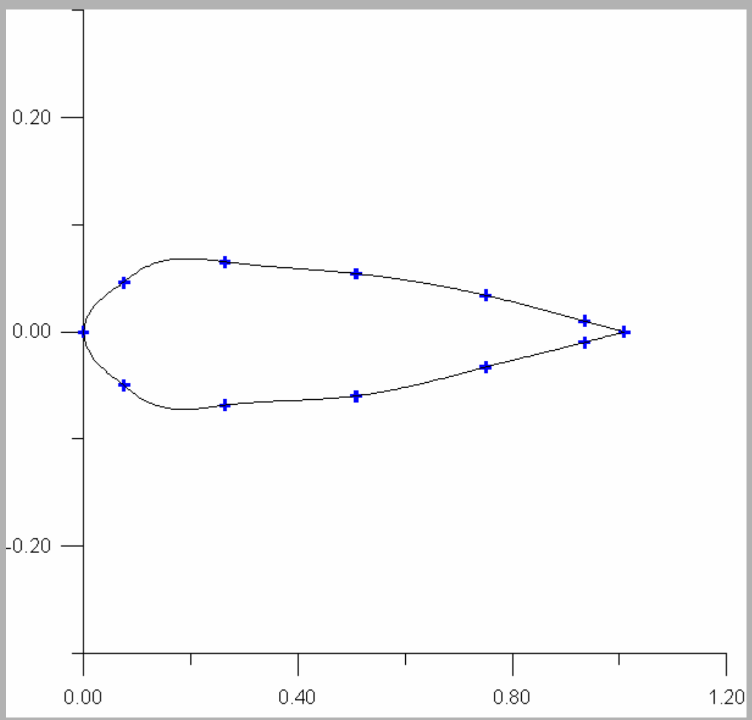
Addition of new SB* points and deletion of previous SB points.





\+ previous

\+ new

*SB = Solid Boundary

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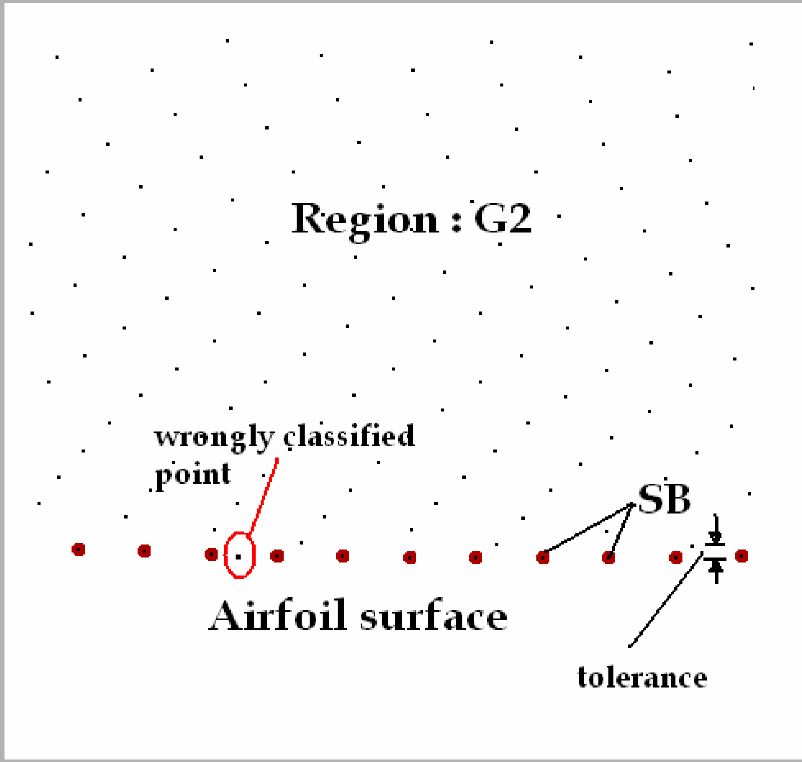
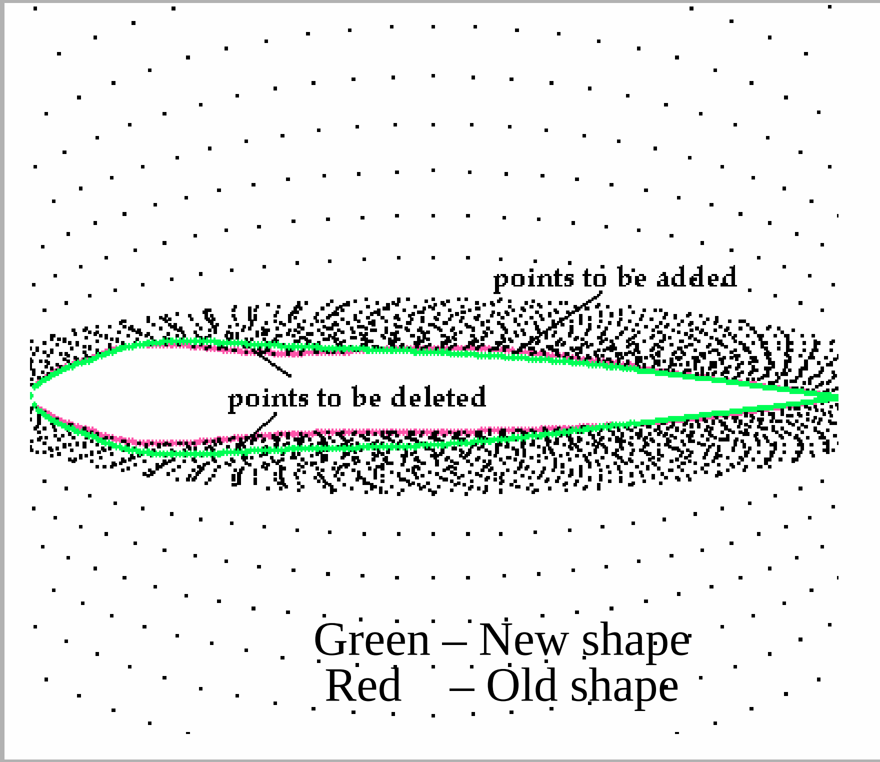
Difficulties encountered (3 Scenarios identified)

1.Wrongly classified points e.g. a blanked point very near the SB gets the flag of interior point.
   Solution : specify tolerance for deletion
2. Points which are blanked in one shape many become interior points in the next shape.
3. Interior points for one shape become blanked points in the next shape.
   Regenerating connectivity in the Region **G2** solves the problem due to 2 and 3.

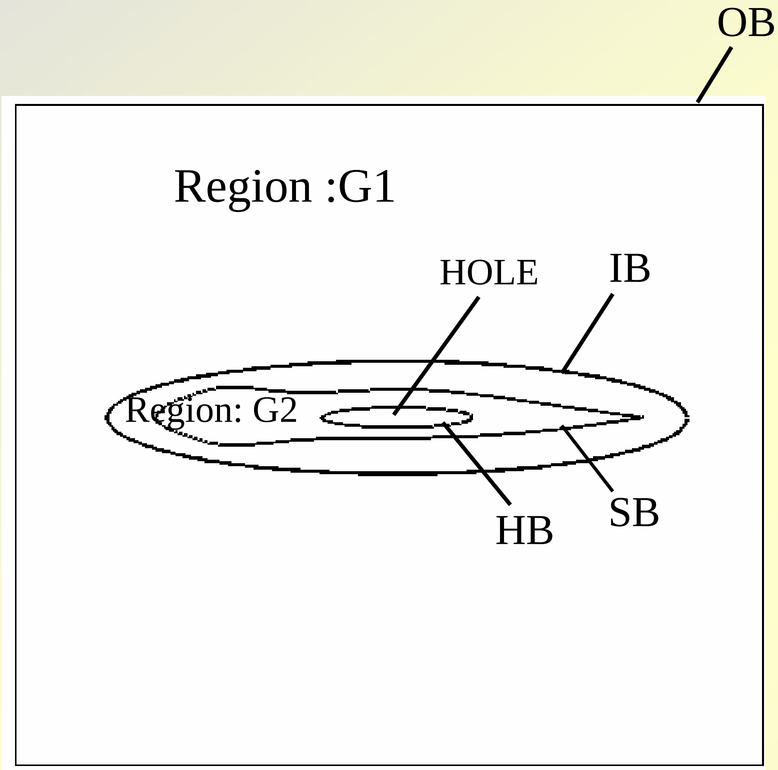


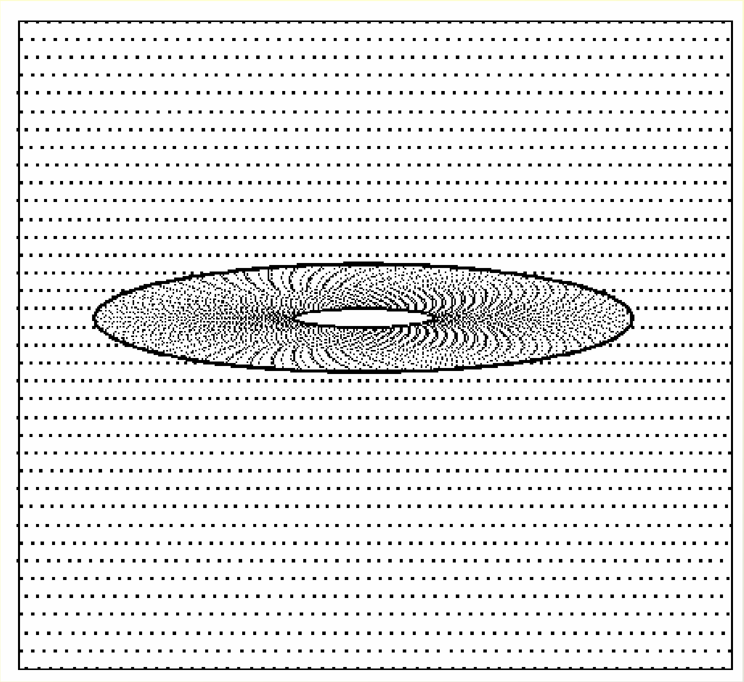


LSKUM HAS THE ADVANTAGE OF OPERATING ON ANY TYPE OF GRID

A cartesian grid can be chosen in the Region G1.

Point Generation





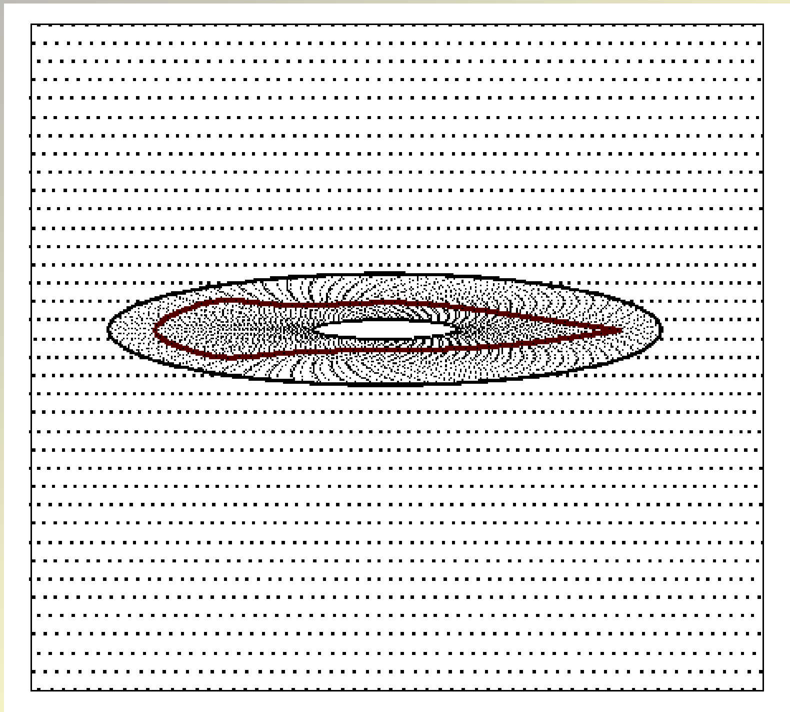
OB = OUTER BOUNDARY

IB = INTERMEDIATE BOUNDARY

SB = SOLID BOUNDARY

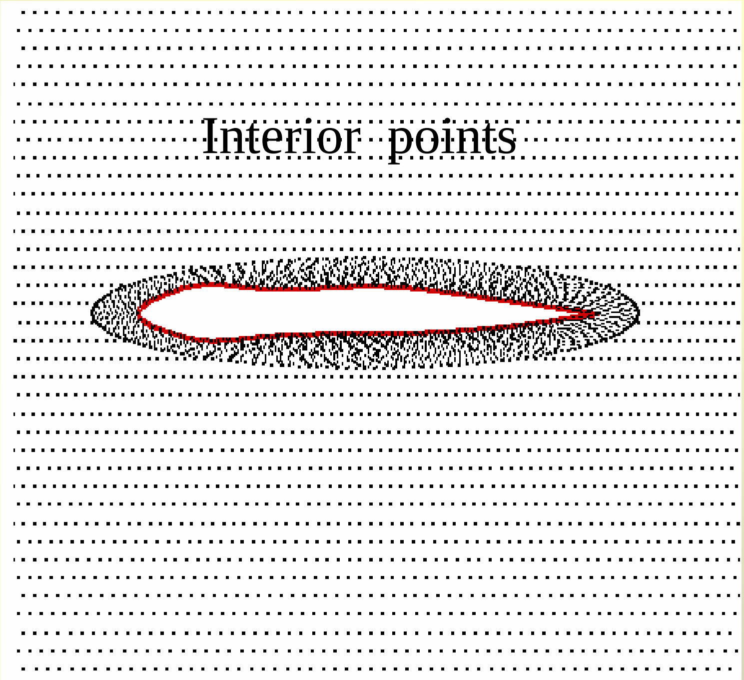
HB = HOLE BOUNDARY

POINT ADDITION



- The points of the airfoil are added into the previous cloud of points.
- These are the points of intersection between gridlines and SB.

POINT DELETION

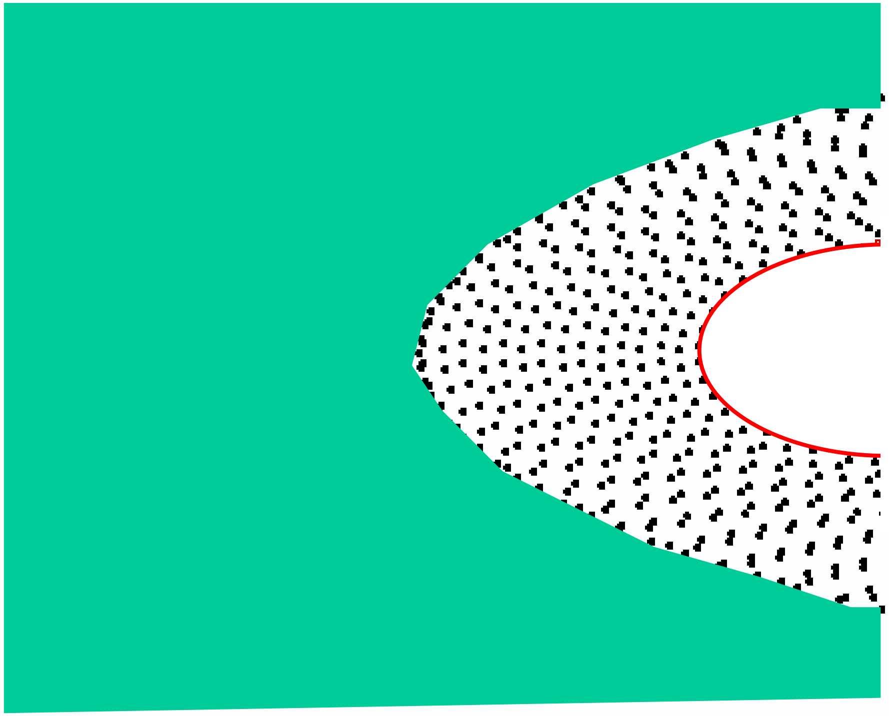


- Flagging using the ray tracing algorithm.
- All points except the SB points are here after referred as interior points.
- Points deleted will be referred as blanked points

CONNECTIVITY GENERATION

Two levels of connectivity generation used.

1. Connectivity of points in Region G1. Its connectivity is not changed during computation.



2. Connectivity in Region G2 is obtained as explained earlier.

In First Generation number of shapes = 16

Fitness function definition*

$$\mathbf{OF} = \frac{0.4}{C_d^{\,2}} \cdot \exp(-10 \times Max\{|0.4 - C_l|, 10^{-4}\})$$

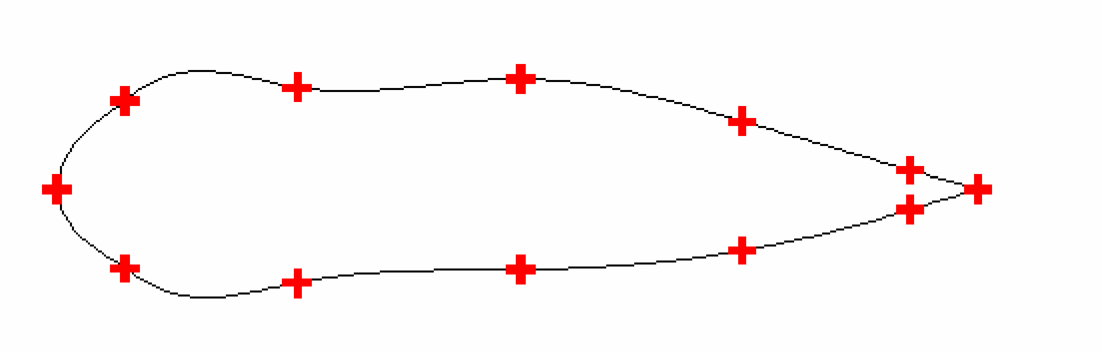
Generation Number	(OF)avg	(OF)max
1	16.044	31.79241
2	21.18463	42.56156
3	34.63741	60.45795
4	47.83723	60.45795
5	47.51205	67.38612
6	53.87326	67.38612
7	56.90183	67.38612
8	64.68613	74.90163
9	66.91092	77.06178
10	67.83563	81.81162
11	71.88188	85.69412
12	72.25507	89.90558
13	72.36669	90.00023
14	79.39845	91.2577
15	76.72232	91.2577

$(OF)_{avg}$ = average of OF for 16 members in the generation

$(OF)_{max}$ = maximum OF among the 16 members in a generation

*Shigeru Obayashi," Target pressure Optimization using MOGA" in "Inverse design and optimization methods" , Lecture series 1997-05, edited by R.A.Vanden Braembussche, M.Manna, Von Karman Institute for Fluid Dynamics, Chausse de Waterloo, 72 B-1640 Rhode Saint Genese, Belgium, April 21-25, 1997.

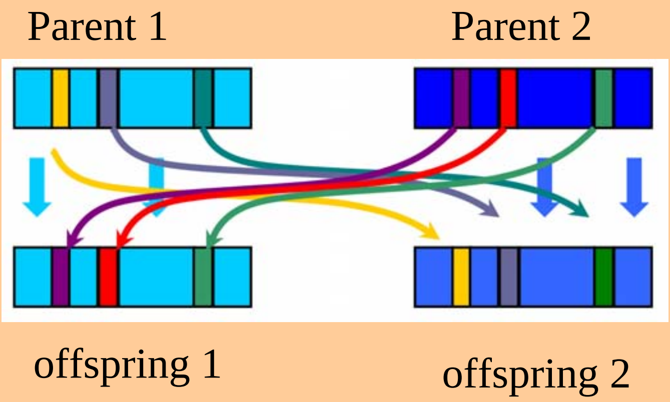
- ANY AIRFOIL CAN BE PARAMETERIZED IN THE CONTROL POINTS.
- ALL THE CONTROL POINTS OF A SINGLE SHAPE ARE CODED INTO BINARY FORM AND APPENDED INTO A SINGLE STRING. THIS STRING IS CALLED THE CHROMOSOME.
- A TYPICAL SHAPE AND ITS CHROMOSOME ARE SHOWN BELOW.



1 1 1 1 0 0 1 0 0 1 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 1 0 0
0 0 0 0 1 1 1 0 0 0 0 1 0 0 1 1 0 0 0 1 1 1 0 1 1 1 1 1 1 1
1 1 1 0 1 1 0 1 0 0 0 0 1 1 1 1 0 1 1 0 1 0 1 0 0 1 0 1 1 1
1 1 1 0 0 1 1 1 0 0 1 1 0 1 0 0 0 0 0 1 1 1 1 0 1 1 0 0 1 0
1 0 1 0 1 1 1 1 0 0 0 0 1 1 0 1 1 0 1 0 0 0 1 1 0 1 1 1 0 0

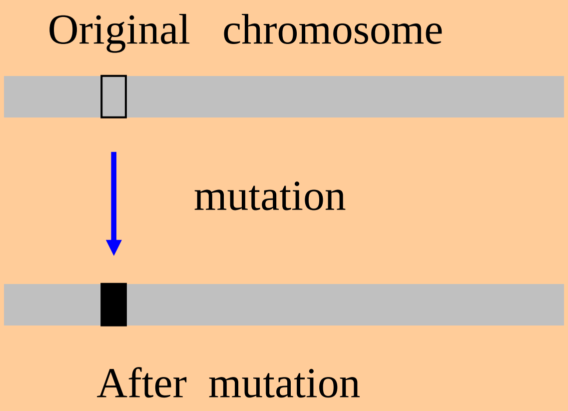
CROSSOVER

- Provides random information exchange (Works on couples of individuals)
- Uniform crossover



MUTATION

- Mutation - preserves population diversity (Works on single individual)



- Selection is made based on method of tournament
- Uniform crossover was taken with probability = 0.5
- Jump mutations are done with probability = 0.02
- If the best individual not necessarily replicated from one generation to the next there would be loss of computation as some of the potential candidates to reproduce have been lost. To avoid such a situation best individual are replicated into next generation which is termed as elitism.

Generation 1

Number of Crossovers (bits) = 602

Number of Jump Mutations (bits)= 47

Elitist Reproduction on Individual = 9 of 16

2 children for pair of selected parents

Generation 2

Number of Crossovers (bits) = 596

Number of Jump Mutations (bits) = 53

Elitist Reproduction on Individual = 6 of 16

Generation 3

Number of Crossovers (bits) = 588

Number of Jump Mutations (bits) = 49

Elitist Reproduction on Individual = 10 of 16

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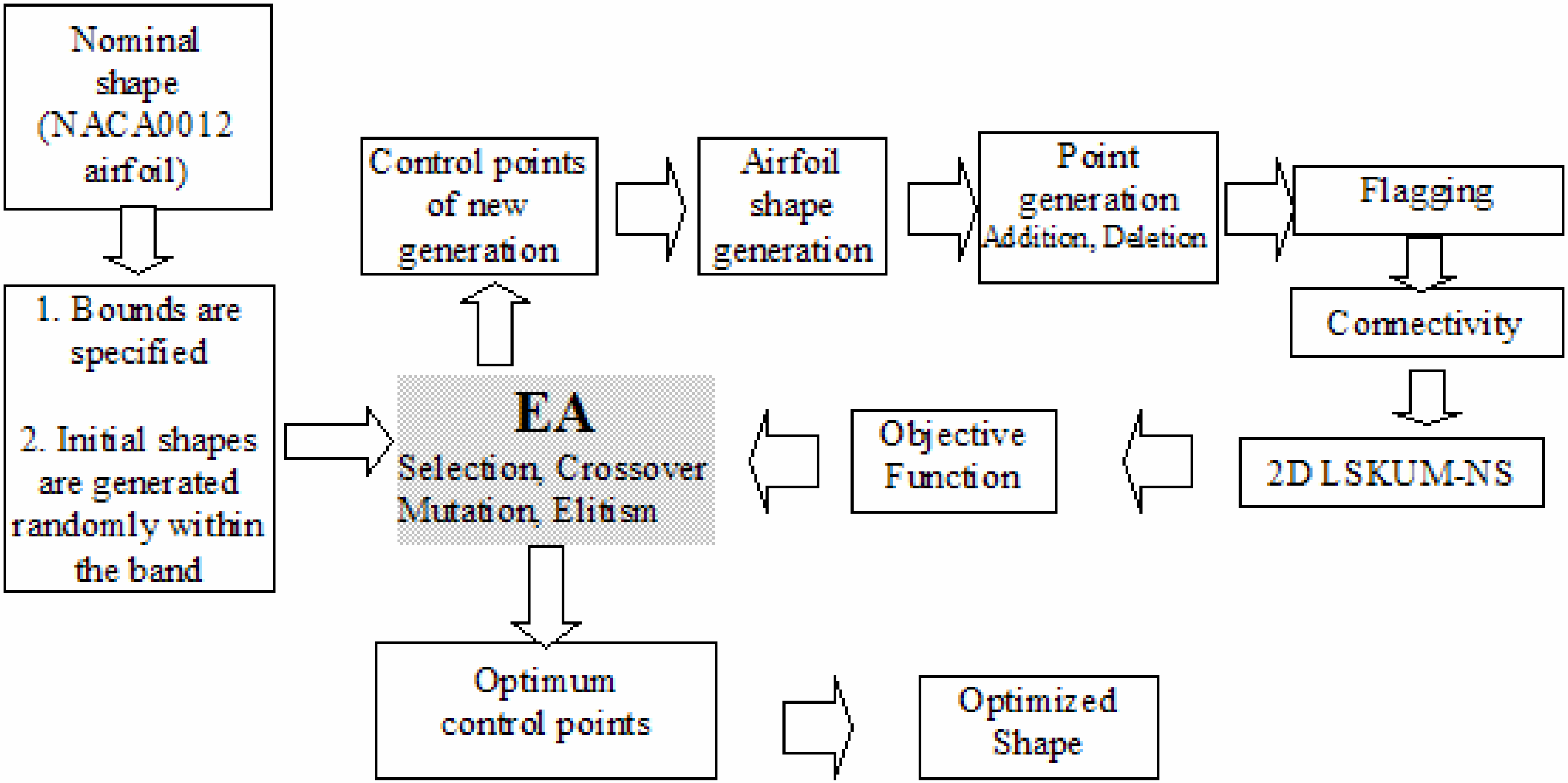
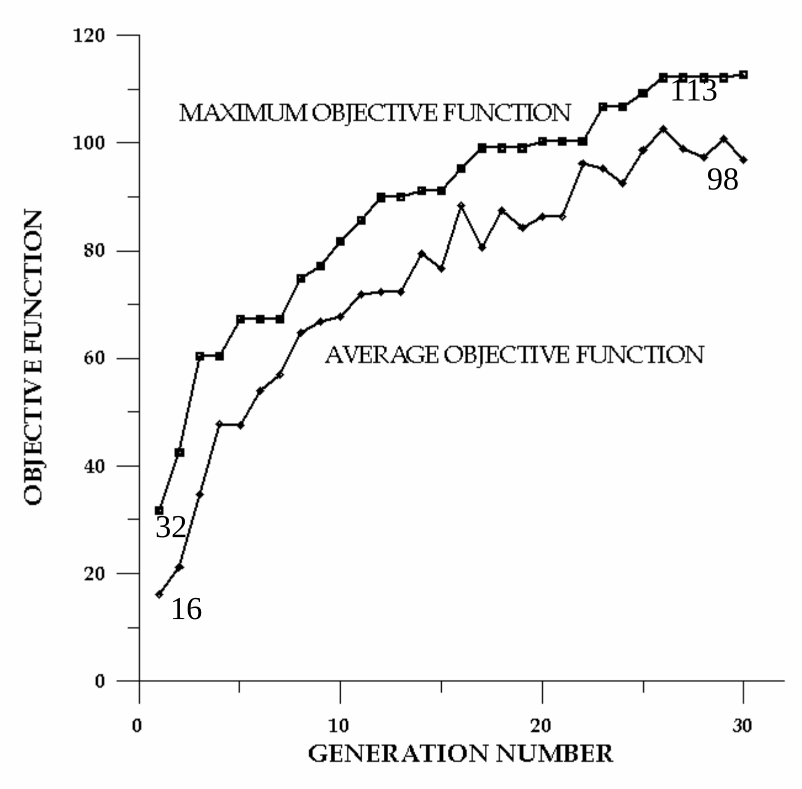
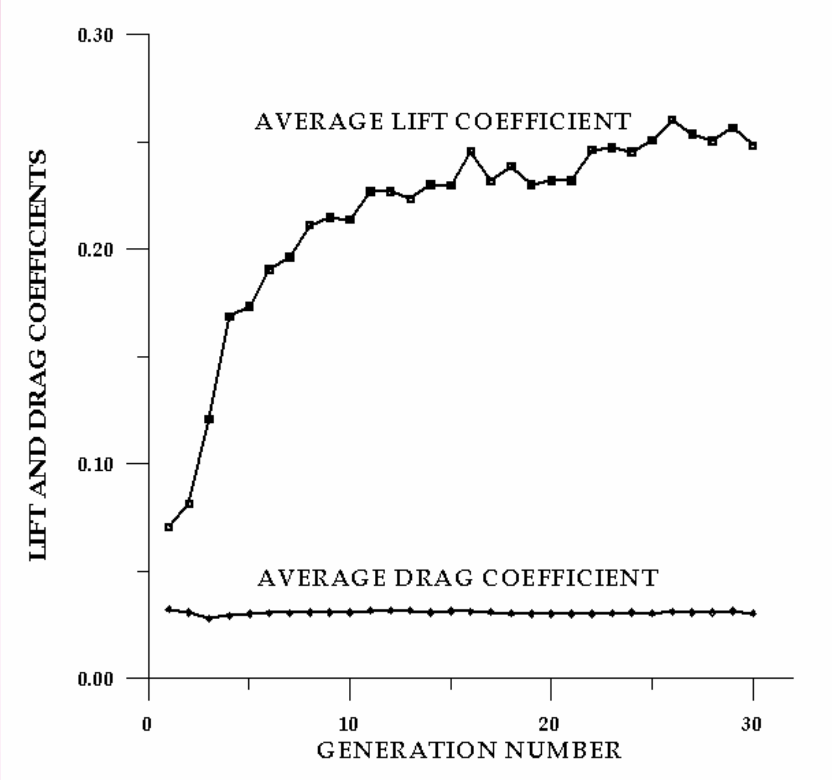
RESULTS



Schematic of Grid free solver coupled with EA





$(OF)_{Max}$ and $(OF)_{avg}$ Vs Generation.

Cl_{avg} and Cd_{avg} in each Generation

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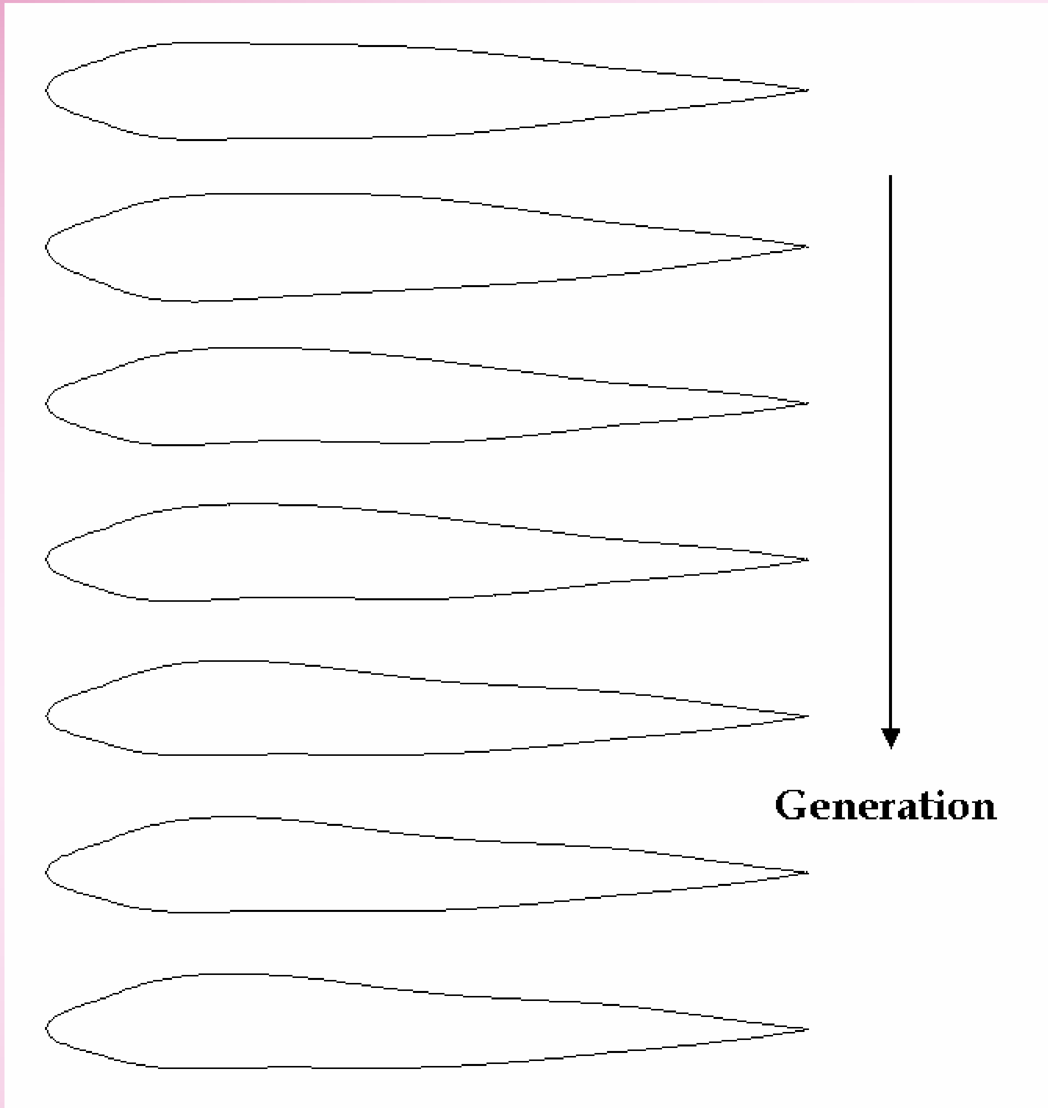
BEST SHAPE IN EACH GENERATION

(1ST, 5TH, 10TH, 15TH, 20TH,
25TH AND 30TH GENERATION)



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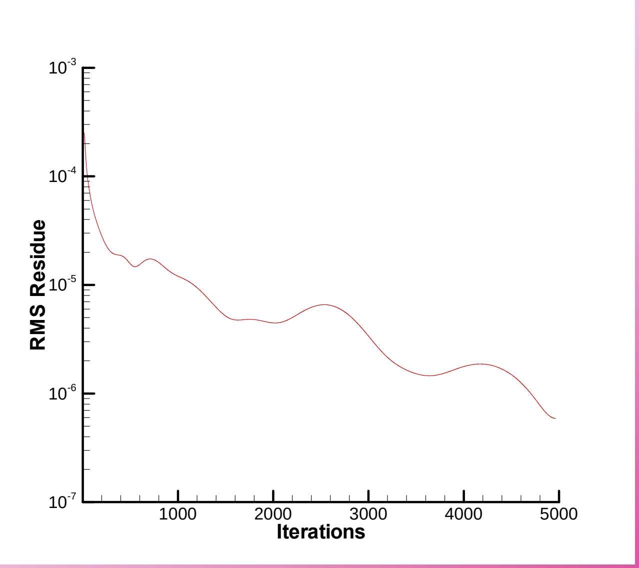
SIZE OF THE PROBLEM

Number of points in the cloud : 19548 to 22102
Machine used for calculation : DEC-Alpha @ 750 MHz

No. of CFD Solver calls for 30 generations = 480
@ 16 calls/generation

Three order drop in residue



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TIME REQUIREMENT

	Minute	%
For GA	0.02	0.02
For Grid Generation (neighborhood generation only)	4.8 to 6.4	~5.39
For LSKUM-NS Solver (3-order drop in Residue)	95.2 to 101.3	~94.59

Total time taken = 836 hr, 12 min.
(30 generations of simulation) (34days, 8hr, 12min.)

CONCLUSIONS

- GRID FREE SOLVER LSKUM-NS COMBINED SUCCESSFULLY WITH EA FOR OPTIMIZATION OF 2D SHAPES
- THE POTENTIAL OF LSKUM + EA NEEDS TO BE EXPLOITED FURTHER