Efficient Aerodynamic Shape Designs by Adjoint Approaches

Nicolas Gauger [1,2]

Joint work with J. Brezillon [1] and A. Fazzolari [1,3]

[1] DLR Braunschweig
Institute of Aerodynamics and Flow Technology
Numerical Methods Branch

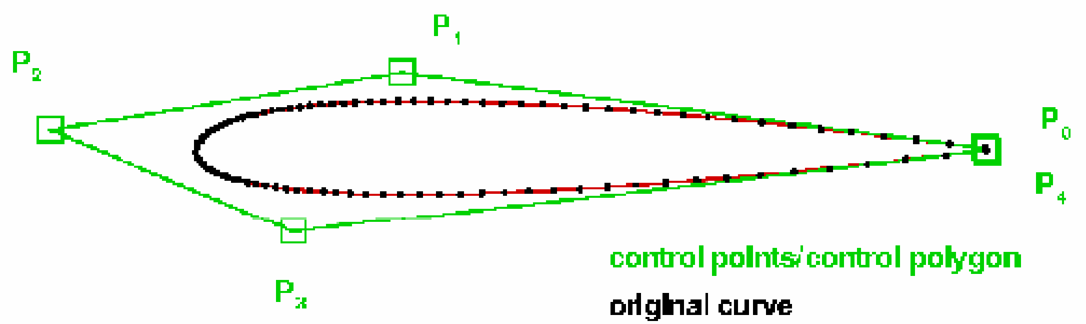
[2] Humboldt University Berlin
Department of Mathematics

[3] Technische Universität Carolo-Wilhelmina zu Braunschweig
Graduiertenkolleg Wechselwirkung von Struktur und Fluid

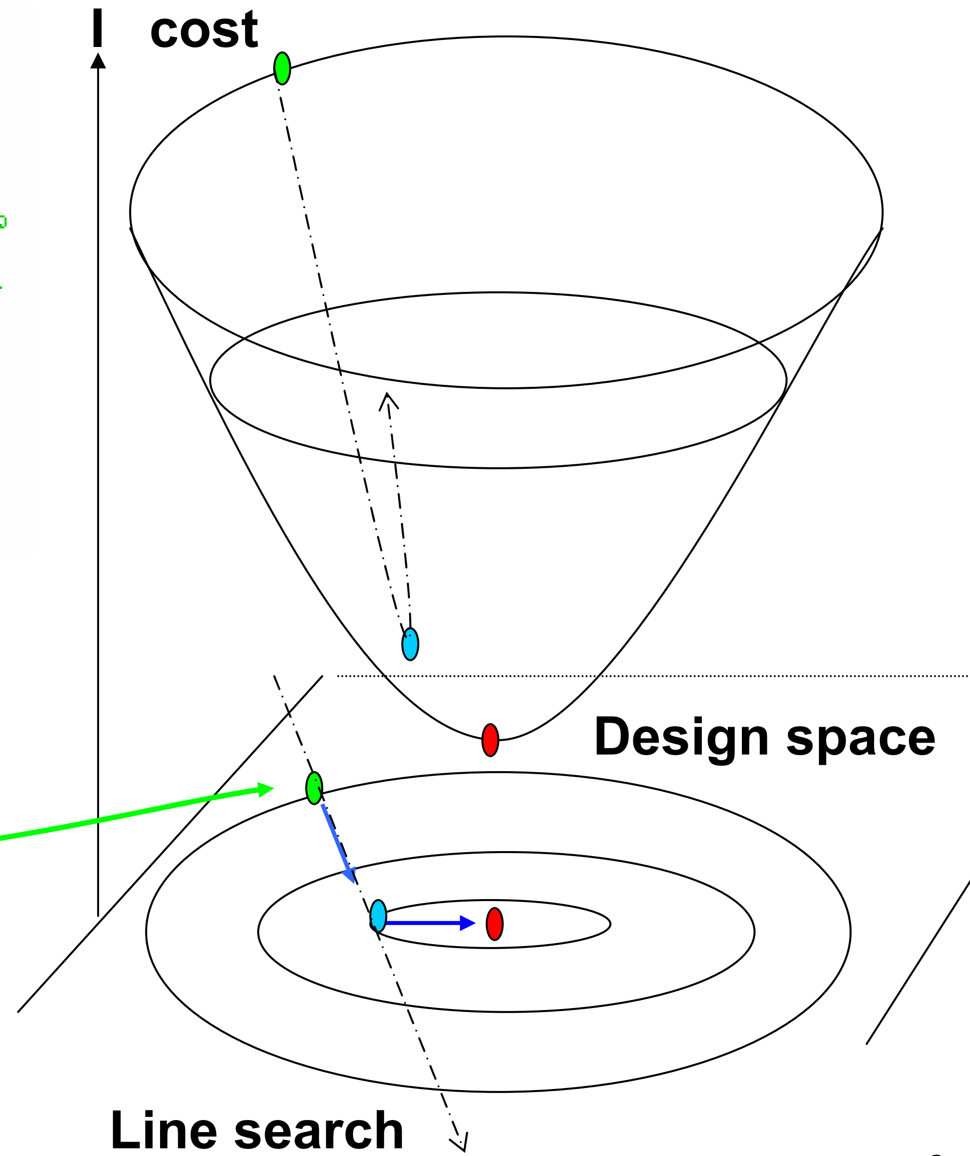
Outline

- Aerodynamic shape optimization
- Continuous adjoint approach
- Validation and application in 2D and 3D
- Coupled aero-structure adjoint approach
- Validation and application in MDO context
- (One shot approach)
- Conclusions

Aerodynamic Shape Optimization



Parametrized airfoil



Search direction

" ↘ " : $$-\nabla I = -\left(\ldots, \frac{\delta I}{\delta P_i}, \ldots \right)^T_{i=1,\ldots,n}$$

Governing Equations and Aerodynamic Coefficients

Compressible 2D Euler-Equations

$$\frac{\partial w}{\partial t}+\frac{\partial f}{\partial x}+\frac{\partial g}{\partial y}=0$$

while

$$w=\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho E\end{pmatrix},\quad f=\begin{pmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ \rho uH\end{pmatrix},\quad g=\begin{pmatrix}\rho v\\ \rho vu\\ \rho v^2+p\\ \rho vH\end{pmatrix}$$

Pressure (ideal gas)

$$p=(\gamma-1)\rho(E-\frac{1}{2}\vec{v}^2)$$

Dimensionless pressure

$$C_p=\frac{2(p-p_\infty)}{\gamma M_\infty^2 p_\infty}$$

Drag, lift, pitching moment coefficients

$$C_D=\frac{1}{C_{ref}}\int_C C_p(n_x\cos\alpha+n_y\sin\alpha)dl$$

$$C_L=\frac{1}{C_{ref}}\int_C C_p(n_y\cos\alpha-n_x\sin\alpha)dl$$

$$C_m=\frac{1}{C_{ref}^2}\int_C C_p(n_y(x-x_m)-n_x(y-y_m))dl$$

Finite Differences

i-loop
i=1,...,n

Variation of i-th design variable

$$\delta C_D = \frac{2}{\gamma M_\infty^2 p_\infty C_{ref}} \int_C \delta p \, (n_x \cos\alpha + n_y \sin\alpha) dl$$
$$+ \frac{1}{C_{ref}} \int_C C_p (\delta n_x \cos\alpha + \delta n_y \sin\alpha) dl \,,$$

Metric sensitivities → **pressure variation** → **aerodynamic sensitivity**

i-th component of cost function‘s gradient

- **Finite Differences** ➡ **n design variables require n+1 flow calculations**

Motivation of Adjoint Approach

High number of design variables

- **Finite Differences** → **n design variables require n+1 flow calculations**

- **Adjoint Approach** → **n design variables require 1 flow and 1 adjoint flow calculation**

  Independent of number of design variables

  High accuracy

Continuous Adjoint Approach

Adjoint Euler-Equations:

$$-\frac{\partial \psi}{\partial t}-\left(\frac{\partial f}{\partial w}\right)^T\frac{\partial \psi}{\partial x}-\left(\frac{\partial g}{\partial w}\right)^T\frac{\partial \psi}{\partial y}=0$$

Ψ: Vector of adjoint variables

Boundary conditions:

Wall: $n_x\psi_2+n_y\psi_3=\underline{-d(I)}$

Farfield: $\delta x_\xi,\ldots,\delta y_\eta=0,\ \delta w=0$

Adjoint formulation of cost function's gradient:

$$\delta I=-\int_C p(-\psi_2\delta y_\xi+\psi_3\delta x_\xi)dl+\underline{K(I)}$$

$$-\int_D \psi_\xi^T(\delta y_\eta f-\delta x_\eta g)+\psi_\eta^T(-\delta y_\xi f+\delta x_\xi g)dA$$

Continuous Adjoint Approach

$$d(C_D) = \frac{2}{\gamma M_\infty^2 p_\infty C_{ref}} (n_x \cos\alpha + n_y \sin\alpha)$$

Drag

$$K(C_D) = \frac{1}{C_{ref}} \int_C C_p (\delta n_x \cos\alpha + \delta n_y \sin\alpha) dl$$

$$d(C_L) = \frac{2}{\gamma M_\infty^2 p_\infty C_{ref}} (n_y \cos\alpha - n_x \sin\alpha)$$

Lift

$$K(C_L) = \frac{1}{C_{ref}} \int_C C_p (\delta n_y \cos\alpha - \delta n_x \sin\alpha) dl$$

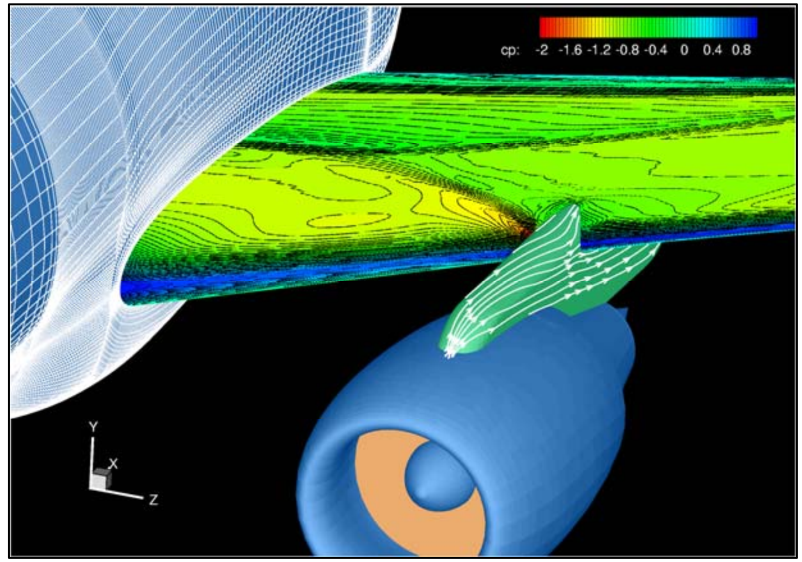
$$d(C_m) = \frac{2}{\gamma M_\infty^2 p_\infty C_{ref}^2} (n_y (x - x_m) - n_x (y - y_m))$$

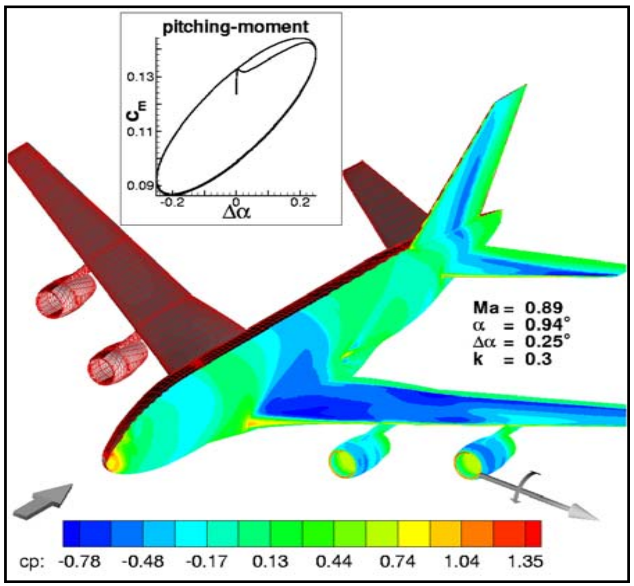
Pitching moment

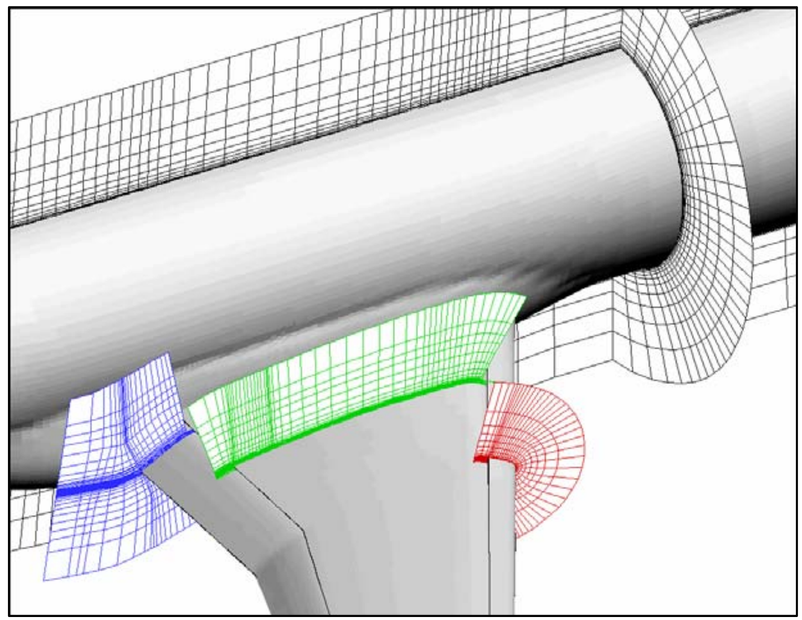
$$K(C_m) = \frac{1}{C_{ref}^2} \int_C C_p \delta (n_y (x - x_m) - n_x (y - y_m)) dl$$

Block-Structured RANS Solver FLOWer
(Reynolds-Averaged Navier-Stokes)

- advanced turbulence and transition models
- 2nd order finite volume discretization (cell centered & cell vertex option)
- central and upwind schemes
- multigrid
- implicit treatment of turbulence equations
- implicit schemes for time accurate flows
- preconditioning for low speed flow
- design option (inverse design, adjoint)



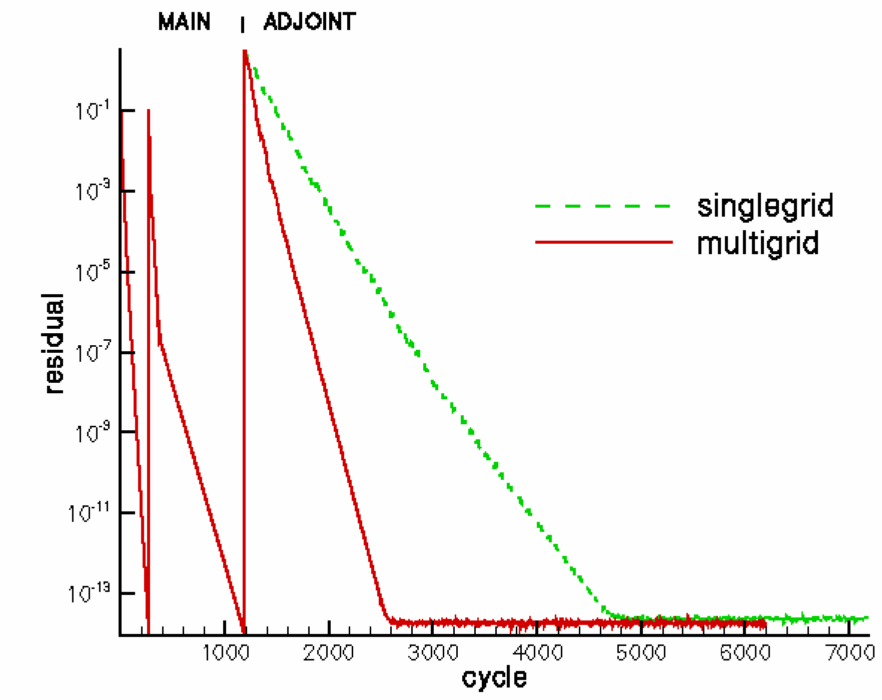




Adjoint Flow Solver

Adjoint solver on block-structured grids

- **continuous adjoint approach**
- **implemented in FLOWer**
- **cost functions: lift, drag & moment and combinations**
- **adjoint solver based on multigrid**
- **Euler & Navier-Stokes (frozen μ)**
- **AD for turbulence equations (*Fast*Opt)**



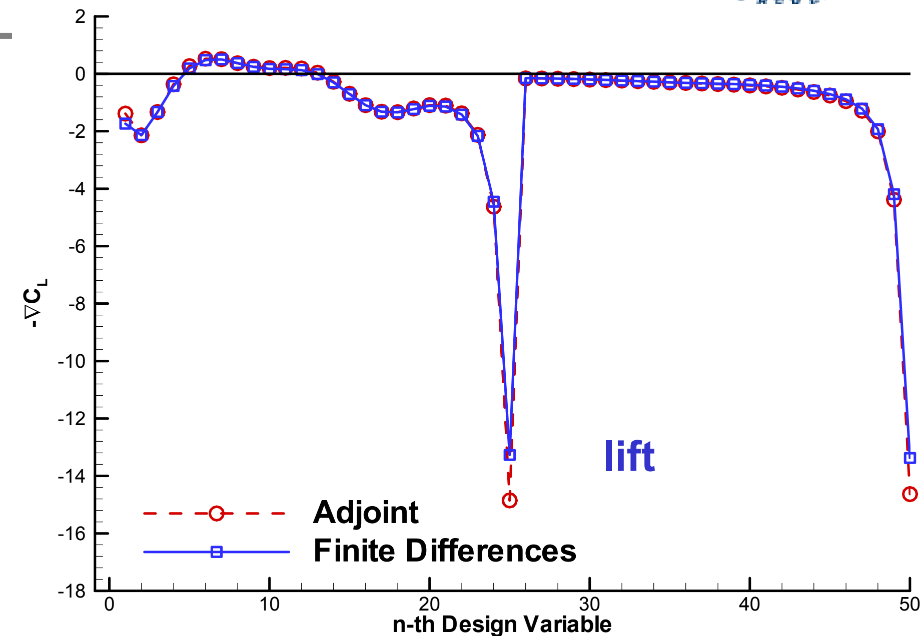
convergence history, FLOWer

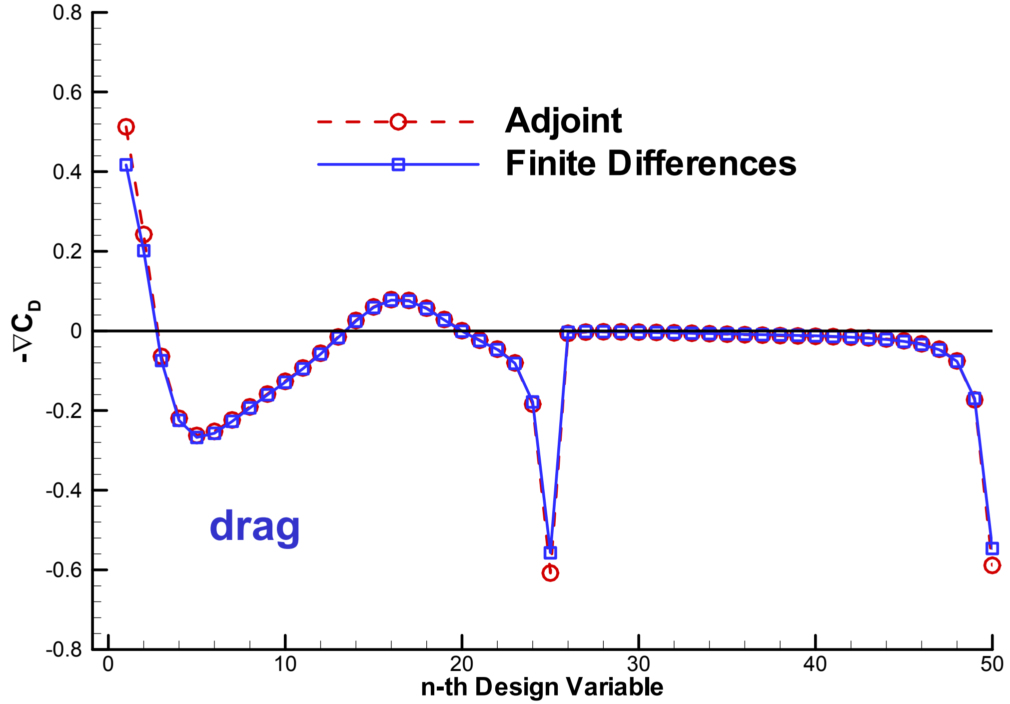
Validation of Euler Adjoint

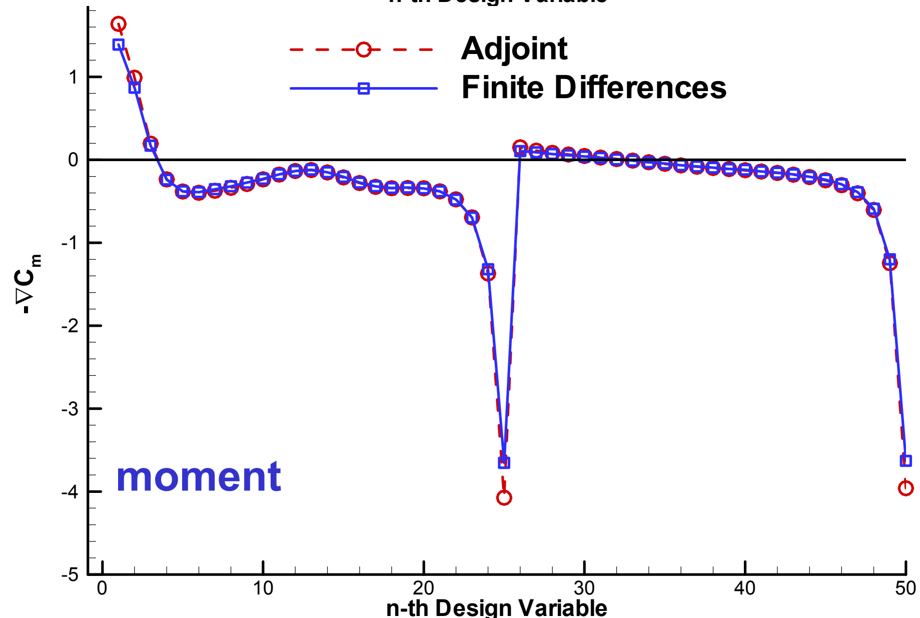
adjoint gradient vs. finite differences‘ gradient

finite differences:
51 calls of FLOWer MAIN
adjoint approach:
1 call of FLOWer MAIN
3 calls of FLOWer ADJOINT

RAE2822
M_∞=0.73, α = 2.0°
50 design variables
(B-spline)







Validation of adjoint gradient based optimization

Objective function

- Drag reduction for RAE 2822 airfoil
- $M_\infty = 0.73$, $\alpha = 2.00°$

Constraints

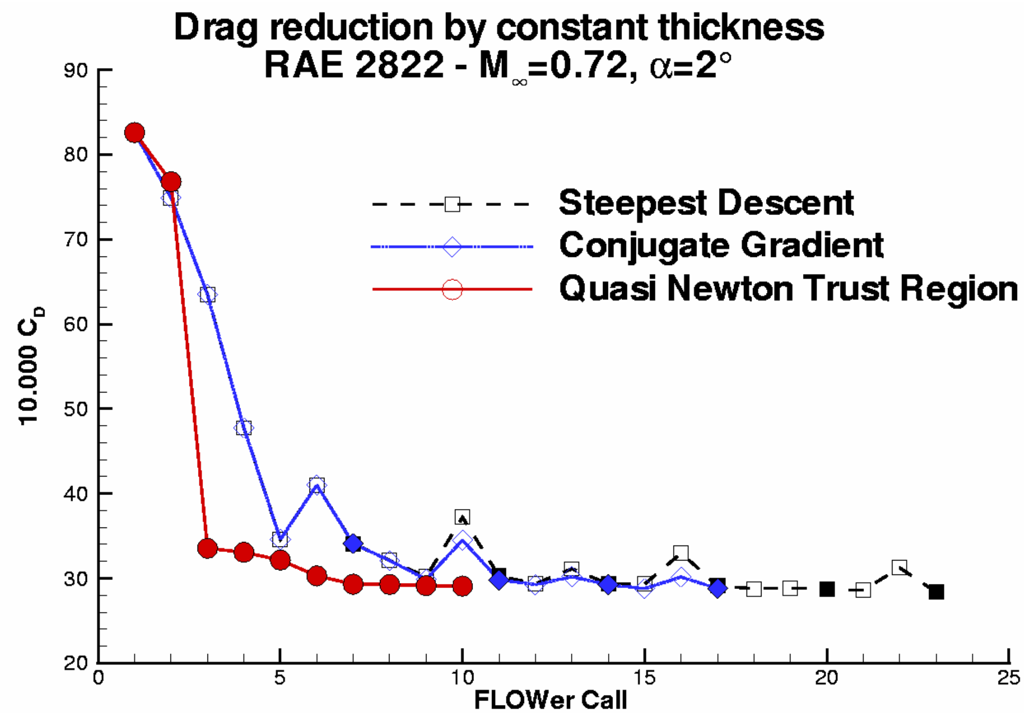
- Constant thickness

Approach

- FLOWer Euler Adjoint
- Deformation of camberline (20 Hicks-Henne functions)

Optimizer

- Steepest Descent
- Conjugate Gradient
- Quasi Newton Trust Region



Validation of adjoint gradient based optimization

MEGAFLOW

Objective function

- Drag reduction for RAE 2822 airfoil
- $M_\infty = 0.73$, $\alpha = 2.00°$

Constraints

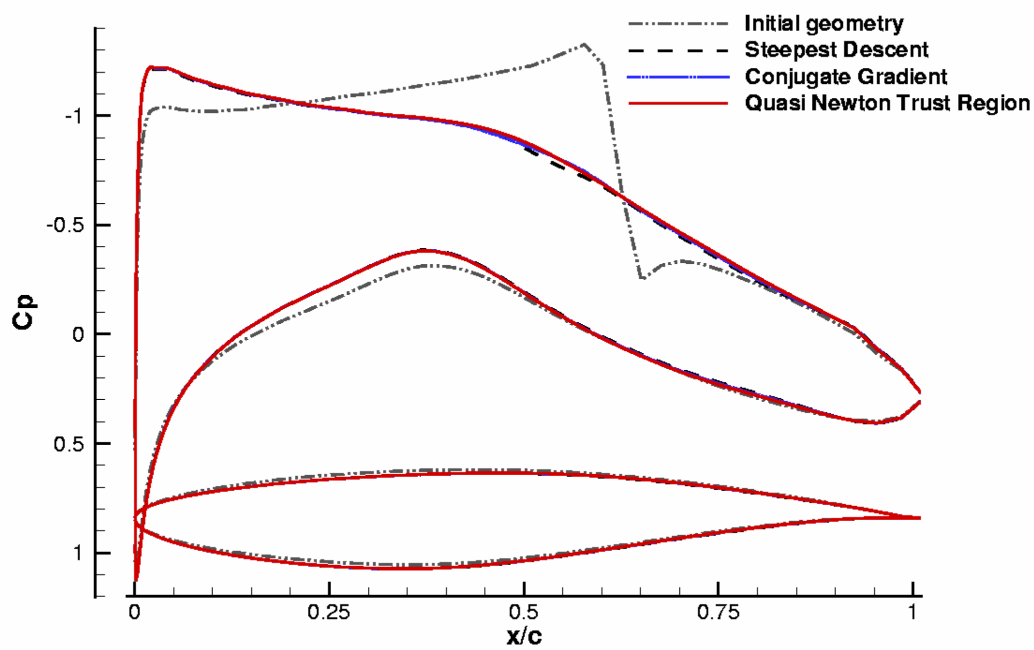
- Constant thickness

Approach

- FLOWer Euler Adjoint
- Deformation of camberline (20 Hicks-Henne functions)

Optimizer

- Steepest Descent
- Conjugate Gradient
- Quasi Newton Trust Region



Multi-constraint airfoil optimization RAE2822

Objective function

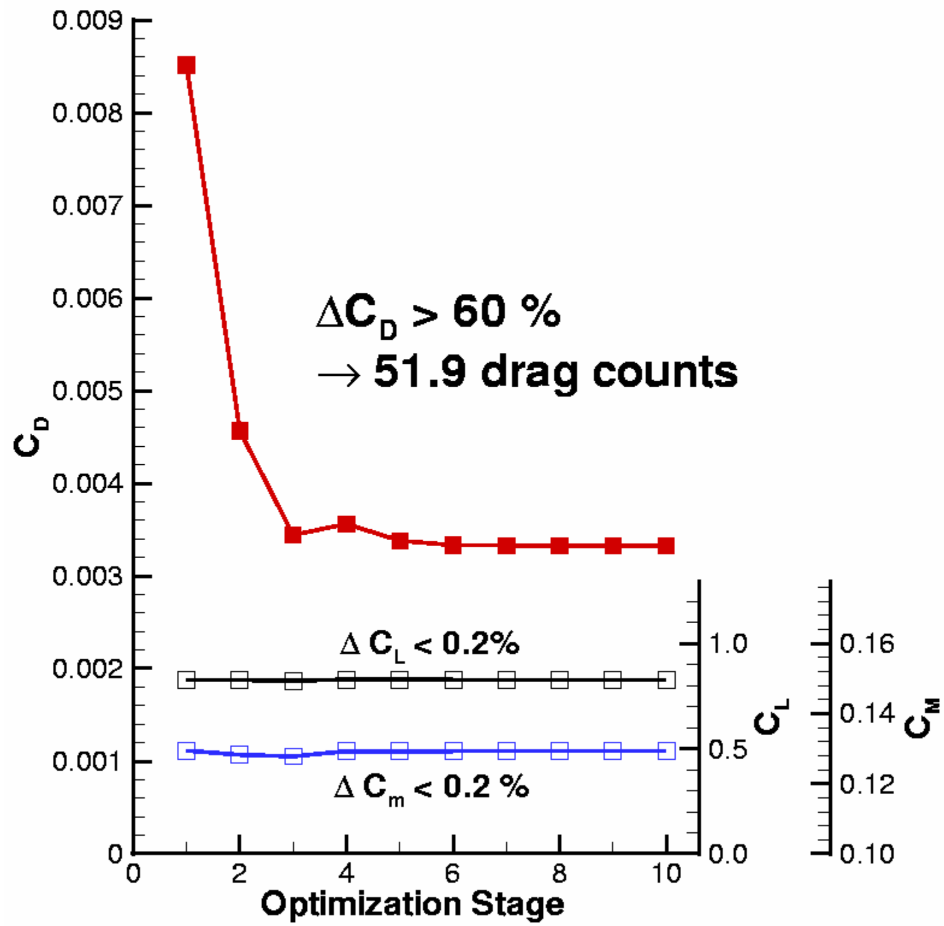
- Drag reduction for RAE 2822 airfoil
- $M_\infty = 0.73$, $\alpha = 2.0°$

Constraints

- Lift, pitching moment and angle of attack held constant
- Constant thickness

Approach

- FLOWer Euler Adjoint
- Constraints handled by feasible direction
- Deformation of camberline



Multi-constraint airfoil optimization RAE2822

Objective function

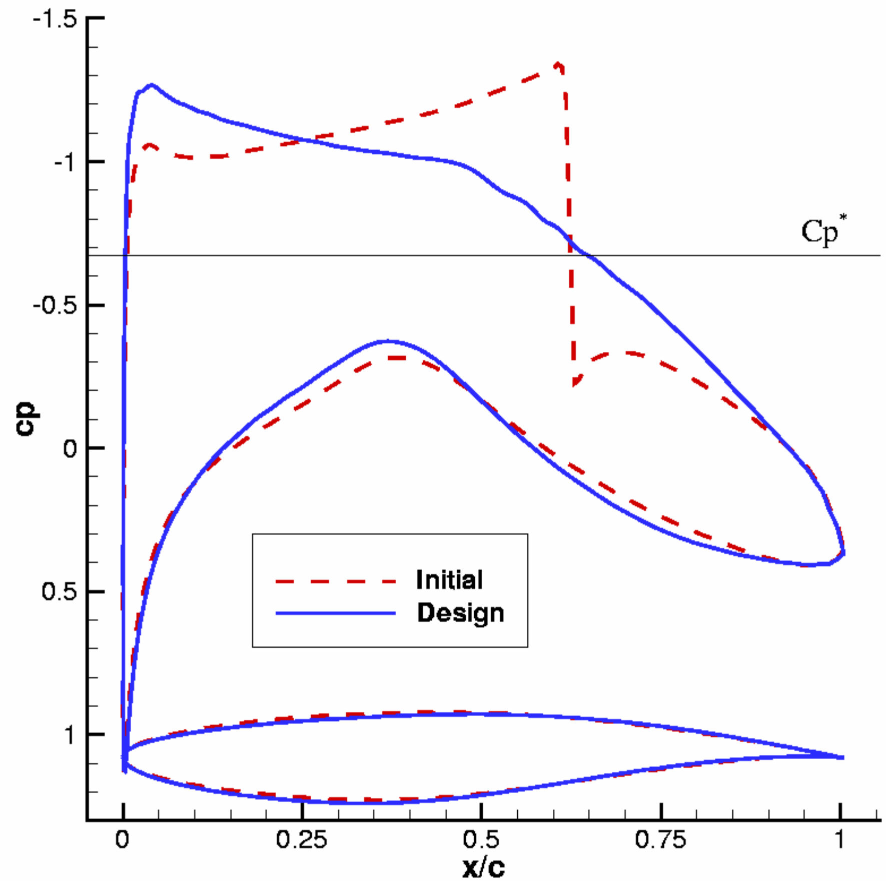
- Drag reduction for RAE 2822 airfoil
- M_∞ =0.73, α=2.0°

Constraints

- Lift, pitching moment and angle of attack held constant
- Constant thickness

Approach

- FLOWer Euler Adjoint
- Constraints handled by feasible direction
- Deformation of camberline



surface pressure distribution

Multipoint airfoil optimization RAE2822

Objective function

- **Reduction of drag in 2 design points**

$$I = \sum_{i=1}^{2} W_i C_d(\alpha_i, M_i)$$

Design points

- **1 : M_∞=0.734, CL = 0.80 , α = 2.8°, Re=6.5x10^6, xtrans=3%, W_1=2**
- **2 : M_∞=0.754, CL = 0.74 , α = 2.8°, Re=6.2x10^6, xtrans=3%, W_2=1**

Constraints

- **No lift decrease, no change in angle of incidence**
- **Variation in pitching moment less than 2% in each point**
- **Maximal thickness constant and at 5% chord more than 96% of initial**
- **Leading edge radius more than 90% of initial**
- **Trailing edge angle more than 80% of initial**

Multipoint airfoil optimization RAE2822

Parameterization

- 20 design variables changing camberline, Hicks-Henne functions

Optimization strategy

- Constrained SQP
- Navier-Stokes solver FLOWer, Baldwin/Lomax turbulence model
- Gradients provided by FLOWer Adjoint, based on Euler equations

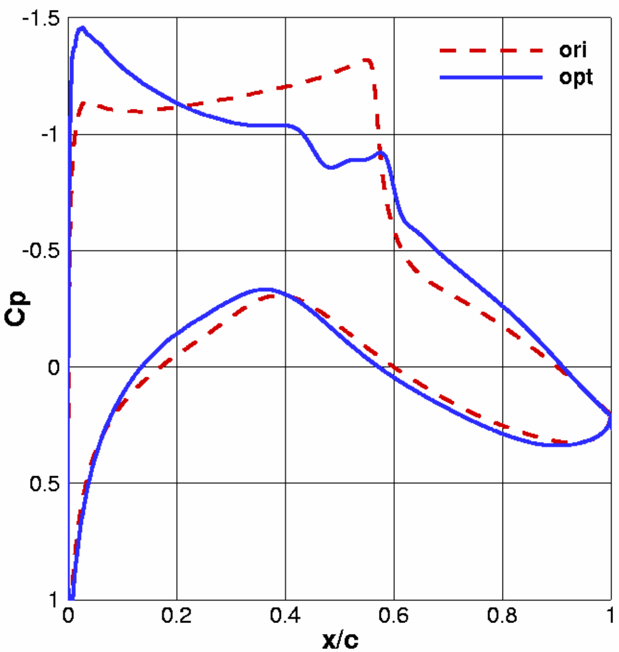
Results

Pt	α	M_i	cl^t	cd^t $(.10^{-4})$	cl	cd^t $(.10^{-4})$	$\Delta cd/cd^t$	$\Delta cl/cl^t$	$\Delta cm/cm^t$
1	2.8	0.734	0.811	197.1	0.811	135.5	-31.2%	0%	+1.6%
2	2.8	0.754	0.806	300.8	0.828	215.0	-27.4%	+2.7%	+2.0%

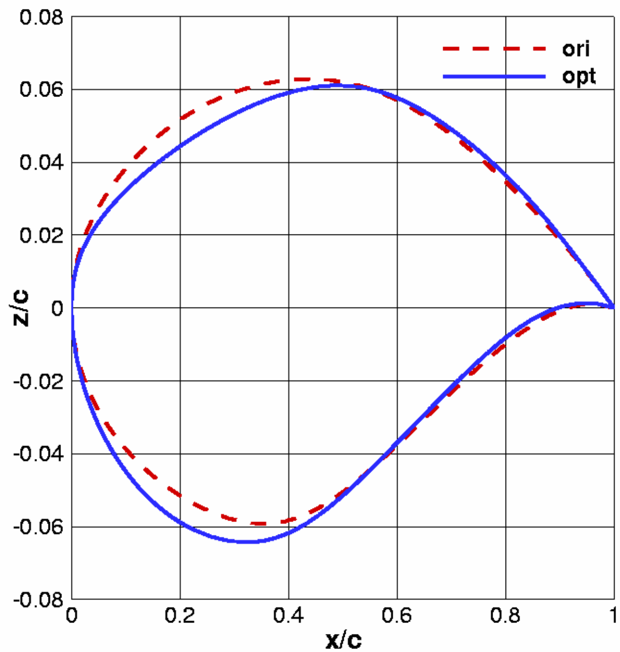
Multipoint airfoil optimization RAE2822

1. design point

M_∞=0.734, α=2.8°



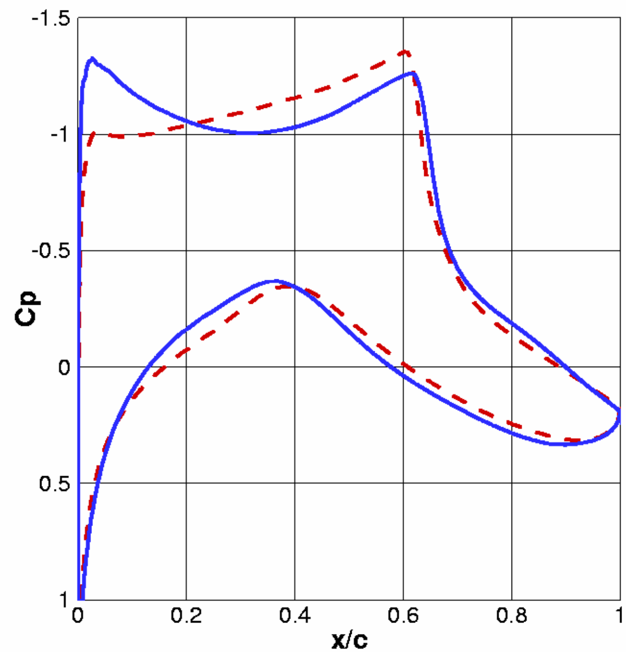
Airfoil Geometry



shape geometry

2. design point

M_∞=0.754, α=2.8°



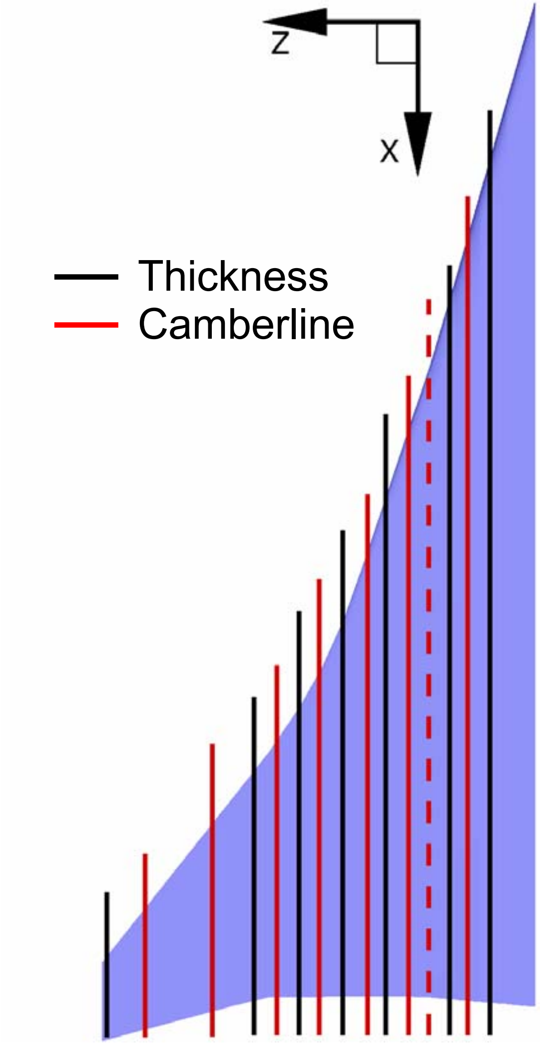
Optimization of SCT Configuration

Drag reduction at constant lift

- Mach number = 2.0
- lift coefficient = 0.1207

Design variables

Fuselage	10 parameters
twist deformation:	10 parameters
camberline (8 sections):	32 parameters
thickness (8 sections) :	32 parameters
angle of attack:	1 parameter
	85 parameters



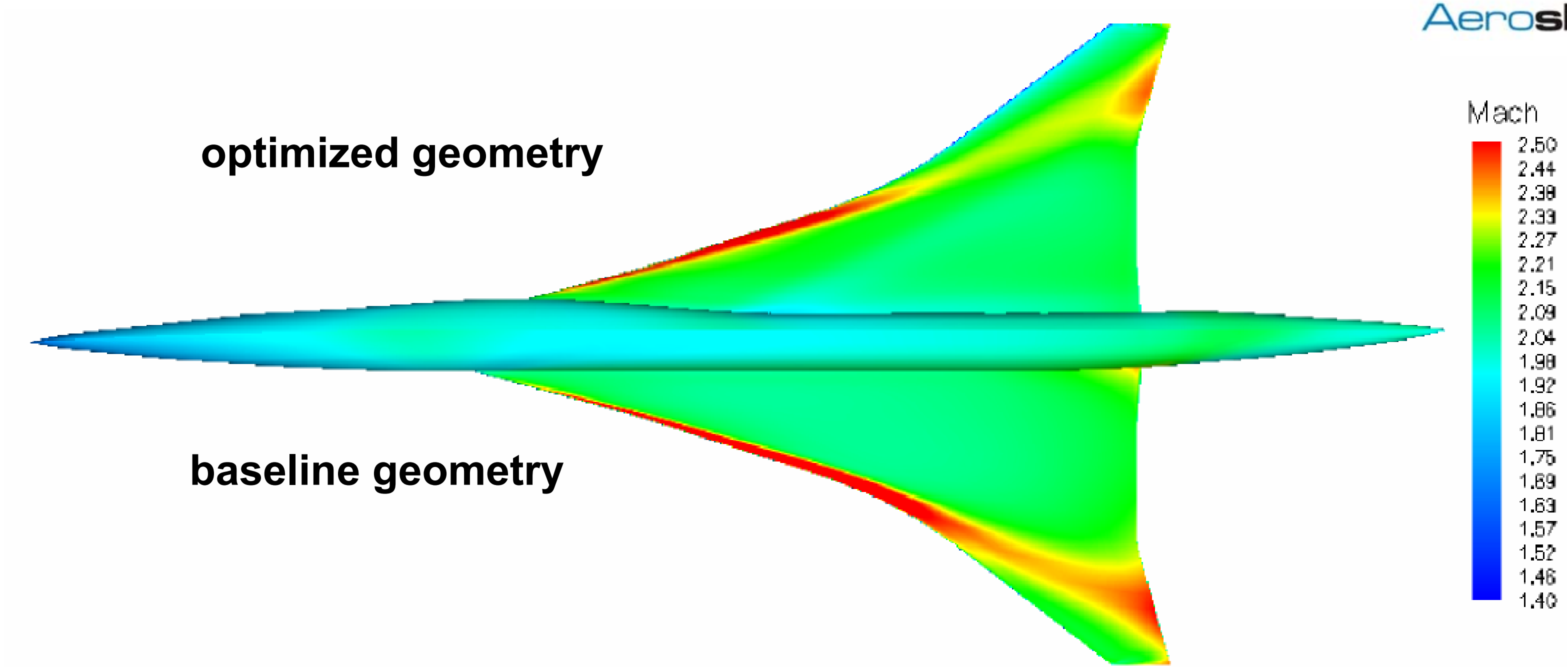
Geometric constraints

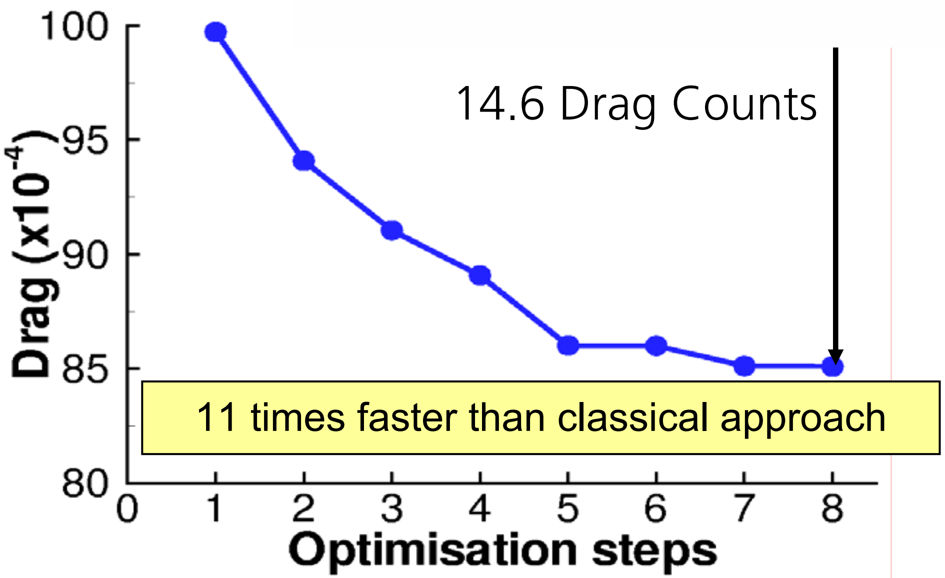
- minimum wing thickness distribution along the spanwise direction
- minimum fuselage radius

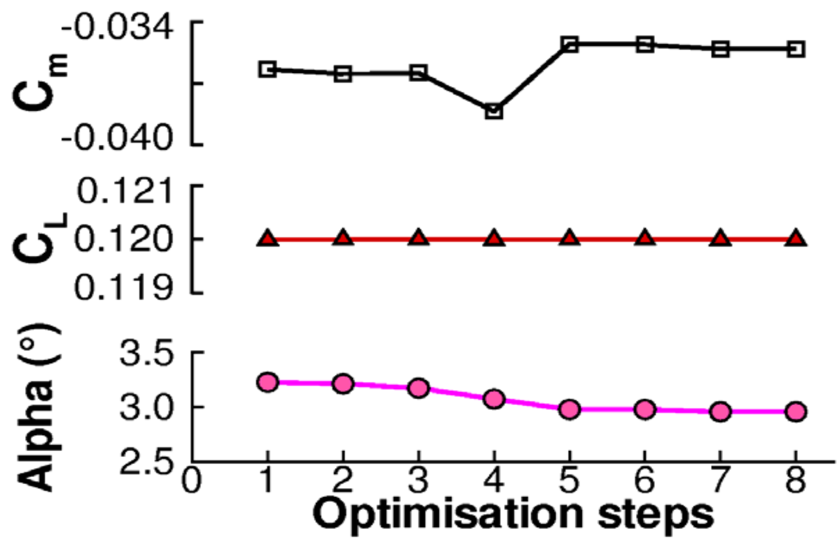
Approach

- inviscid flow computation
- Euler adjoint for calculation of flow sensitivities
- 230.000 points

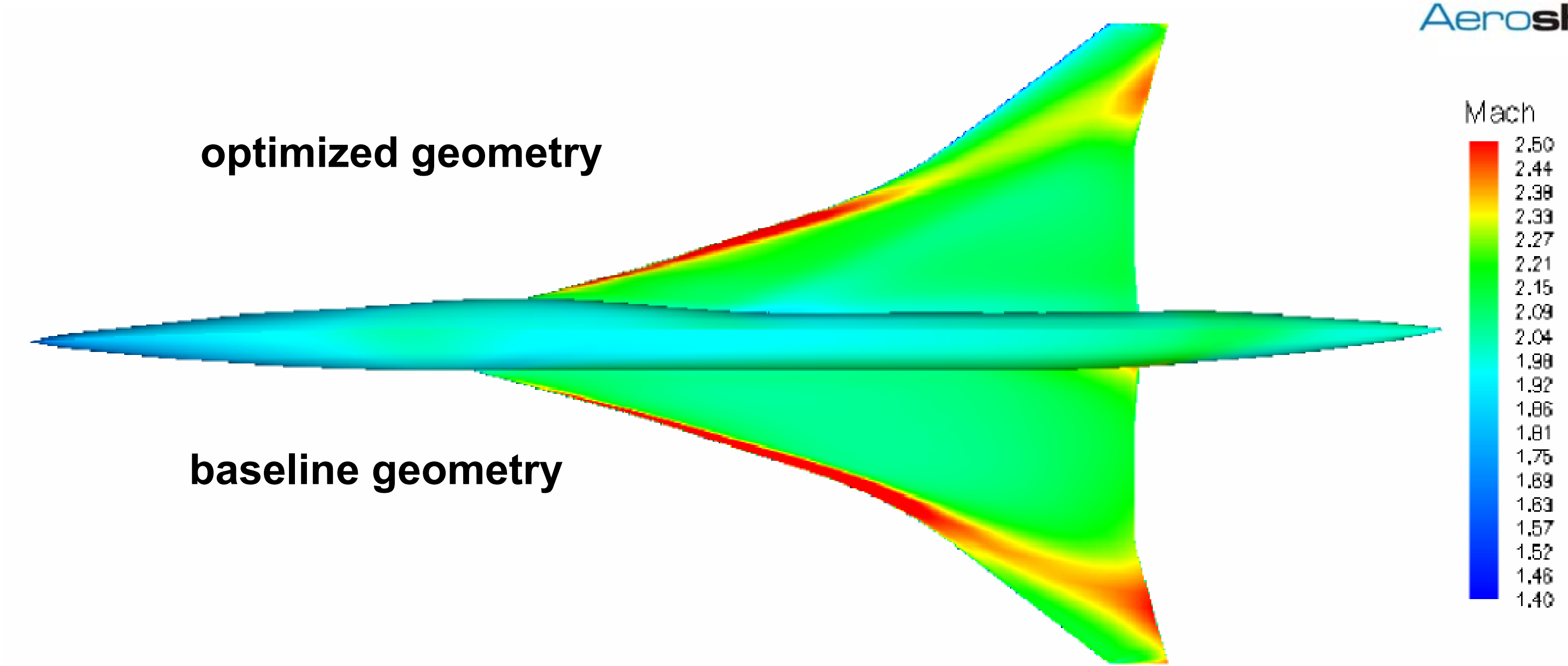
Optimization of SCT Configuration

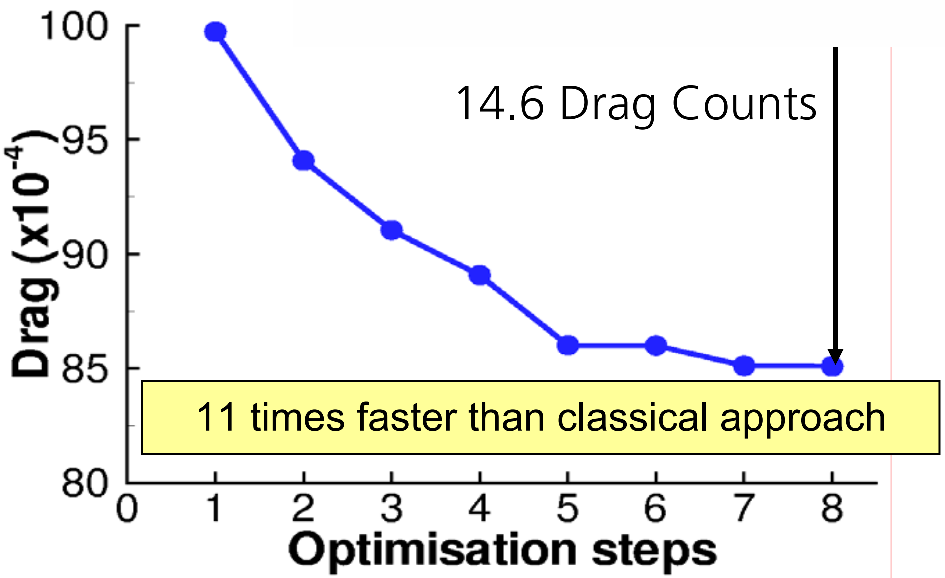


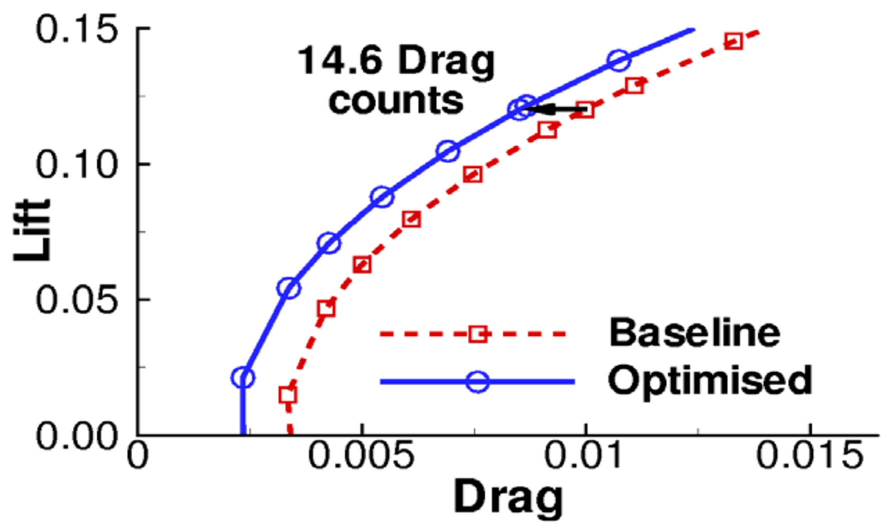




Optimization of SCT Configuration







Coupled Aero-Structure Adjoint

Motivation

Wing deflection up to 7% of wing span!

Deflected aerodynamic optimal shape can be worse than the initial …

Boeing 737-800 at ground and in cruise (Ma = 0.76)

Coupled Aero-Structure Adjoint

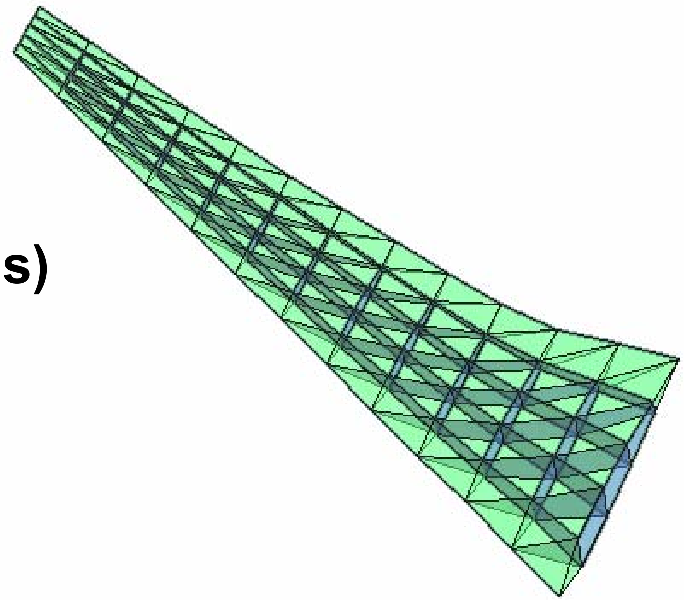
AMP wing

15 design variables (shape bumping functions based on Bernstein polynomials)

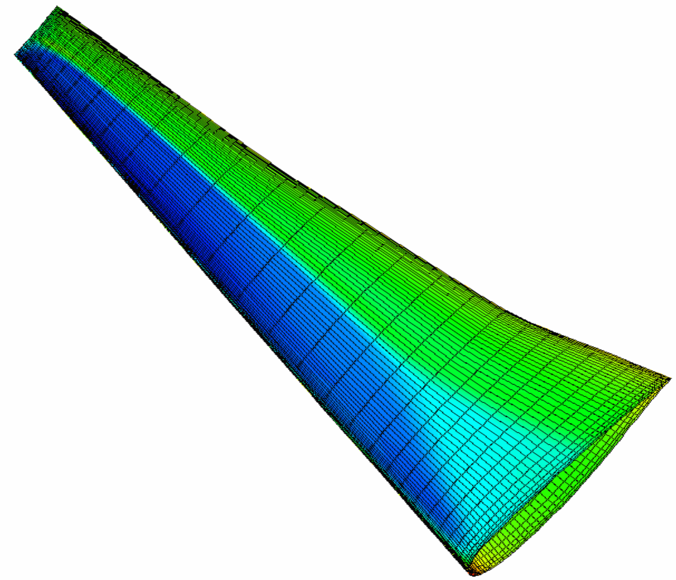
Ma=0.78
alpha=2.83

Drag reduction by constant lift

Taking into account static deformation



NASTRAN
shell/beam model
126 nodes



FLOWer MAIN/ADJOINT
15 design variables
Ma=0.78
alpha=2.83
(300.000 cells)

Coupled Aero-Structure Adjoint

Aerodynamics,
e.g Euler Eqn.: $R_A = 0$

Structure:

$$R_S = Ku - f = 0$$

K: Symmetric stiffness matrix
f: Aerodynamic force
u: Displacement vector
x: Vector of Design variables

ψ_A **: Aerodynamic Adjoint**

ψ_S **: Structure Adjoint**

~: Lagged ...

Conventional Gradient:

$$\frac{dC_D}{dx} = \frac{\partial C_D}{\partial x} + \frac{\partial C_D}{\partial w}\frac{\partial w}{\partial x} + \frac{\partial C_D}{\partial u}\frac{\partial u}{\partial x}$$

Aero/Structure Adjoint System:

$$\left(\frac{\partial R_A}{\partial w}\right)^T \psi_A = \frac{\partial C_D}{\partial w} - \left(\frac{\partial R_S}{\partial w}\right)^T \tilde{\psi}_S$$

$$\left(\frac{\partial R_S}{\partial u}\right)^T \psi_S = \frac{\partial C_D}{\partial u} - \left(\frac{\partial R_A}{\partial u}\right)^T \tilde{\psi}_A$$

Adjoint Gradient:

$$\frac{dC_D}{dx} = \frac{\partial C_D}{\partial x} - \psi_A^T \frac{\partial R_A}{\partial x} - \psi_S^T \frac{\partial R_S}{\partial x}$$

Coupled Aero-Structure Adjoint

$$\frac{\partial R_A}{\partial u}, \frac{\partial R_A}{\partial x}$$ **: perturb shape by u,x → calculate change in CFD residual**

$$\frac{\partial C_D}{\partial u}, \frac{\partial C_D}{\partial x}$$ **: perturb shape by u,x → calculate change in drag coefficient**

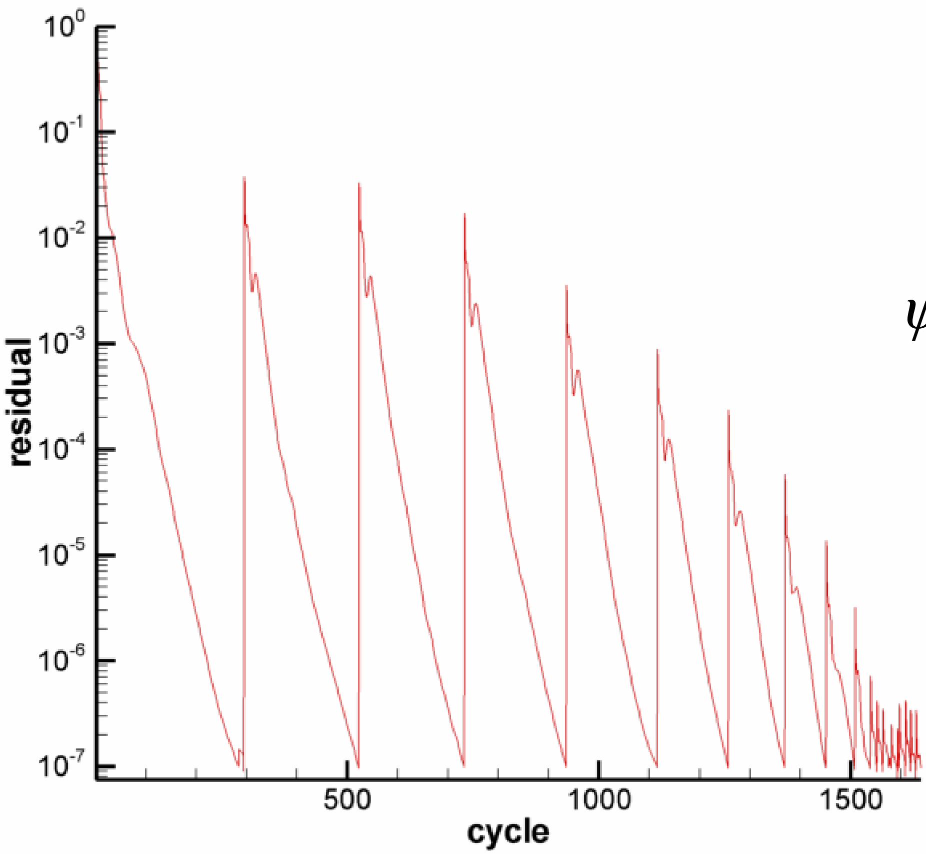
$$\frac{\partial C_D}{\partial w}$$ **: treat** $\int_C \ldots \frac{\partial p}{\partial w}(n_x \cos\alpha + n_y \sin\alpha)\ldots$ **→ boundary condition**

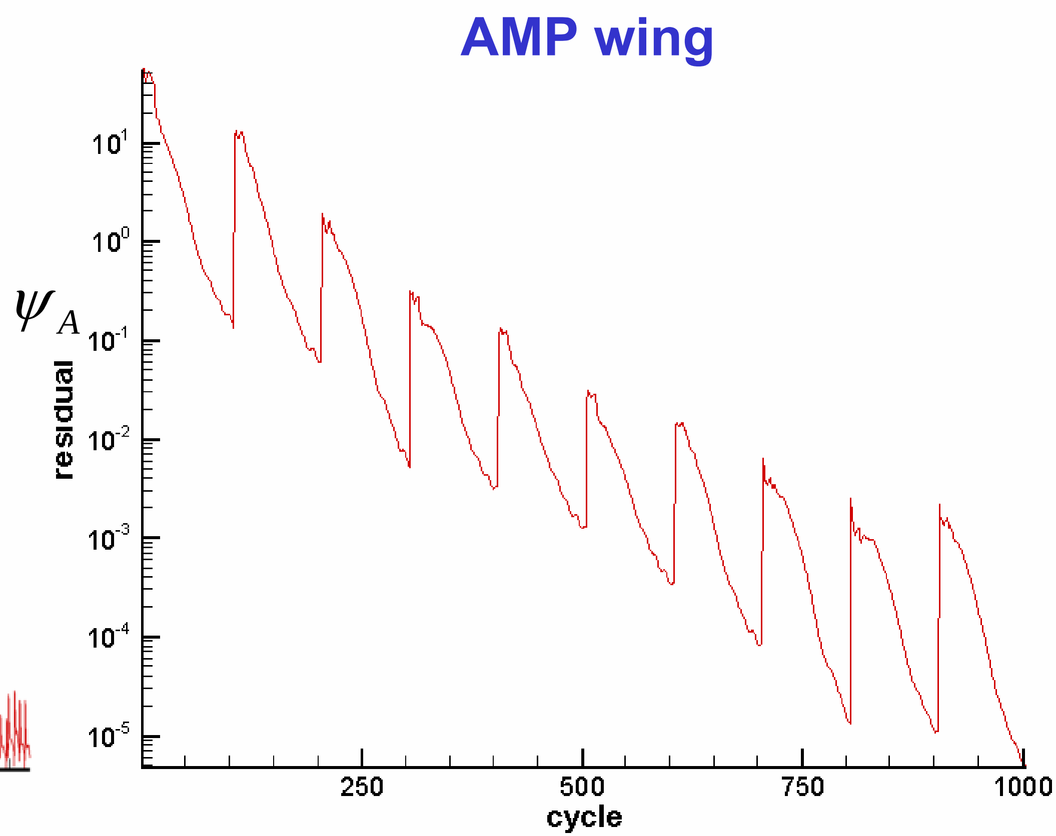
$$\frac{\partial R_S}{\partial w} = \frac{\partial (Ku - f)}{\partial w} = -\frac{\partial f}{\partial w}$$ **: treat** $\int_C \ldots \frac{\partial p}{\partial w} \ldots$ **→ boundary condition**

$$\frac{\partial R_S}{\partial u} = \frac{\partial (Ku - f)}{\partial u} = K = K^T$$

$$\frac{\partial R_S}{\partial x} = \frac{\partial (Ku - f)}{\partial x} = \frac{\partial K}{\partial x} u - \frac{\partial f}{\partial x}$$

Coupled Aero-Structure Adjoint





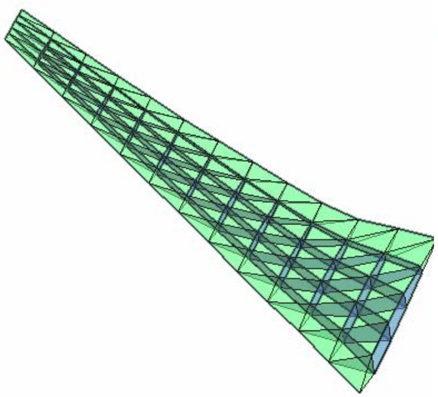
Finite Differences:
Perturb the shape by each design variable and converge the aero-elastic loop until stationary behavior

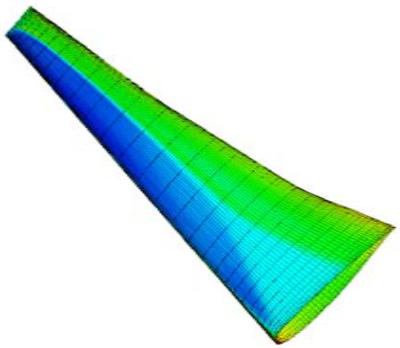
Coupled Aero-Structure Adjoint:
Each 100 iterations the lagged $\tilde{\psi}_S$ is updated ...

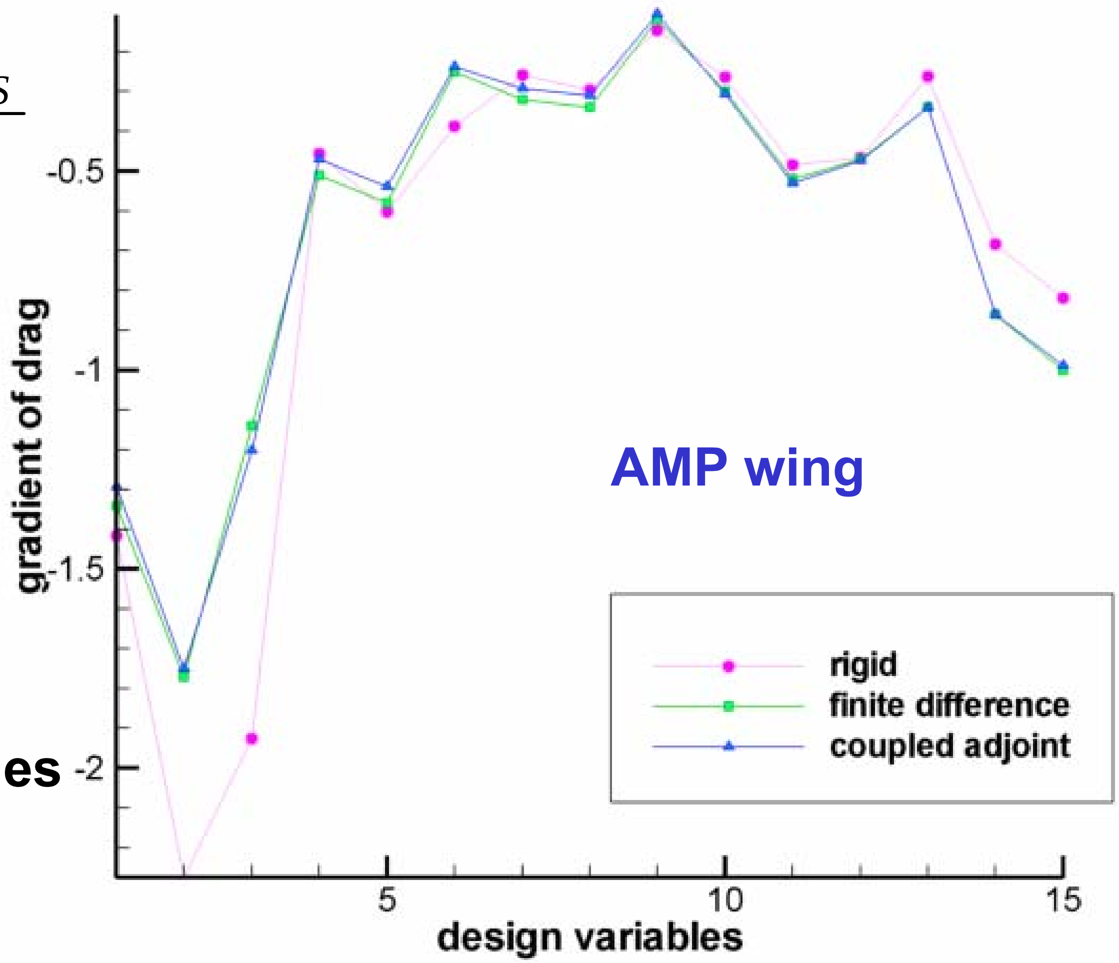
Coupled Aero-Structure Adjoint

Validation of Adjoint Gradient

$$\frac{dC_D}{dx} = \frac{\partial C_D}{\partial x} - \psi_A^T \frac{\partial R_A}{\partial x} - \psi_S^T \frac{\partial R_S}{\partial x}$$







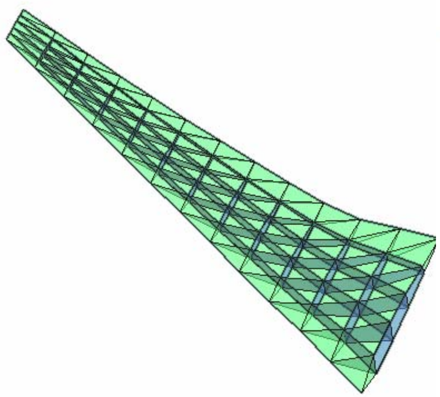
NASTRAN
shell/beam model
126 nodes

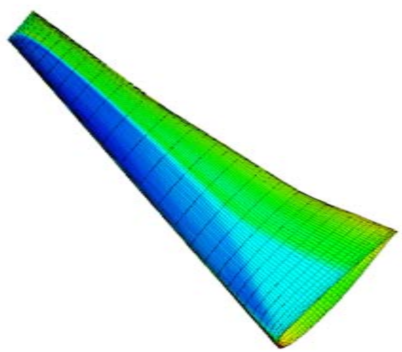
15 design variables
Ma=0.78
alpha=2.83
(300.000 cells)

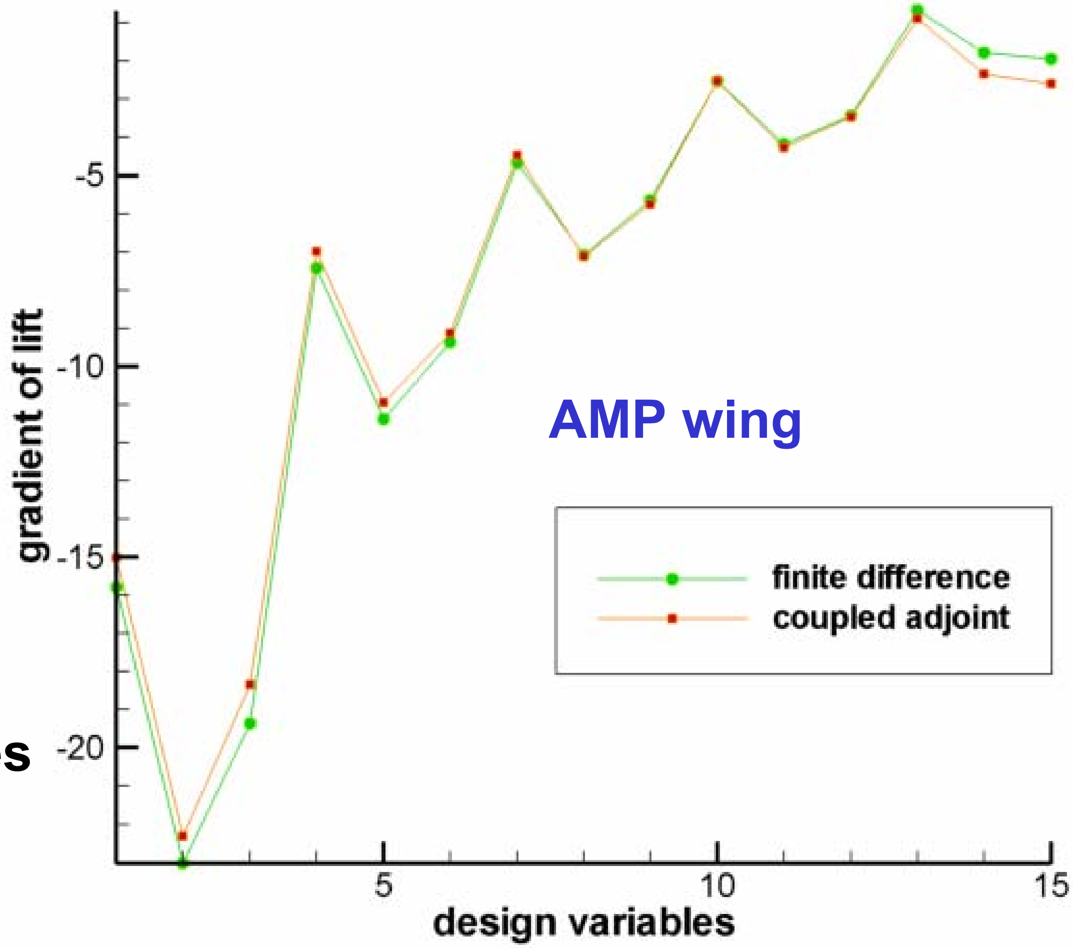
Coupled Aero-Structure Adjoint

Validation of Adjoint Gradient

$$\frac{dC_L}{dx} = \frac{\partial C_L}{\partial x} - \psi_A^T \frac{\partial R_A}{\partial x} - \psi_S^T \frac{\partial R_S}{\partial x}$$







NASTRAN
shell/beam model
126 nodes

15 design variables
Ma=0.78
alpha=2.83
(300.000 cells)

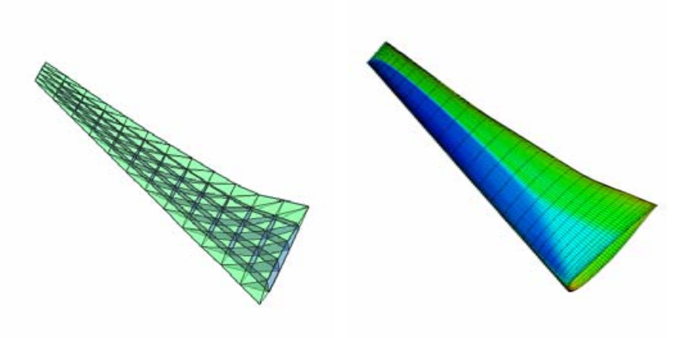
Coupled Aero-Structure Adjoint

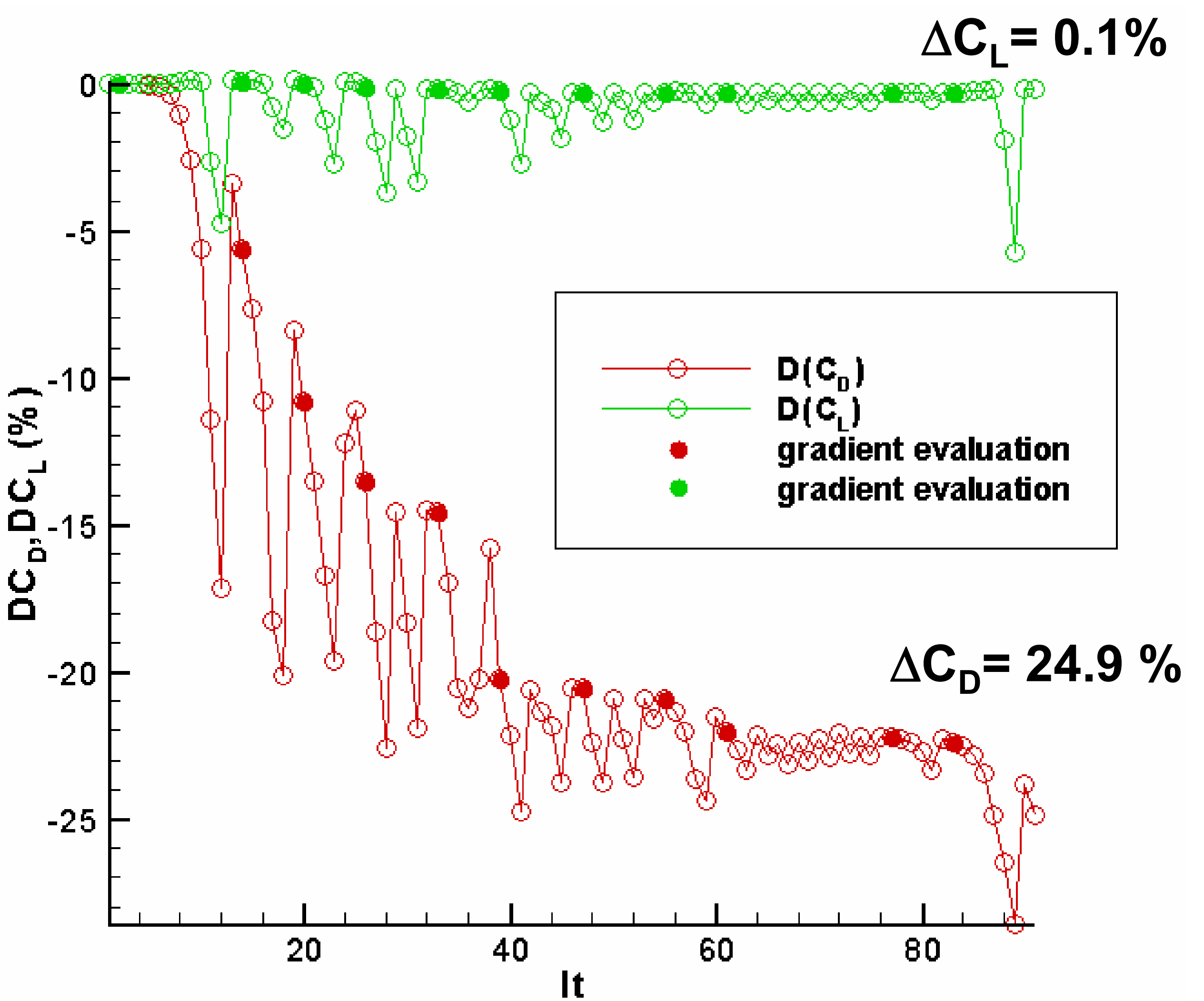
AMP wing

240 design variables (control points free form deformation)

Ma=0.78
alpha=2.83

Drag reduction by constant lift





feasible direction method

A. Fazzolari, N. Gauger

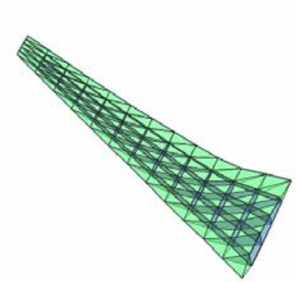
Coupled Aero-Structure Adjoint

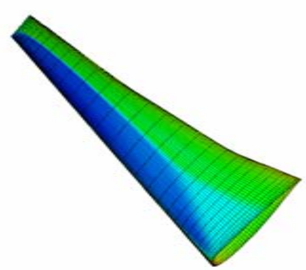
AMP wing

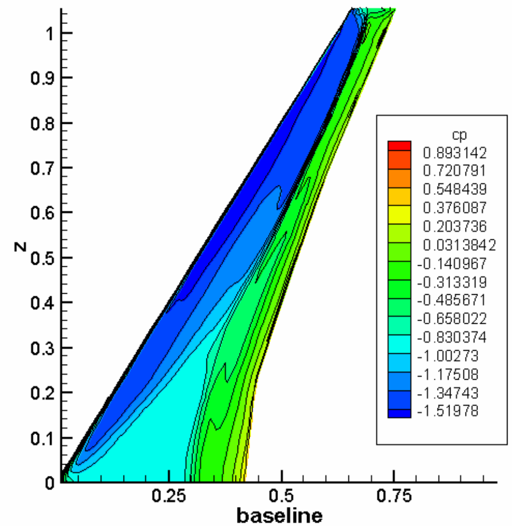
240 design variables (control points free form deformation)

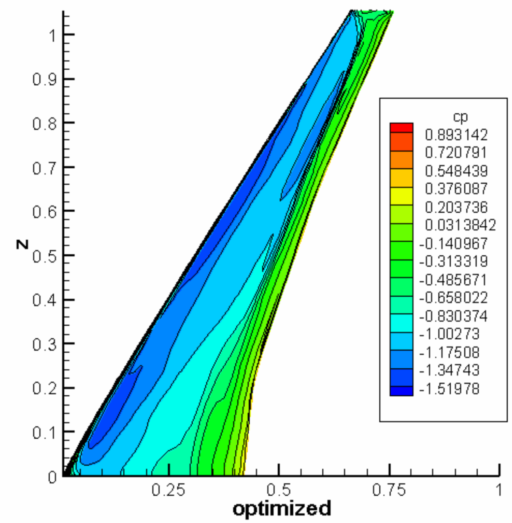
Ma=0.78
alpha=2.83

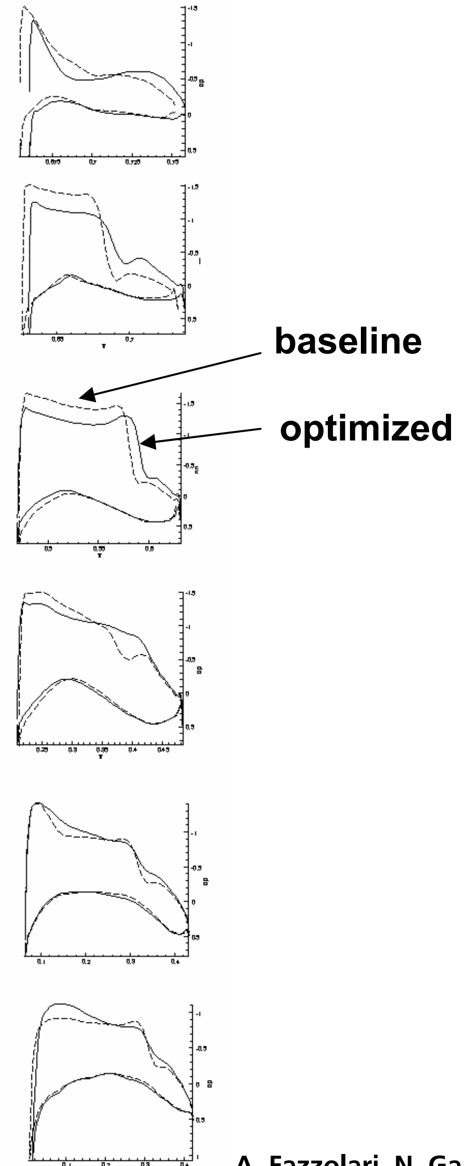
Drag reduction by constant lift











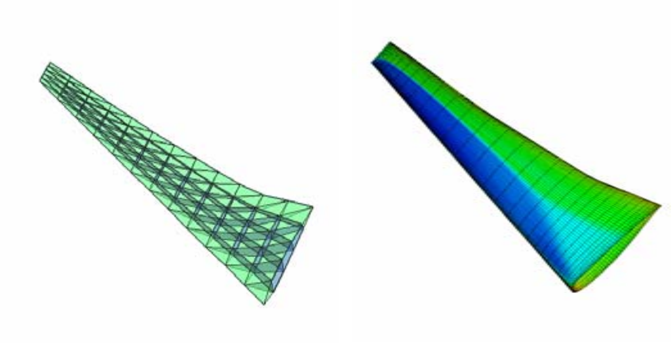
Coupled Aero-Structure Adjoint

AMP wing

240 design variables (control points free form deformation)

Ma=0.78
alpha=2.83

Drag reduction by constant lift



Comparison of numerical effort:
(PC Pentium IV, 2.6 GHz, 2GB RAM)

- **Coupled adjoint: 15 days**
  (11 gradient and 91 state evaluations)
- **Finite differences: 227 days**

Aero-Structure MDO

Range R:

$$R \propto \frac{C_L}{C_D} \ln \frac{W}{W-F} = \frac{C_L}{C_D} \ln \left(\frac{1+\lambda\, ks}{1+\lambda\, ks - \frac{F}{W_0}} \right)$$

Fuel Weight F

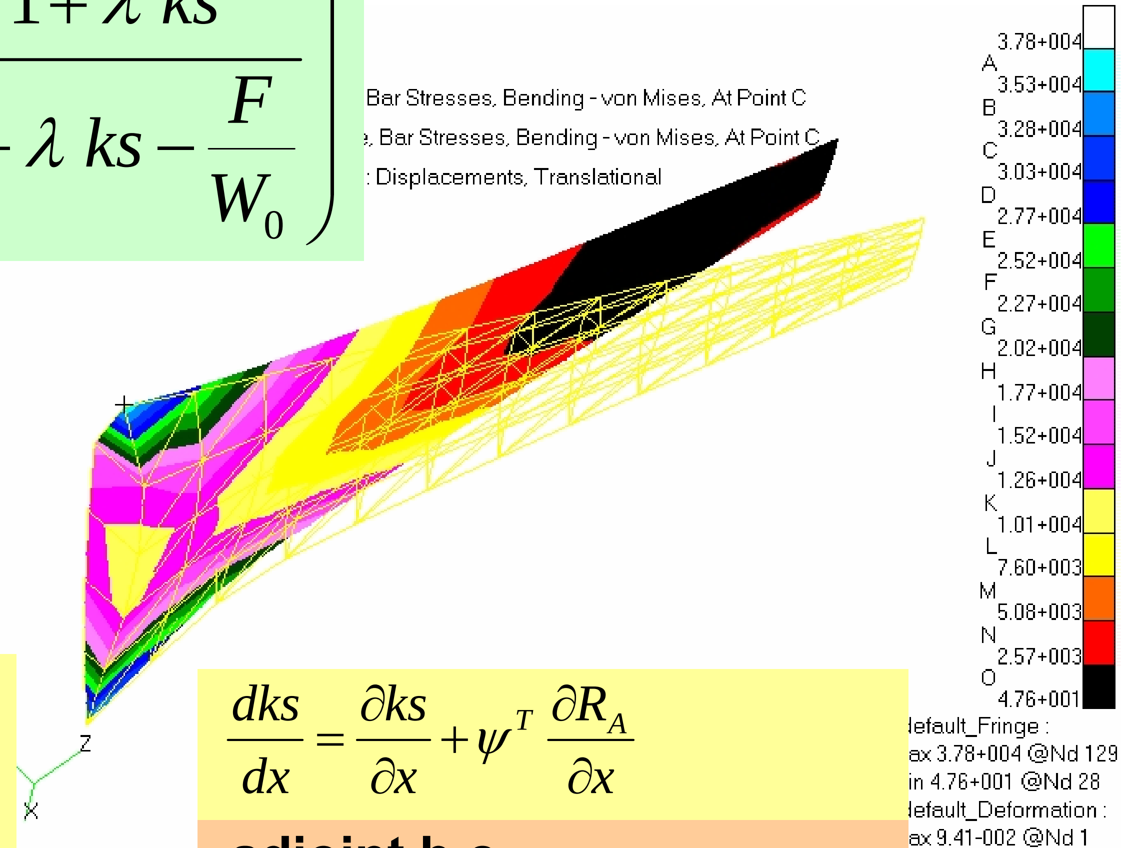
Weight W:

$$W = W_0 (1+\lambda\, ks)$$

Kreisselmeier-Steinhauser:

$$ks = \frac{1}{\beta} \ln\left(\sum_n \exp\left(\beta \frac{\sigma_n - \sigma_0}{\sigma_0}\right)\right)$$

λ=0.2 , σ_0 =30.000 and β=40



$$\frac{dks}{dx} = \frac{\partial ks}{\partial x} + \psi^T \frac{\partial R_A}{\partial x}$$

adjoint b.c.

$$n_x \psi_2 + n_y \psi_3 + n_z \psi_4 = -\frac{\partial ks}{\partial p}$$

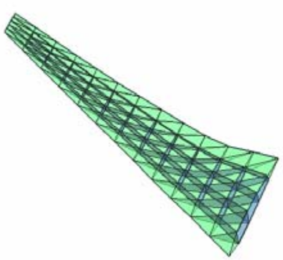
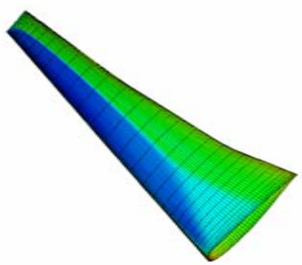
Aero-Structure MDO

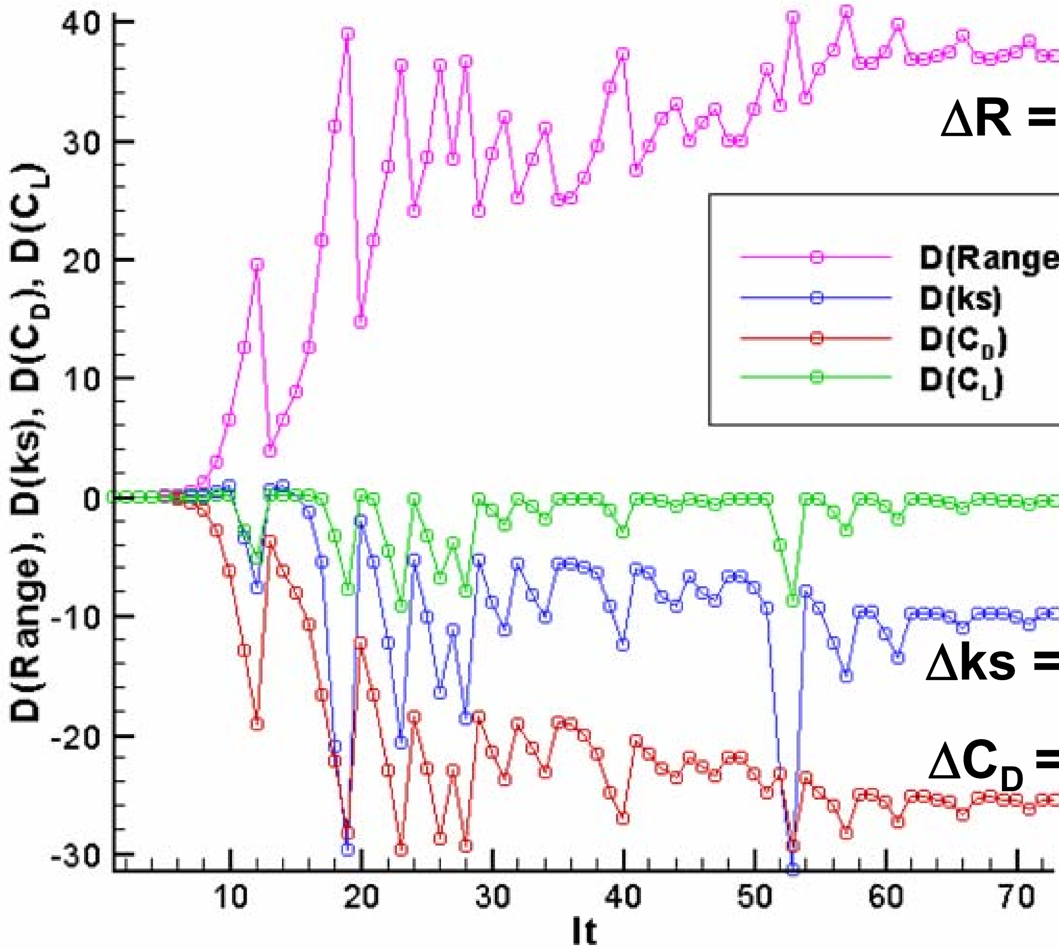
AMP wing

240 design variables (control points free form deformation)

Ma=0.78
alpha=2.83

Range maximization by constant lift



ΔR = +37 %

Δks = -10 %

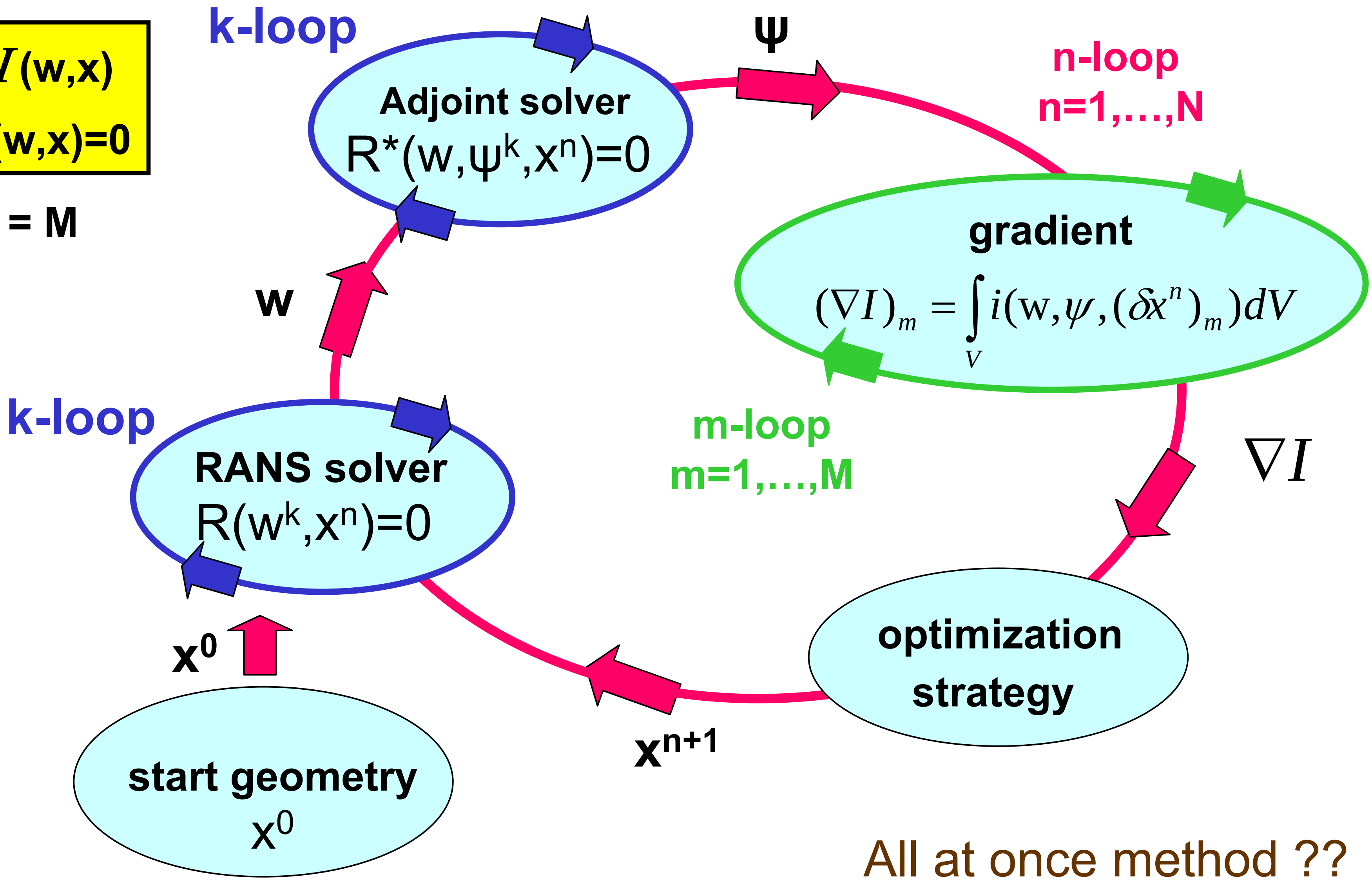
ΔC_D = -25 %

feasible direction method

Adjoint Based Optimization

min I(w,x)
s.t. R(w,x)=0

dim x = M



All at once method ??

Simultaneous Pseudo-Time Stepping
One Shot Approach

University of Trier

min $I(w,x)$

s.t. R(w,x)=0

dim x = M

$$L(w,x,\psi) = I(w,x) - \psi^T R(w,x)$$

KKT

$$\nabla_w L(w,x,\psi) = 0 \quad \textbf{(adjoint equation)}$$

$$\nabla_x L(w,x,\psi) = 0 \quad \textbf{(design equation)}$$

$$R(w,x) = 0 \quad \textbf{(state equation)}$$

$$\begin{bmatrix} w+\Delta w \\ x+\Delta x \\ \psi+\Delta\psi \end{bmatrix} = \begin{bmatrix} w \\ x \\ \psi \end{bmatrix} - \begin{bmatrix} L_{ww} & L_{wx} & (\partial R/\partial w)^T \\ L_{xw} & L_{xx} & (\partial R/\partial x)^T \\ \partial R/\partial w & \partial R/\partial x & 0 \end{bmatrix}^{-1} \begin{bmatrix} \nabla_w L \\ \nabla_x L \\ R \end{bmatrix}$$

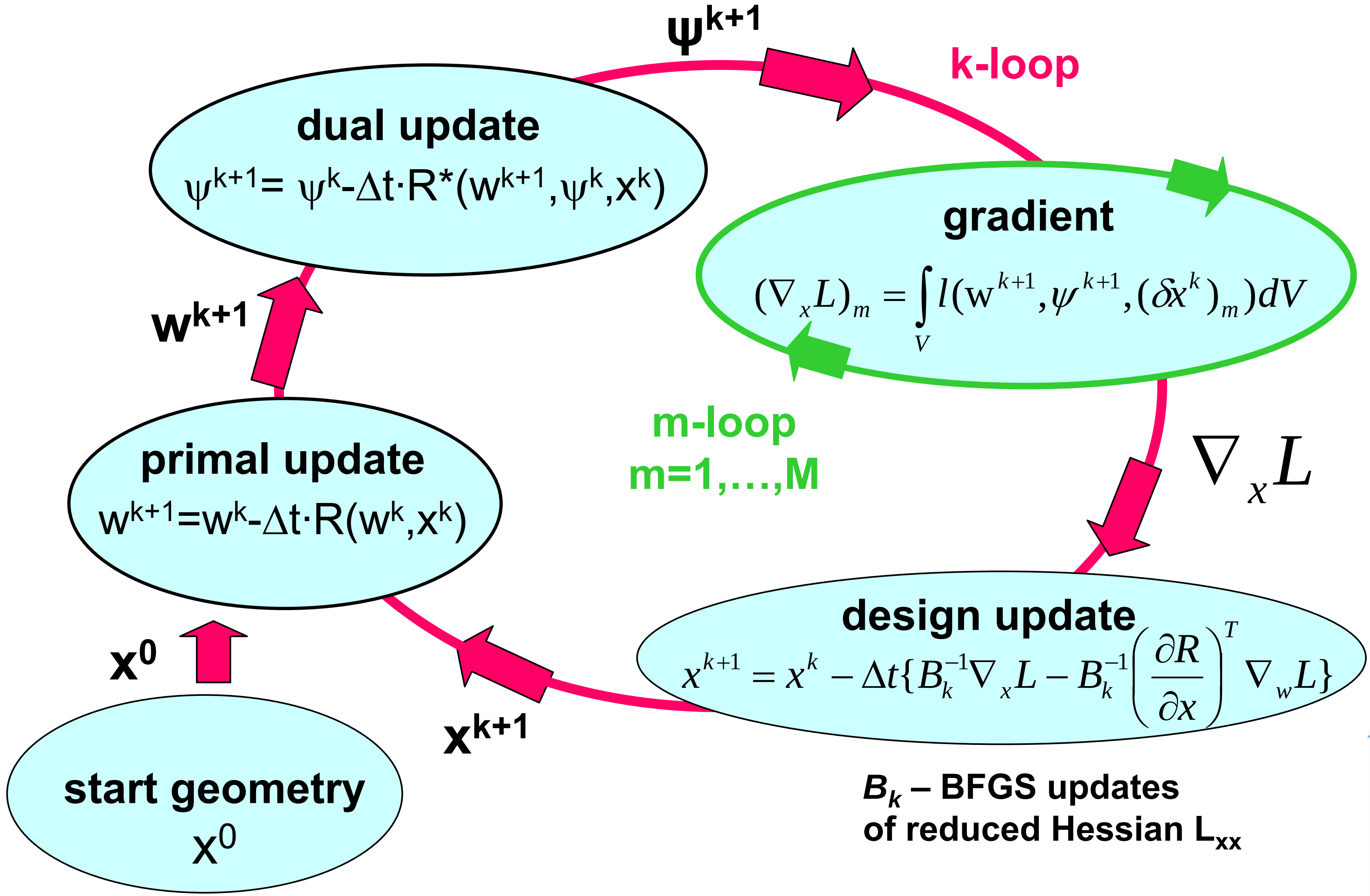
Newton SQP method

$$\begin{bmatrix} \Delta w \\ \Delta x \\ \Delta\psi \end{bmatrix} = \begin{bmatrix} 0 & 0 & I \\ 0 & B & (\partial R/\partial x)^T \\ I & \partial R/\partial x & 0 \end{bmatrix}^{-1} \begin{bmatrix} -\nabla_w L \\ -\nabla_x L \\ -R \end{bmatrix}$$

inexact Newton rSQP method

simultaneous preconditioned pseudo time stepping

Simultaneous Pseudo-Time Stepping
One Shot Approach



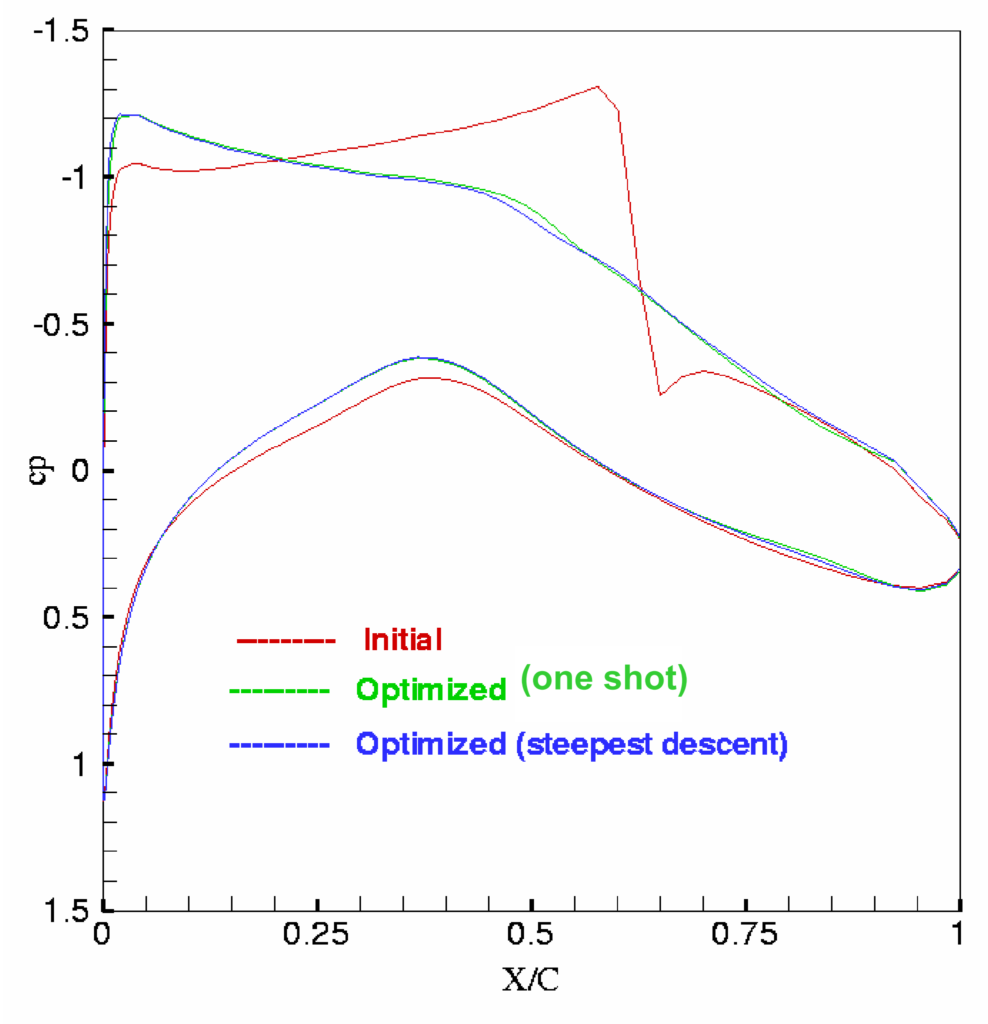
Simultaneous Pseudo-Time Stepping One Shot Approach

Optimization problem

- drag reduction for RAE 2822
- inviscid flow
- M=0.73, a=2^0

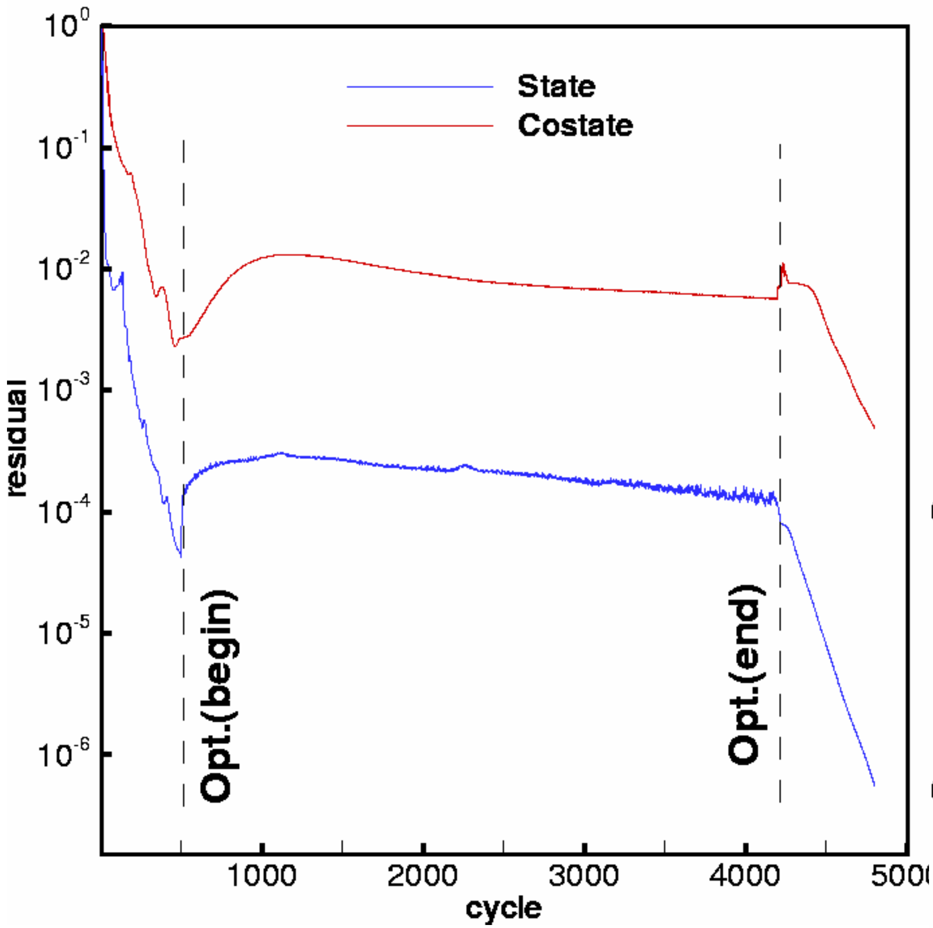
Tools

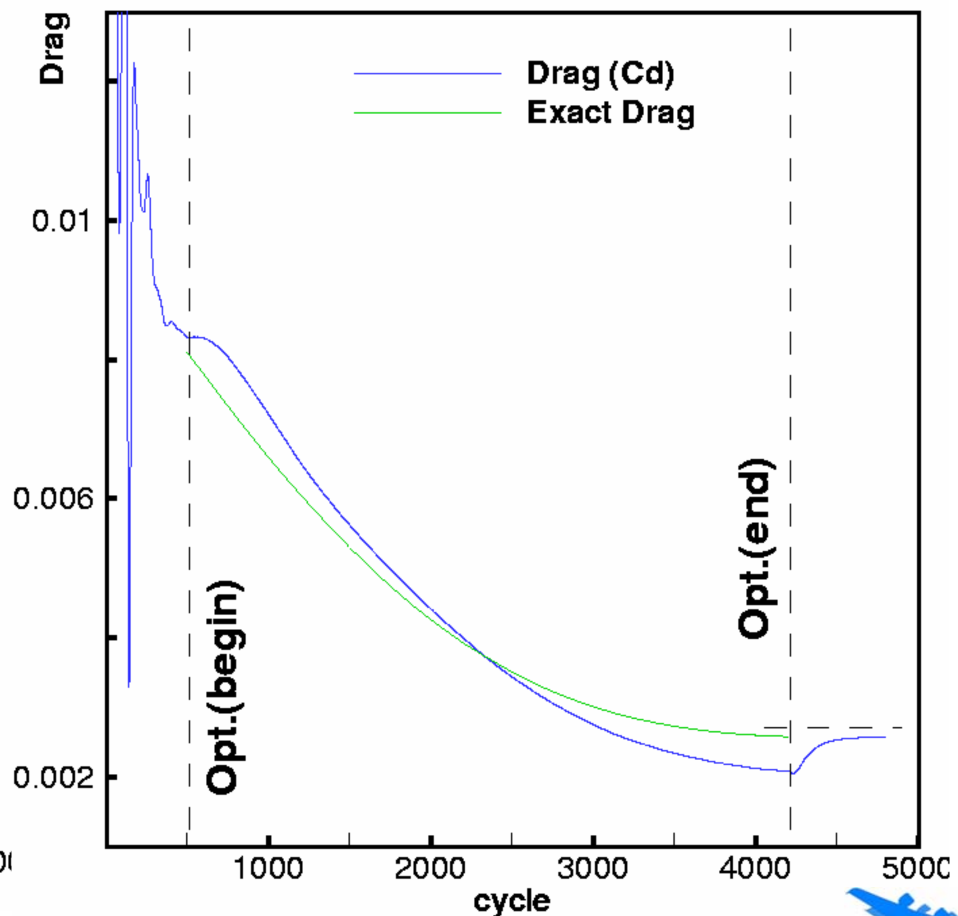
- FLOWer
- FLOWer ADJOINT



Simultaneous Pseudo-Time Stepping
One Shot Approach

University of Trier





Optimization at the cost of 4 flow simulations!

Conclusions

Adjoint approach

- is efficient
- leads to accurate sensitivities
- handles multi-constraints
- handles multi-point designs
- enables innovative optimization strategies
  (e.g. one-shot approaches)
- can be easily extended to MDO
- can handle multi-objectives?!