# Optimization using Adjoint Technique Applied to Problems in Aerodynamics, Electromagnetics and Bio Fluid Mechanics

by

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# Outline

- Aerodynamic Shape Optimisation
- Electromagnetic shape optimization  for reduction of radar cross section
- Data assimilation in Bio Fluid Mechanics

# Aerodynamic Shape Optimisation

## Background

- The aerodynamic shape optimization  system **cadsos** has been developed at SAAB during the last 5-10 years. Part of the work was done within the European project **Aeroshape**.
- The optimization technique is based on gradient calculations using the continuous adjoint technique.
- **Cadsos** can handle nonlinear objective functions and multiple nonlinear physical and geometrical constraints.
- 2D and 3D geometry parametrization och modifications are done in MATLAB. Grid sensitivities are computed by finite difference approximations.

# Optimization technique

Minimize the Drag:

$$C_D = \frac{F_1 \cos\alpha + F_2 \sin\alpha}{\rho E_{k\,\inf} A_{REF}}$$

Constant Lift:

$$C_L = \frac{-F_1 \sin\alpha + F_2 \cos\alpha}{\rho E_{k\,\inf} A_{REF}}$$

Constant pitching moment:

$$C_M = \frac{M_3}{\rho E_{k\,\inf} A_{REF} L_{REF}}$$

Euler flow equations

$$\frac{\partial f_i}{\partial x_i} = 0$$

$$f_i = \begin{pmatrix} \rho \\ \rho \bar{u} \\ \rho H \end{pmatrix} u_i + p I_i$$

$$w = \begin{pmatrix} \rho \\ \rho \bar{u} \\ \rho E \end{pmatrix}$$

# Optimization technique

Pressure Force and Moment:

$$\bar{F} = \int_{B_W(a)} p d\bar{S} \qquad \bar{M} = \int_{B_W(a)} p(\bar{x} - \bar{x}_0) \times d\bar{S}$$

# Optimization technique

Gradient of the Force:

$$\frac{\partial F_n}{\partial a} = \int_{B_W(a)} \frac{\partial}{\partial x_i}(p\,n_i + \psi^t w_H u_i)\frac{\partial x_k}{\partial a}\,dS_k$$

Adjoint solid wall bc:

$$(\psi^t I_i - n_i)dS_i = 0$$

Gradient of the Moment:

$$\frac{\partial M_n}{\partial a} \int_{B_W(a)} \frac{\partial}{\partial x_i}(p\,\varepsilon_{kji}x_k n_j + \psi^t w_H u_i)\frac{\partial x_k}{\partial a}\,dS_k$$

Adjoint solid wall bc:

$$(\psi^t I_i - \varepsilon_{kji}x_k n_j)\,dS_i = 0$$

# Optimization technique

Linearized optimization problem:

$$\begin{cases} \min\limits_{c}(c, g^0) \\ (c, g^m) = \Delta^m \quad m = 1,\ldots,M \\ (c, h^n) = \Delta^n \quad n = 1,\ldots,N \end{cases}$$

Physical constraints (for $m = 1,\ldots,M$)

Geometrical constraints (for $n = 1,\ldots,N$)

Modified optimization problem:

$$\begin{cases} \min\limits_{c} \frac{1}{2}\|c\|^2 \\ (c, g^0) = \Delta^0 \\ (c, g^m) = \Delta^m \quad m = 1,\ldots,M \\ (c, h^n) = \Delta^n \quad n = 1,\ldots,N \end{cases}$$

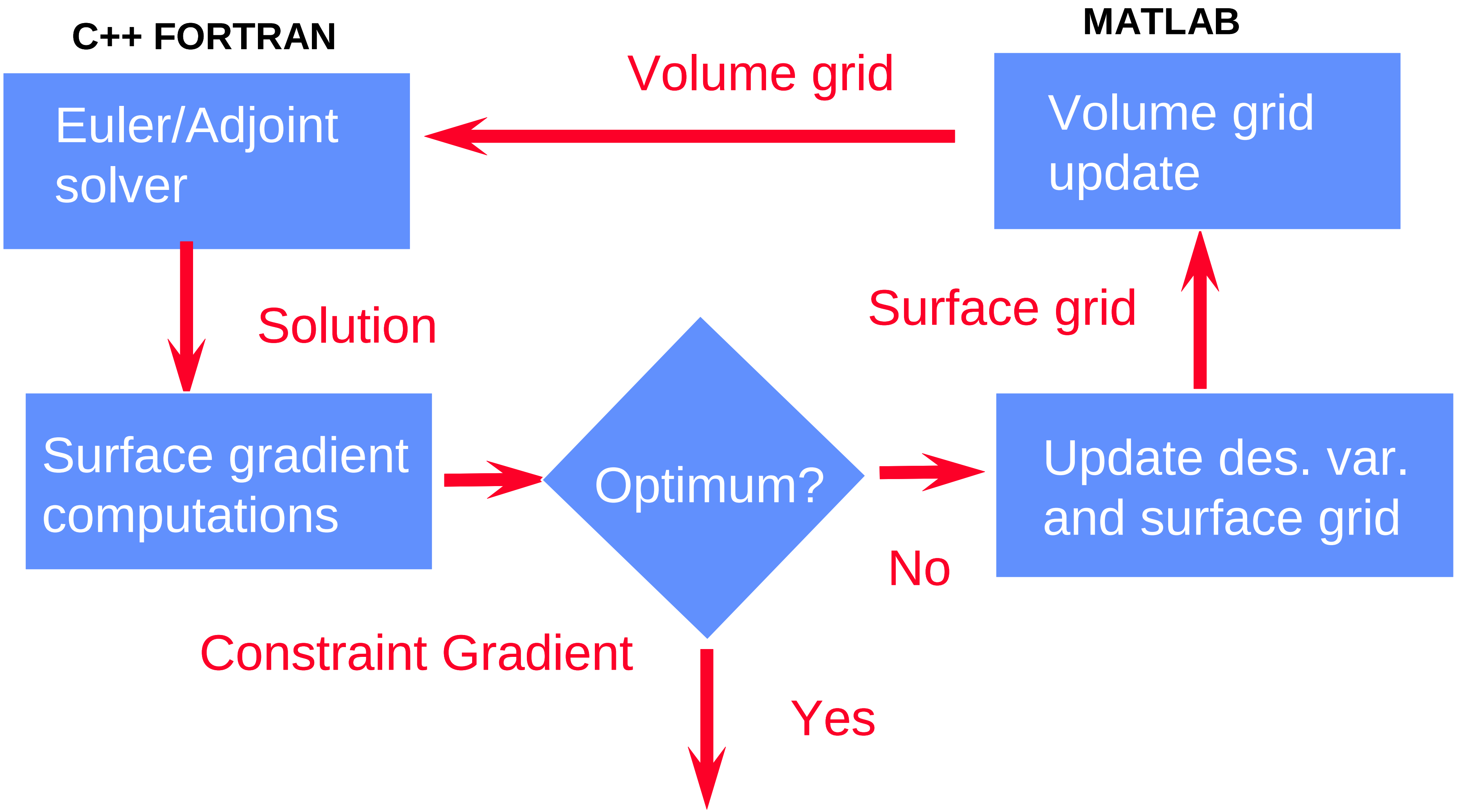
Lagrangian formulation:

$$L = \frac{1}{2}\|c\|^2 + \lambda_0((c, g^0) - \Delta^0) + \lambda_m((c, g^m) - \Delta^m) + \lambda_n((c, h^n) - \Delta^n)$$

$$\frac{\partial L}{\partial c_k} = c_k + \lambda_0 g_k^0 + \lambda_m g_k^m + \lambda_n h_k^n = 0, \quad \frac{\partial L}{\partial \lambda_0} = (c, g^0) - \Delta^0 = 0,$$

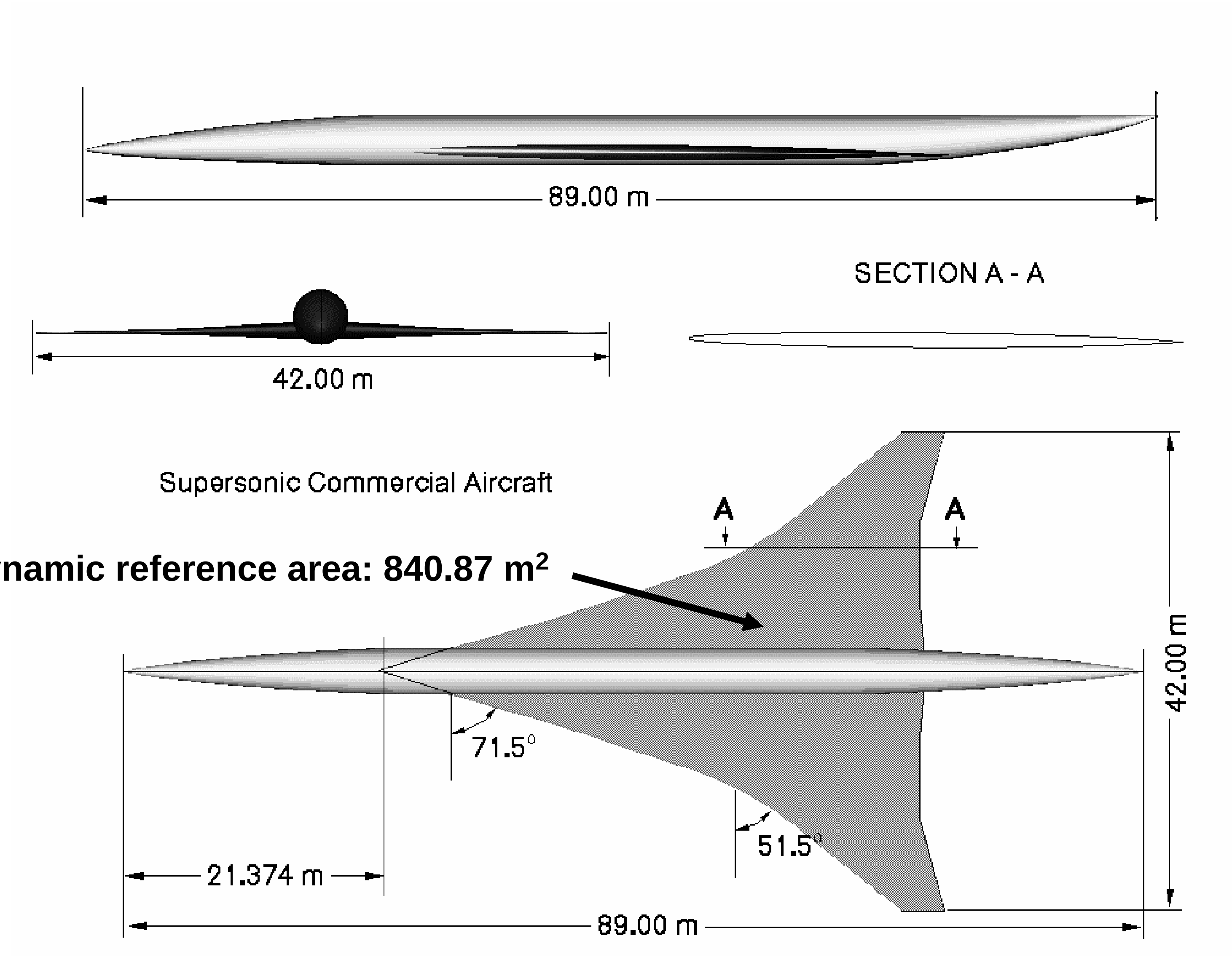
$$\frac{\partial L}{\partial \lambda_m} = (c, g^m) - \Delta^m = 0, \quad \frac{\partial L}{\partial \lambda_n} = (c, h^n) - \Delta^n = 0$$

# Optimization system **cadsos** at SAAB



# Example: Aerodynamic shape optimization of a Supersonic Commercial Transport Aircraft SCT

# Description of the SCT geometry




**Wing aerodynamic reference area: 840.87 m$^2$**

# Surface and volume grid generation

- The CFD grid around the SCT geometry consisted of 5 structured blocks and 196 000 cells.
- The mesh generation system MEGACADS, developed at DLR in Braunschweig, Germany, was used.
- The mesh generation procedure was executed in batch mode by means of script files.
- The multi block topology was kept fixed during the optimisation.

# Formulation of the optimization problem

- Objective function: minimize drag
- Physical Constraint: constant lift
- Flow model: Euler equations (inviscid flow)
- Flow condition: M=2.0 and $C_L$=0.12
- Thickness constraints
- Design variables:
  - 9 spline functions discribing the wing twist distribution in the spanwise direction
  - 36 spline functions discribing the camber line distribution in the spanwise direction
  - 12 spline functions discribing the radial shape of the body in the streamwise direction
  - 5 spline functions discribing the camber line of the body

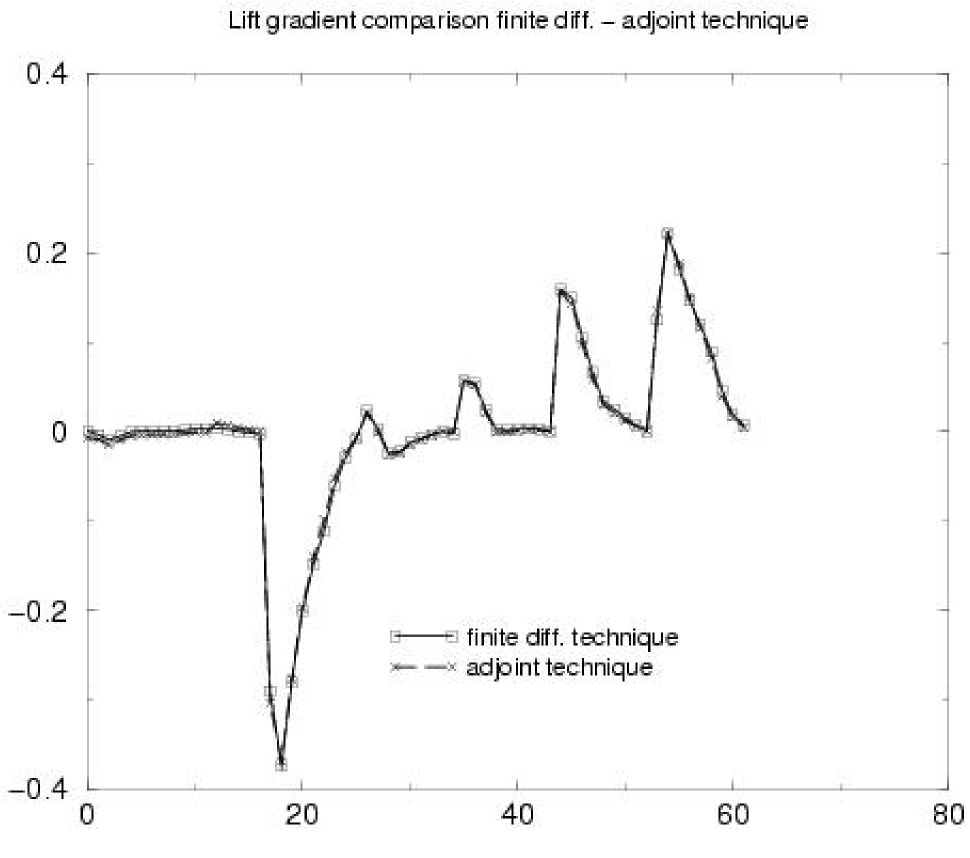
# Gradient validation

For gradient validation, a comparison between gradient computations using adjoint technique and finite difference technique was performed.

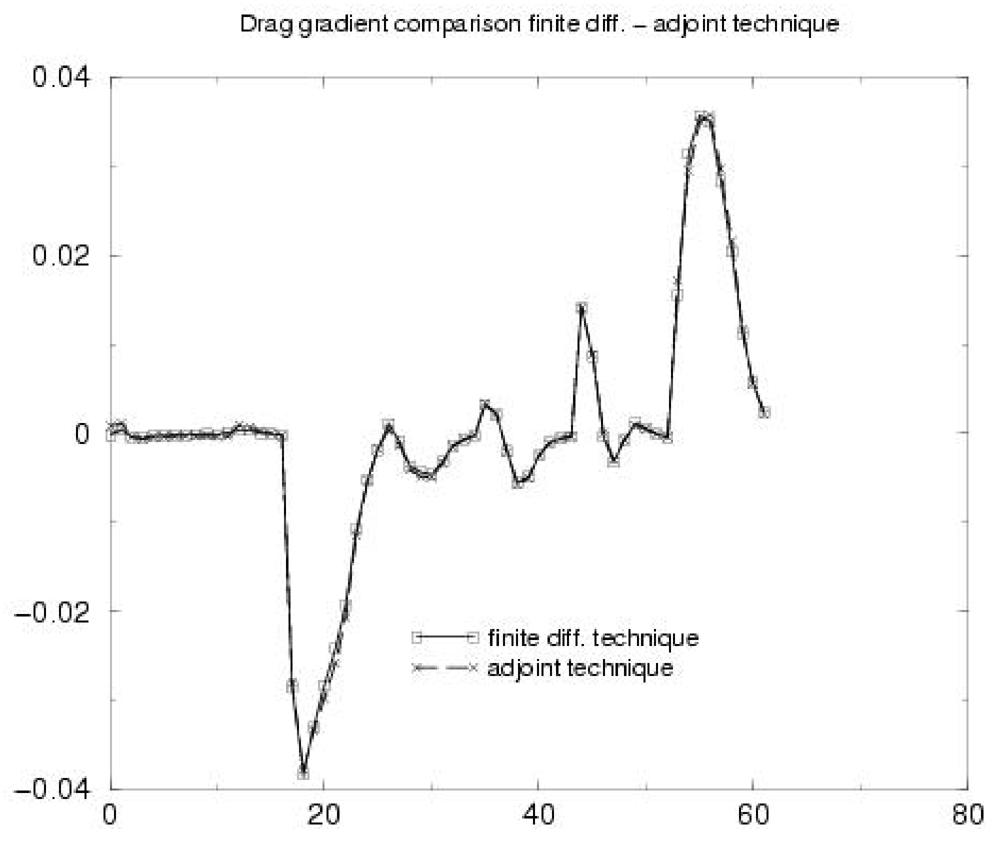
Test case: Inviscid flow over the SCT geometry, at free stream conditions $M=2.0$ and $C_L=0.12$. Disturbance parameter $\varepsilon=0.005$.
The geometry was modified by means of 62 design parameters.

# Gradient validation

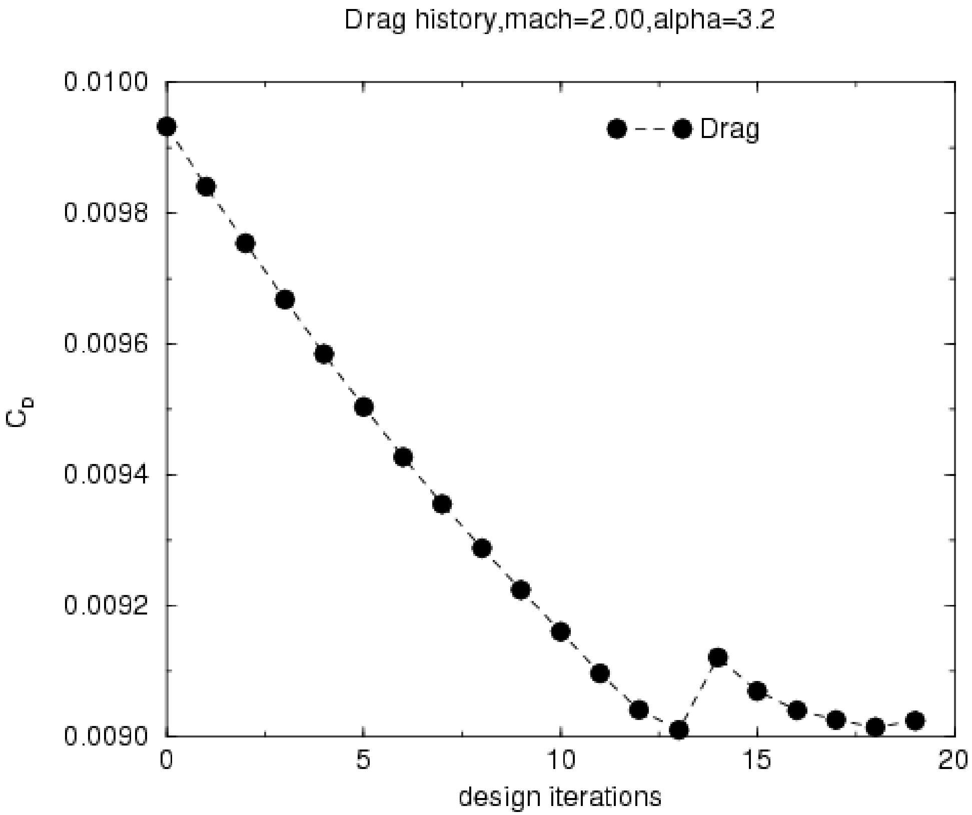
## Lift

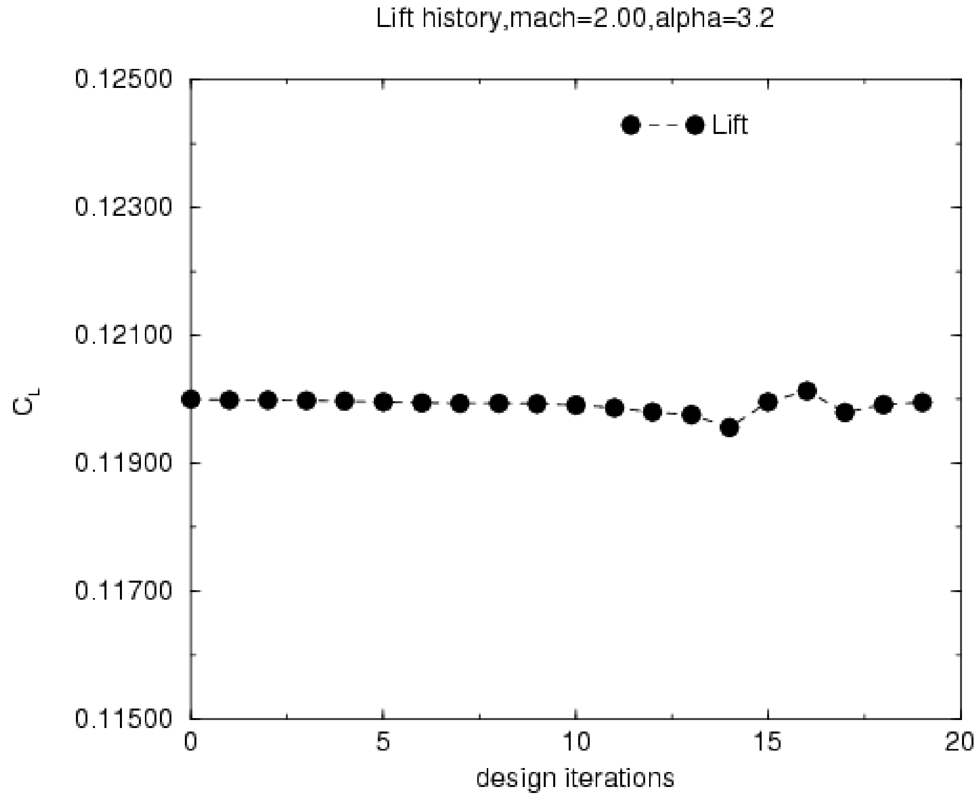



## Drag



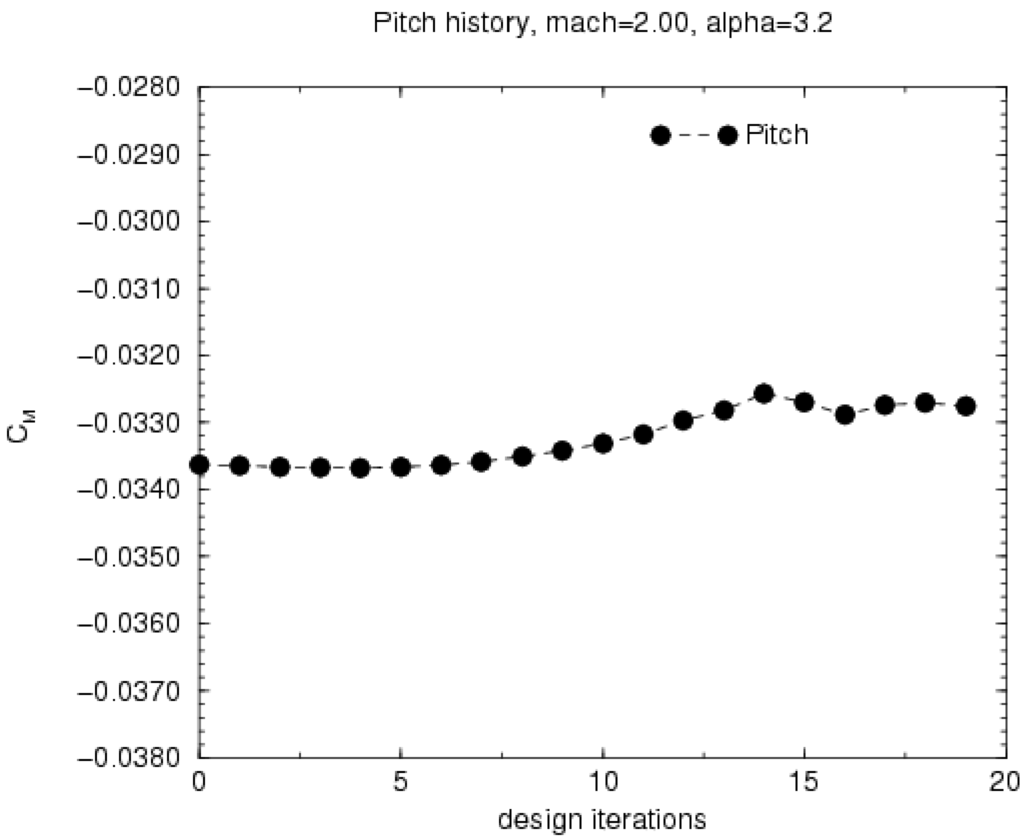
# Optimization results




**9.6 % drag reduction**

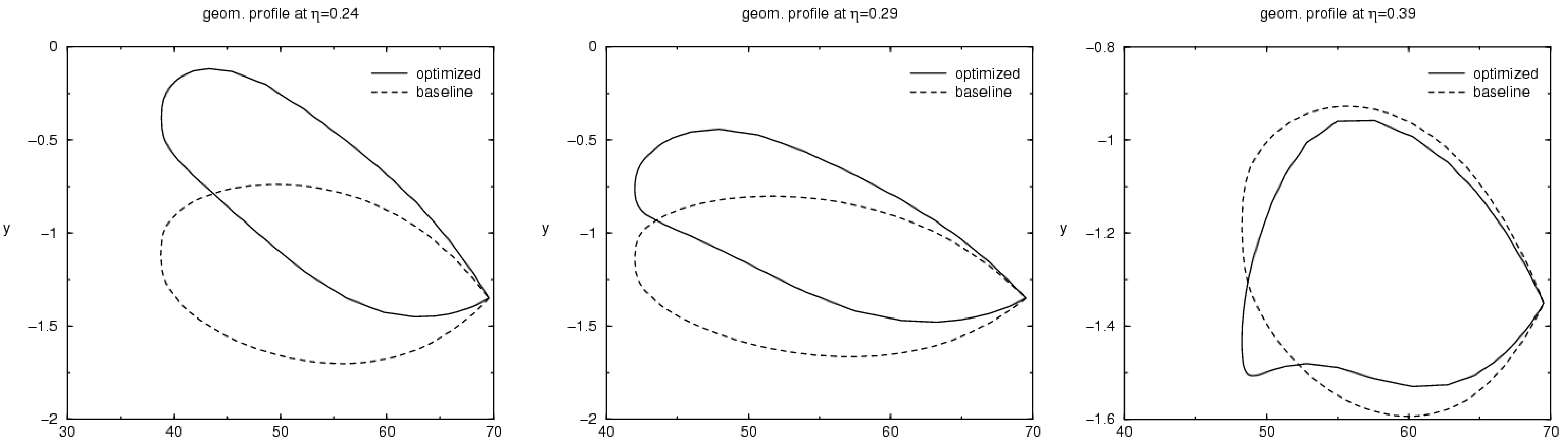
# Optimization results




Pitch history during the optimization

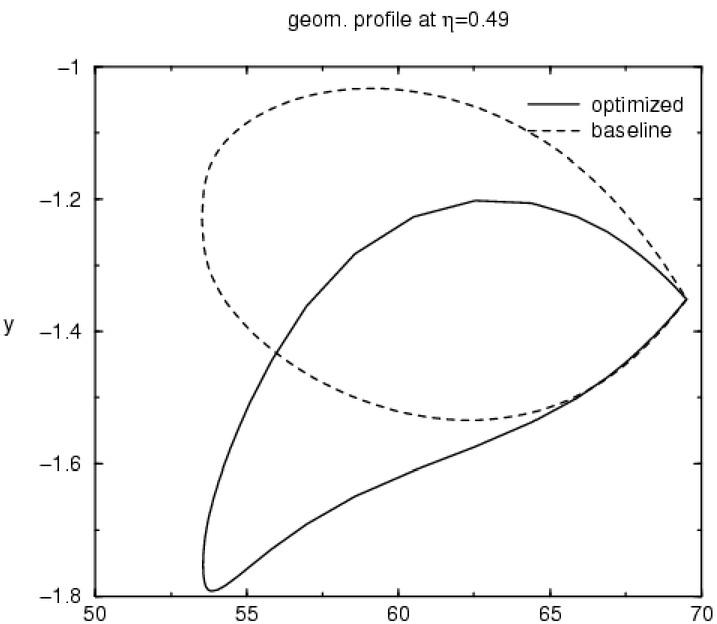
# Optimization results

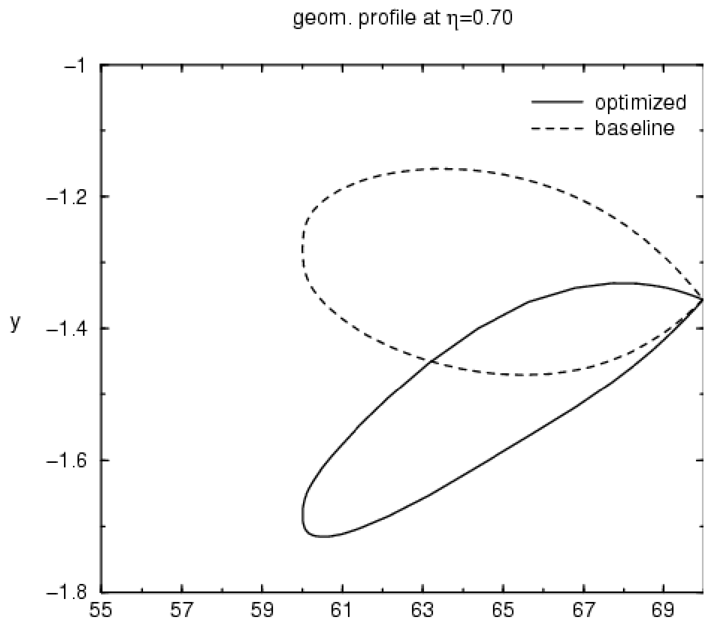
Wing profiles in the wing span direction

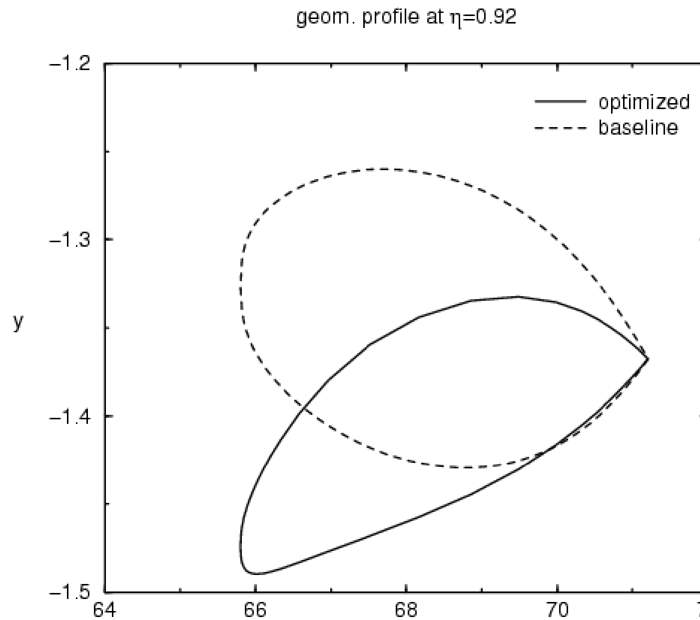


# Optimization results

Wing profiles in the wing span direction

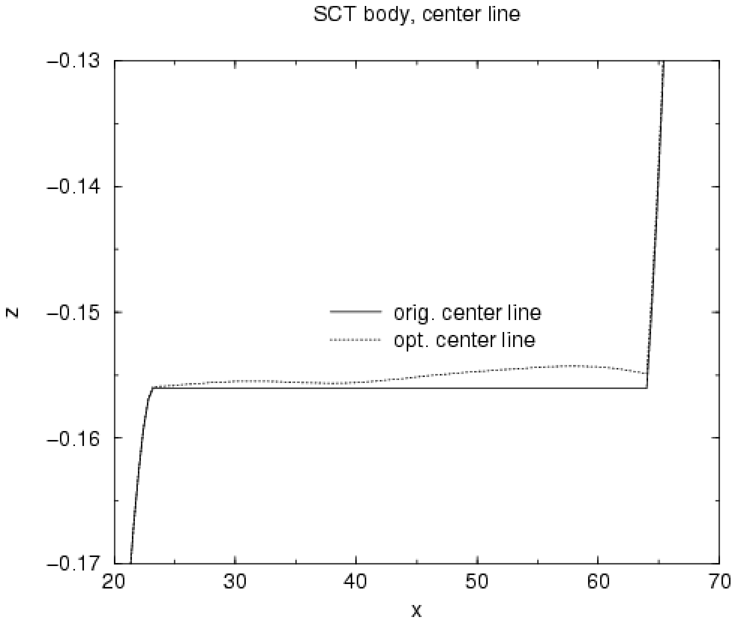


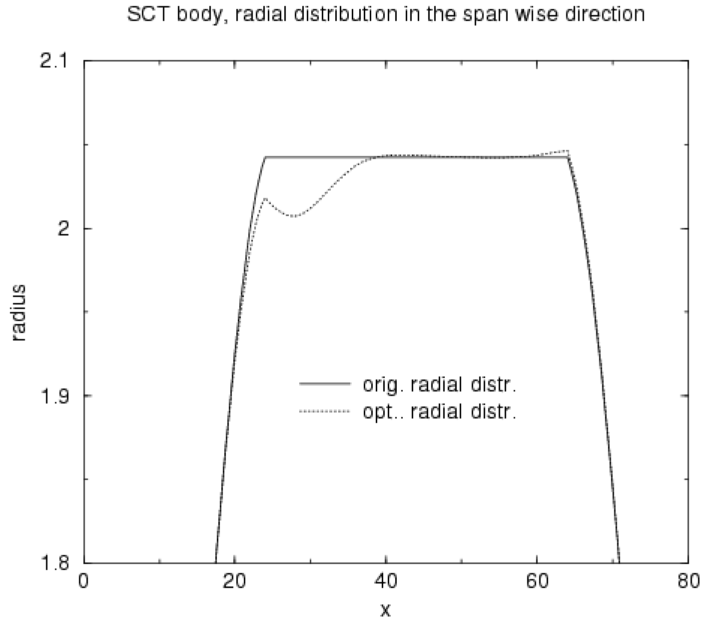



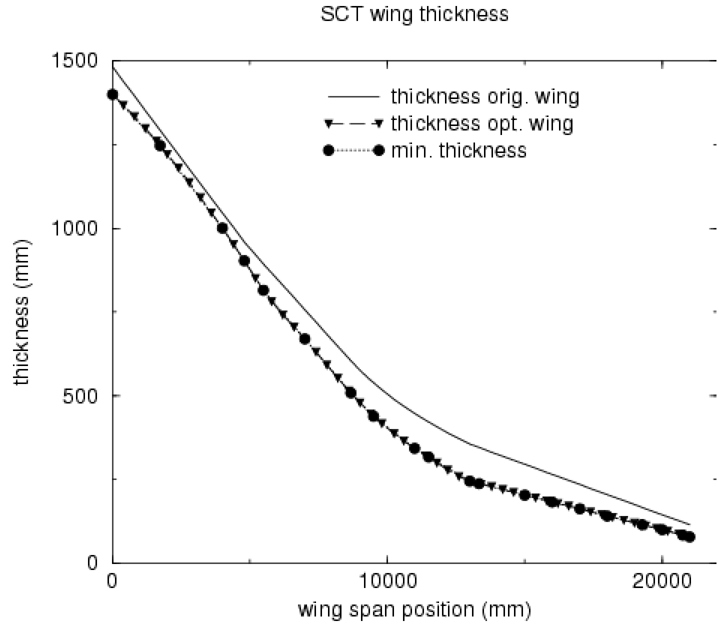



# Optimization results

Centreline. Radius and thickness distribution



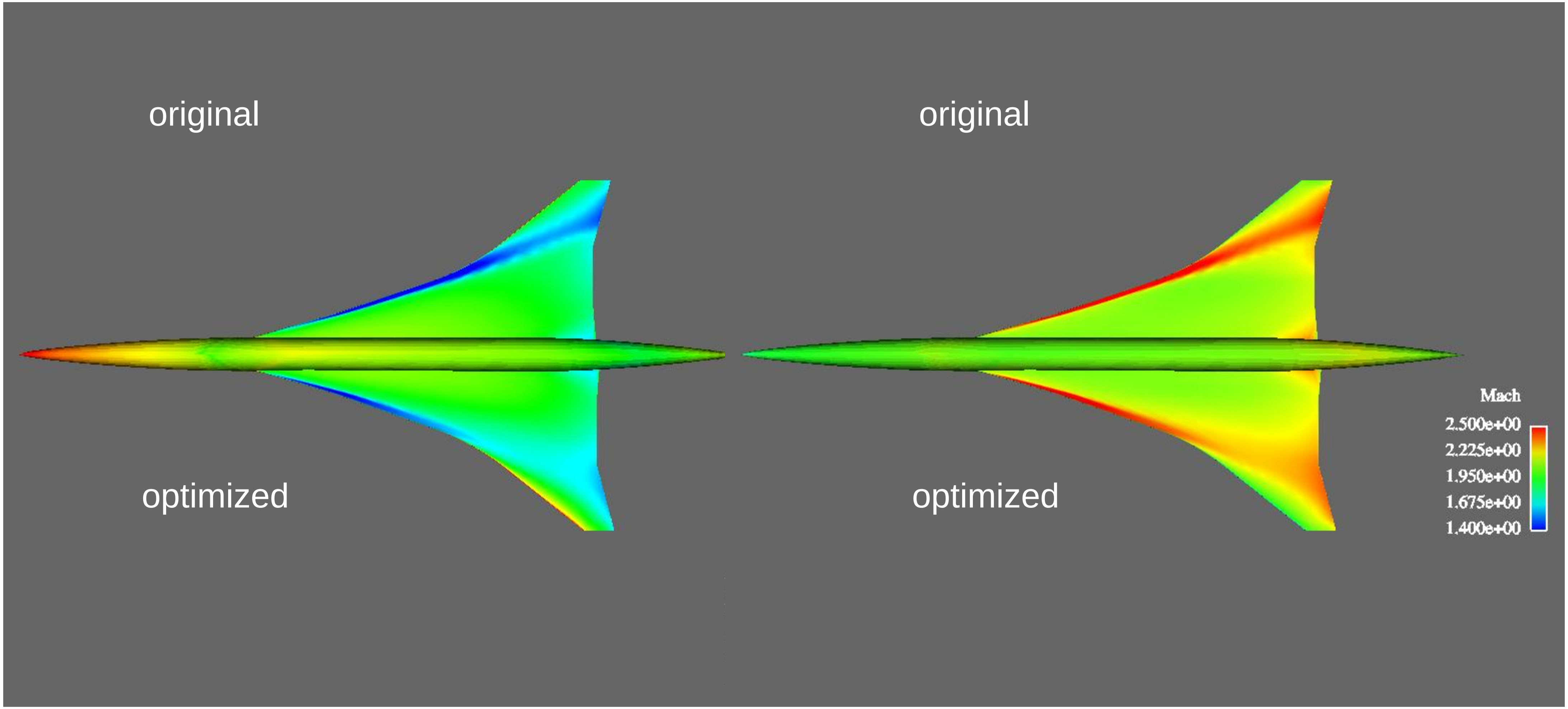




**Drag reduction from decreased thickness: 6.4%**
**Total drag reduction: 16%**

# Optimization results

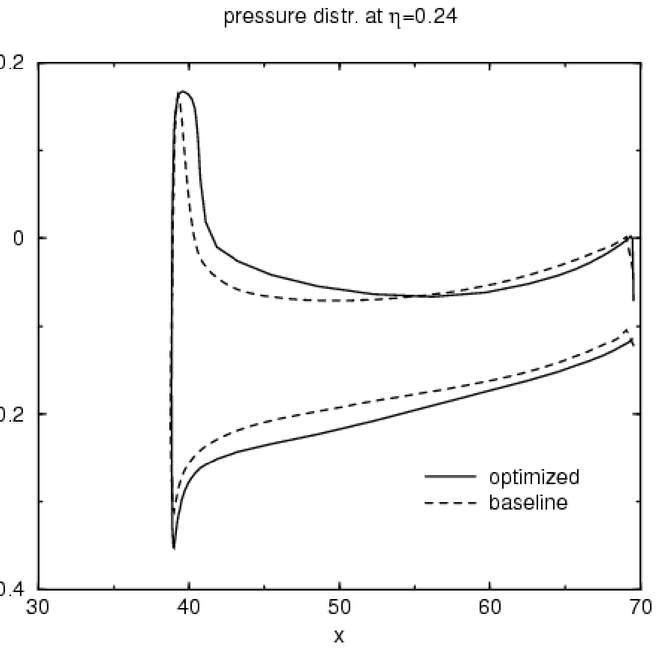
Pressure on the upper surface

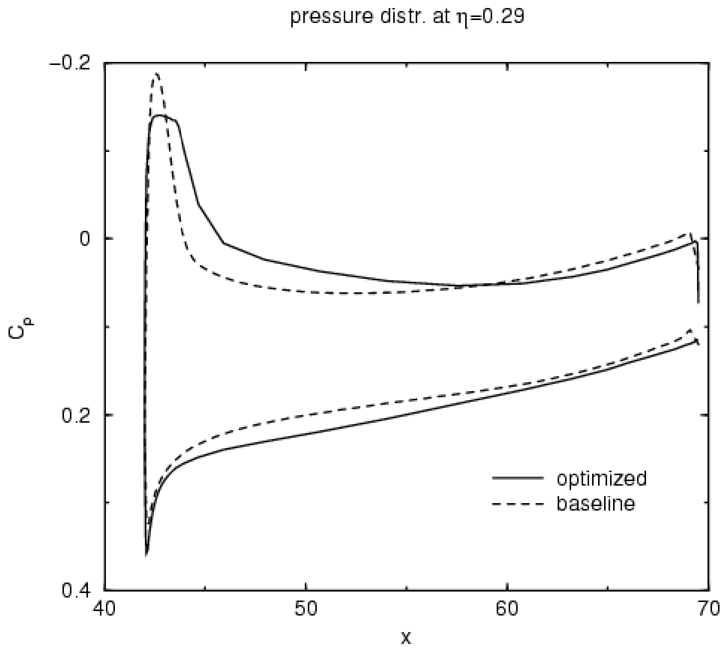
Mach number distr. on the upper surface

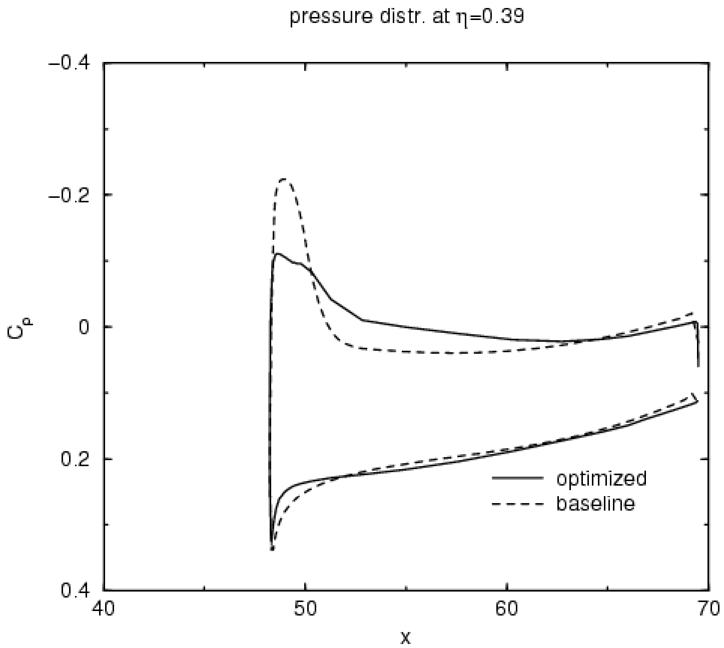


# Optimization results

$C_p$ distribution in the wing span direction

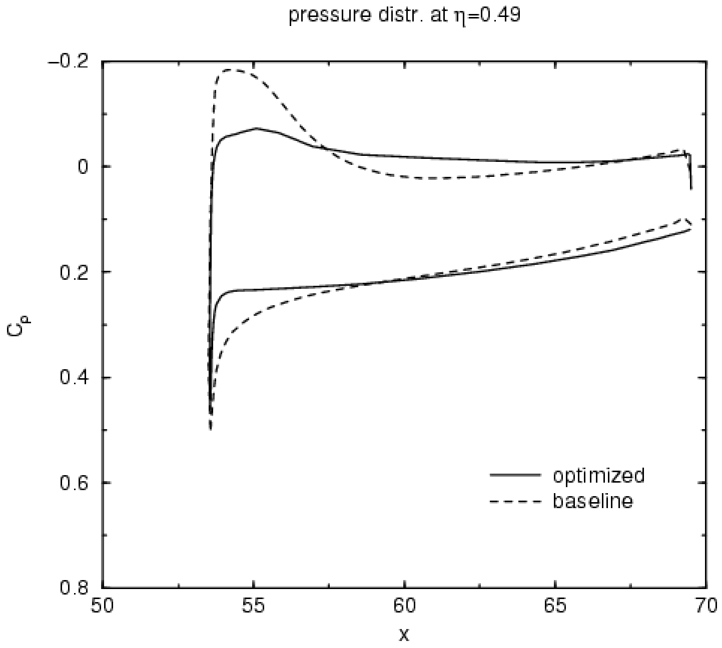


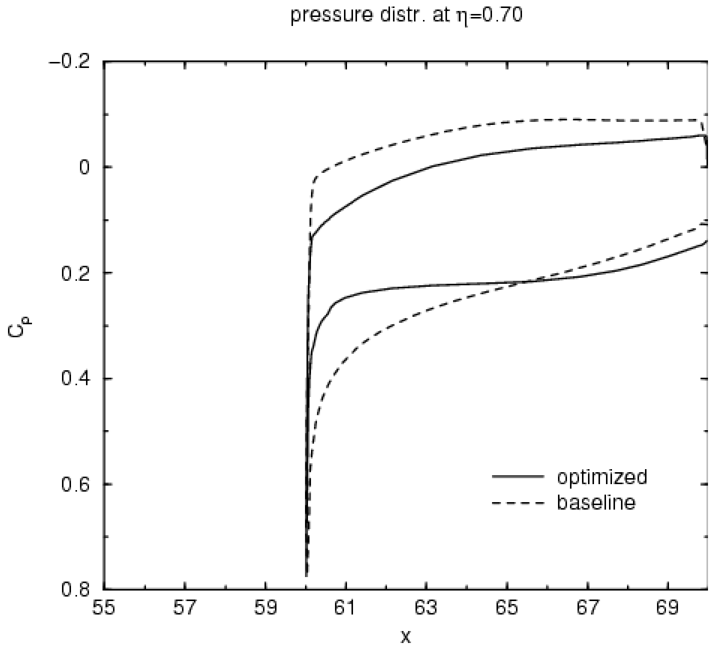



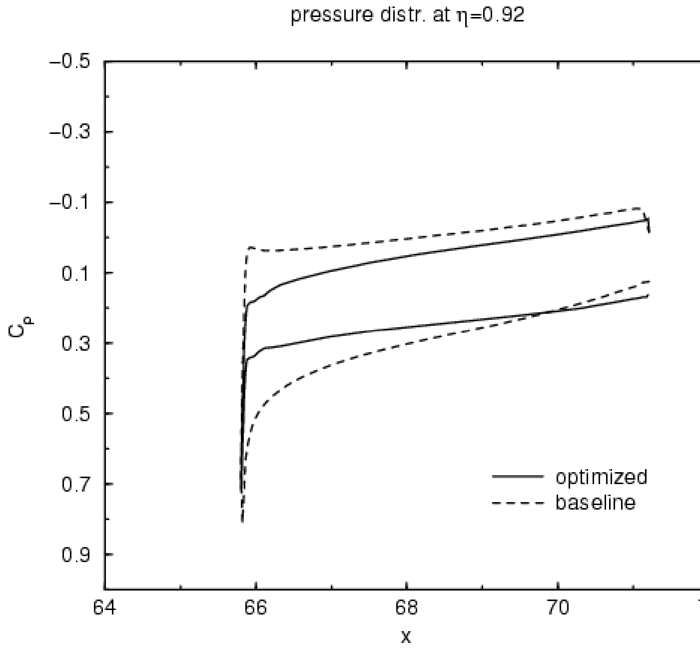



# Optimization results

Cp distribution in the wing span direction



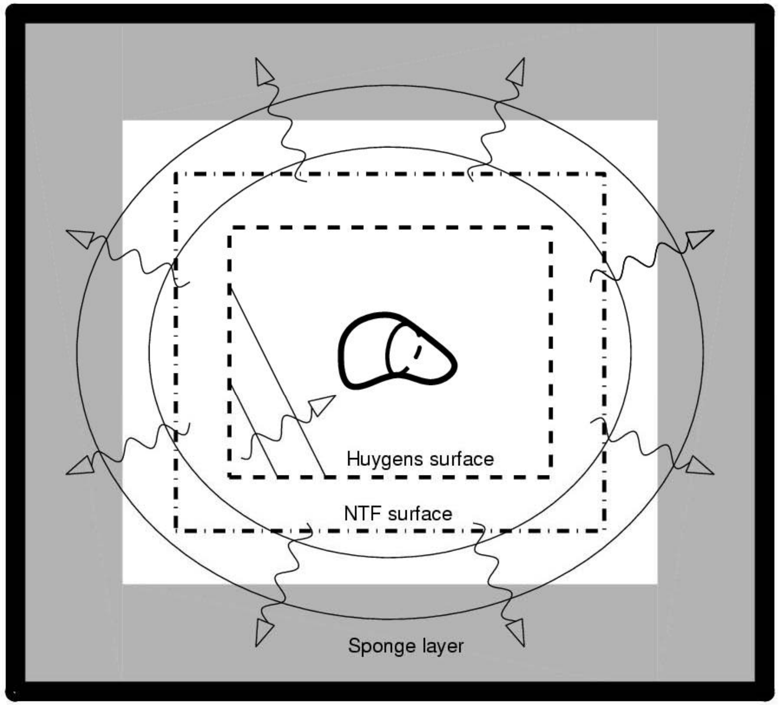



# Electromagnetic shape optimization for reduction of radar cross section

joint work with

A. Bondesson and Y. Yang
Chalmers University of Technology

# Scattering problem




- Incident wave on Huygens surface
- Near-to-Far Field Transformation
- Absorbing boundary condition
- Differential equation formulation used for derivation only
- In reality: solve scattering problem with integral equation to compute surface currents.

# Scattering problem

- **Incident plane wave** $\vec{E}_i = \vec{E}_0 \exp(-j\vec{k}_0 \cdot \vec{r})$
- **Monostatic RCS** $\sigma = \dfrac{4\pi |A_\perp|^2}{E_0^2 k_0^2}$
- **Scattering amplitude**

$$\vec{A} = \vec{a}(\vec{E}) = \frac{k_0}{4\pi} \oint_{NTF} \left[ \hat{n} \times (\nabla \times \vec{E}) - j\vec{k}_0 \times (\vec{E} \times \hat{n}) \right] \exp(-j\vec{k}_0 \cdot \vec{r}) dS$$

- **Scattering problem in total field/scattered field form** $\vec{E} = \vec{E}_s + \sigma(u)\vec{E}_i$

$$\nabla \times \mu_r^{-1} \nabla \times \vec{E} - k_0^2 \varepsilon_r \vec{E} = -\nabla \times [\hat{n} \times \vec{E}_i \, \delta(u)] - \delta(u) \hat{n} \times \nabla \times \vec{E}_i$$

  $u$ **surface label, positive inside Huygens surface,** $dv = du dS$

- $\varepsilon_r$ **and** $\mu_r$ **can be complex**

# Weak form and adjoint problem

$$\mathcal{L}(\vec{w},\vec{E}) = \mathcal{L}_0(\vec{w},\vec{E}) - (4\pi/k_0)\vec{E}_0 \cdot \vec{a}(\vec{w}) = 0, \quad \forall \vec{w} \in H(\text{curl}, \Omega).$$

$$\mathcal{L}_0(\vec{w},\vec{E}) = \int_\Omega \left(\mu_r^{-1}\nabla \times \vec{w} \cdot \nabla \times \vec{E} - k_0^2 \varepsilon_r \vec{w} \cdot \vec{E}\right) dv + \oint_{\partial\Omega} (\hat{n} \times \vec{w}) \cdot (jk_0 \hat{r} \times \vec{E}) dS$$

**$\mathcal{L}_0$ symmetric if $\partial\Omega$ is a large sphere $\hat{n} = \hat{r}$.**

**Variation of $\vec{E}$ gives variation of RCS:** $\delta\sigma = \dfrac{8\pi \text{Re}[\vec{A}_\perp^* \cdot \vec{a}(\delta\vec{E})]}{E_0^2 k_0^2}$

**The adjoint field $\vec{E}_a$ satisfies**

$$\mathcal{L}(\vec{w},\vec{E}_a) = \mathcal{L}_0(\vec{w},\vec{E}_a) - \frac{4\pi}{k_0}\vec{A}_\perp^* \cdot \vec{a}(\vec{w}) = 0$$

**Same scattering problem, but the incident wave is** $\vec{E}_{a,i} = \vec{A}_\perp^* \exp(-j\vec{k}_0 \cdot \vec{r})$.

**Choose $\vec{w} = \delta\vec{E}$ in the weak form of the adjoint equation**

$$\delta\sigma = \frac{8\pi \text{Re}[\vec{A}_\perp^* \cdot \vec{a}(\delta\vec{E})]}{E_0^2 k_0^2} = \frac{2\text{Re}[\mathcal{L}_0(\vec{E}_a, \delta\vec{E})]}{E_0^2 k_0}$$

## Displace PEC scatterer

Evaluate $\mathcal{L}_0(\vec{w}, \delta\vec{E})$ by requiring $(\mathcal{L} + \delta\mathcal{L})(\vec{w}, \vec{E} + \delta\vec{E}) = 0$

$\delta\vec{E}$ from a normal displacement $\xi$ of PEC boundary $\Gamma$ satisfies

$$\mathcal{L}_0(\vec{w}, \delta\vec{E}) = -\delta\mathcal{L}_0(\vec{w}, \vec{E}) = \oint_\Gamma \left(\nabla \times \vec{w} \cdot \nabla \times \vec{E} - k_0^2 \vec{w} \cdot \vec{E}\right) \xi dS.$$

Akel and Webb (IEEE Trans. Magn. 2000) derive discretized design sensitivity, which corresponds to a surface integral.

**Analytic form**

$$\delta\sigma = \frac{2}{E_0^2 k_0} \oint_\Gamma \mathrm{Re}\left(\nabla \times \vec{E}_a \cdot \nabla \times \vec{E} - k_0^2 \vec{E}_a \cdot \vec{E}\right) \xi dS$$

In terms of surface quantities:

$$\delta\sigma = -\frac{2k_0 Z_0^2}{E_0^2} \oint_\Gamma \mathrm{Re}\left(\vec{J}_{\mathrm{adj}} \cdot \vec{J}_{\mathrm{orig}} + c^2 \rho_{\mathrm{adj}} \rho_{\mathrm{orig}}\right) \xi dS$$

# 2D - TM polarization

**TM RCS (a length)** $L = \frac{4|A|^2}{k_0|E_0^2|}$ **where**

$$A = -\frac{1}{4}\oint_{NTF}\left(j\vec{k}_0 E + \nabla E\right)\cdot\hat{n}\exp(-j\vec{k}_0\cdot\vec{r})dl,$$

**Adjoint solution** $E_a = \frac{A^*}{E_0}E.$

**Variation of RCS**

$$\delta L = \frac{8\mathrm{Re}(A^*\delta A)}{k_0|E_0^2|} = \frac{2}{k_0|E_0^2|}\oint_\Gamma \mathrm{Re}(\nabla E_a\cdot\nabla E - k_0^2 E_a E)\xi dl$$

**In terms of surface quantities**

$$\delta L = -\frac{2Z_0^2 k_0}{|E_0^2|}\oint_\Gamma \mathrm{Re}(J_{\mathrm{adj}}J_{\mathrm{orig}})\xi dl$$

# 2D - TE polarization

**RCS** $L = \frac{4|G|^2}{k_0|H_0^2|}$ **where**

$$G = -\frac{1}{4}\oint_{NTF}\left(j\vec{k}_0 H + \nabla H\right)\cdot\hat{n}\exp(-j\vec{k}_0\cdot\vec{r})dl$$

**Adjoint solution** $H_a = (G^*/H_0)H$.

**Variation of the RCS**

$$\delta L = \frac{8\mathrm{Re}(G^*\delta G)}{k_0|H_0^2|} = -\frac{2k_0}{|H_0^2|}\oint_\Gamma \mathrm{Re}\left(J_{\mathrm{adj}}J_{\mathrm{orig}} + c^2\rho_{\mathrm{adj}}\rho_{\mathrm{orig}}\right)\xi dl$$

# Optimization

- Compute shape derivative on all surface nodes.
- Introduce reduced parametrization $\xi = \xi(\vec{r}, p_1, p_2, ..., p_n)$
- Find derivatives wrt parameters by the chain rule $\frac{\partial \sigma}{\partial p_k} = \sum_i \frac{\partial \sigma}{\partial \xi_i} \frac{\partial \xi_i}{\partial p_k}$
- Allows a clear separation of the geometry description (shape parameters), from the RCS calculation (nodes on grid), which fits with CFD optimization developed at Saab.
- Use "black-box" optimizer, e.g., Levenberg-Marquardt least square.
- Solve scattering problem by Complex Field Integral Equation.
  Need many angles: $\Delta\theta < \lambda/(8d)$ for sufficient sampling

# 2D test problem - shape optimization

Reduce average $\sigma$ in $[-\theta_m, \theta_m]$ and $[180^o - \theta_m, 180^o + \theta_m]$.

PEC boundary parametrized as a Fourier expansion for $r(\Theta)$:

$$r(\Theta) = 1 + \sum_{n=1}^{n_{\max}/2} \frac{c_{2n}}{(2n)^\alpha} \cos 2n\Theta.$$

EM solver uses integral formulation (Method of Moments).

$\alpha$ influences gradients in parameter space.

There are several local optima, i.e., optimization problem is ill posed.

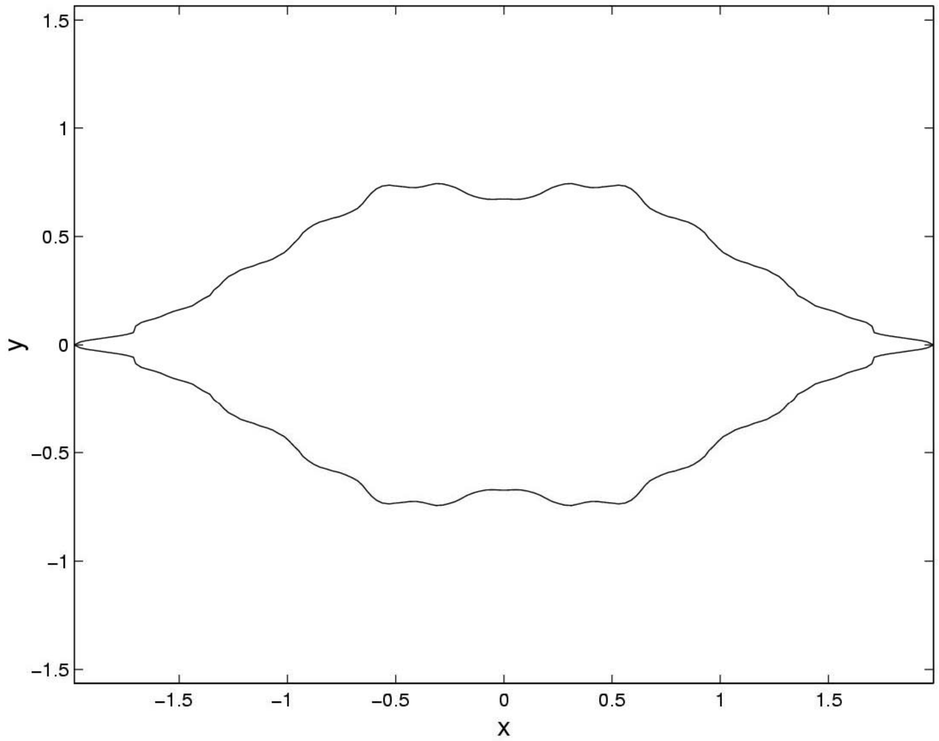
Tikhonov regularization: add penalty term $\nu R$ to goal function

$$R = \frac{1}{\pi} \int_0^{2\pi} \left( \frac{\partial^2 r}{\partial \Theta^2} \right)^2 w_P(l) d\Theta$$

Regularized problem converges well to “nice” shapes for $\alpha \sim 0.5$

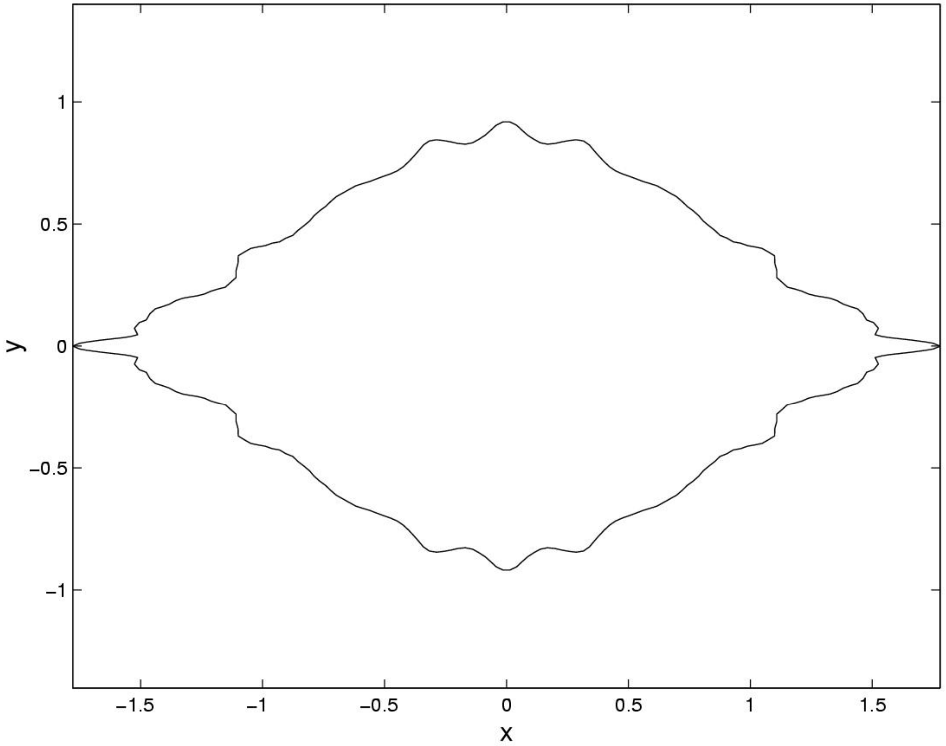
but the optimization problem is ill posed.

# TE only, no penalty, $\theta_m = 45^o$



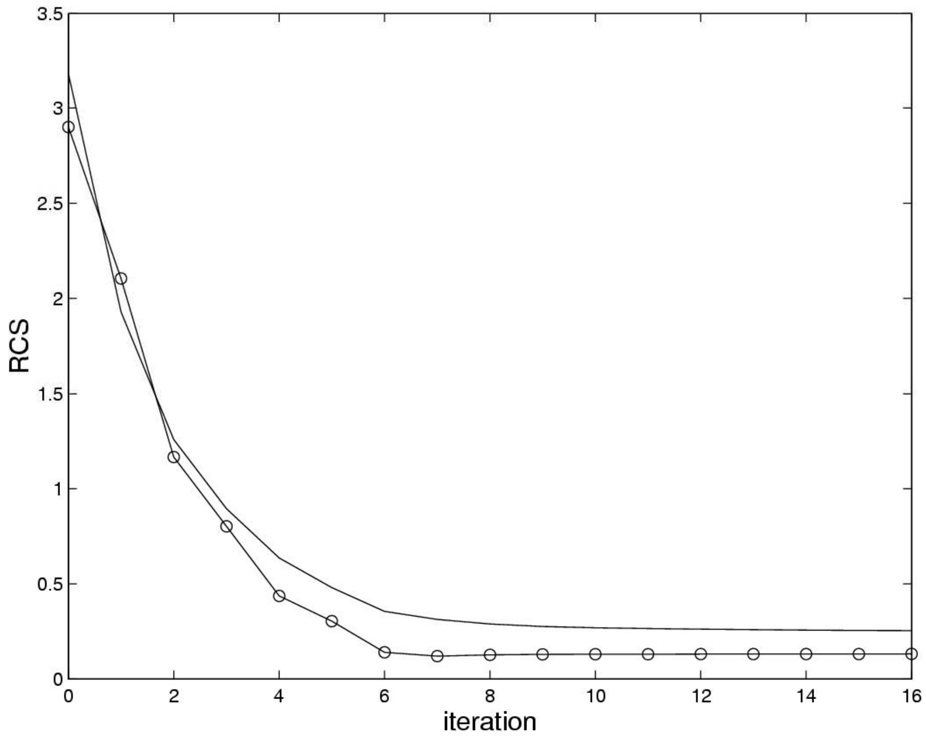

**Optimized shape for TE without penalty has corrugations. The RCS in $[-45^o, 45^o]$ is reduced 400 times from a circle to $I_2^{TE}(\pi/4) = 0.00823$ and $I_2^{TM} = 0.213$.**

# TE + TM, no penalty, $\theta_m = 45^o$



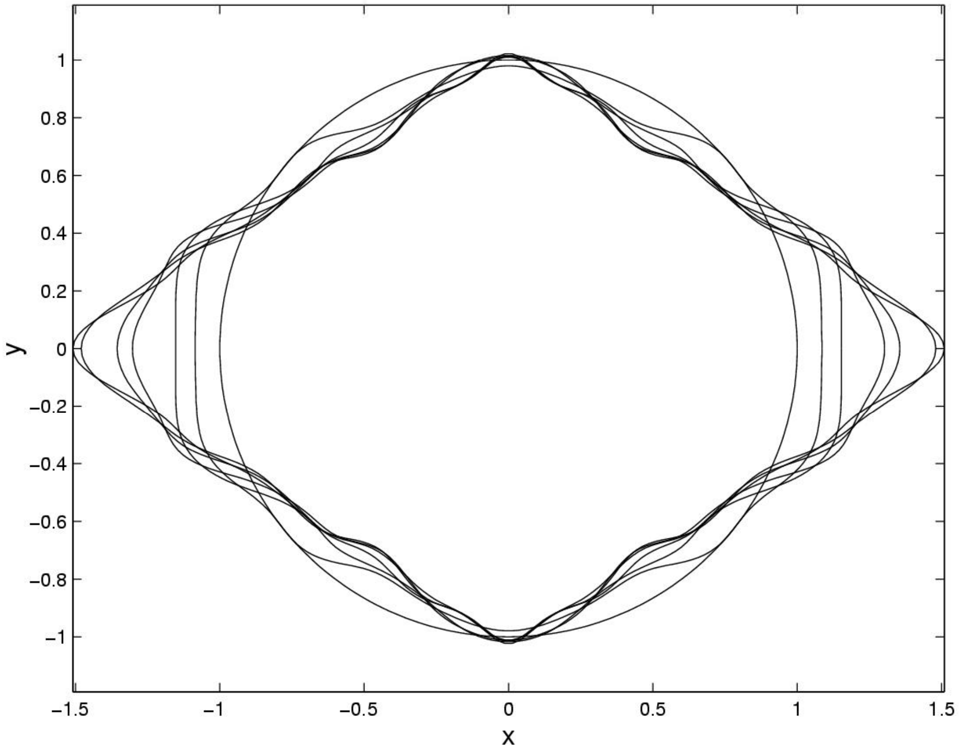

**Final shape of the combined TE and TM optimization without penalty.** $I_2^{\text{TE}} = 0.118$ **and** $I_2^{\text{TM}} = 0.179$. **Optimized shape has corrugations and sharp tips.**

# TE + TM, penalty except at the tips, $\theta_m = 45^{\text{o}}$



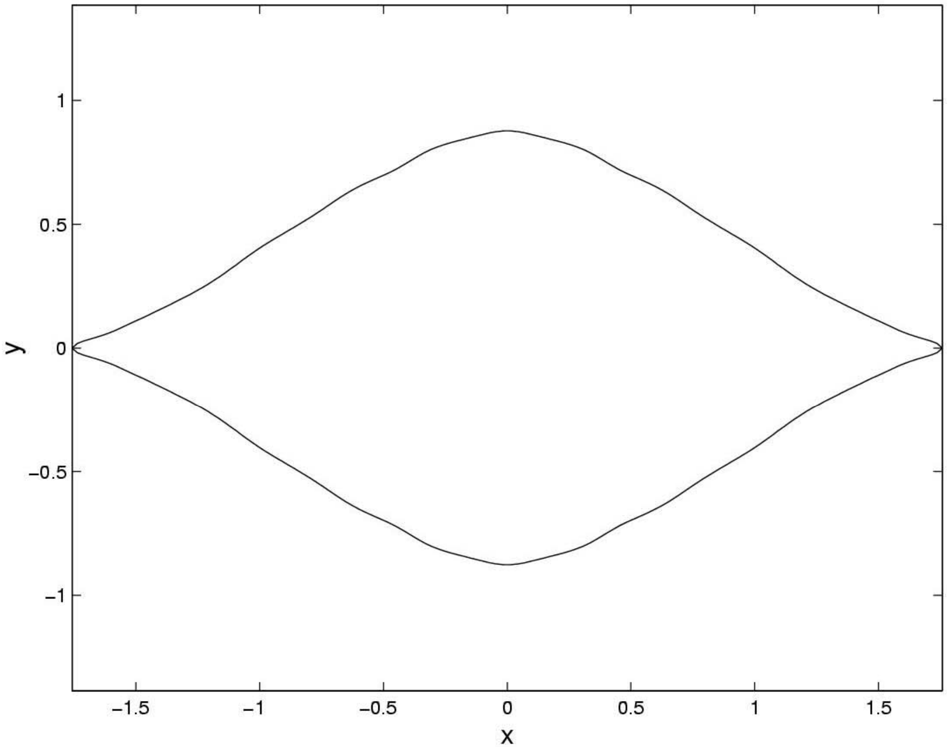

**Evolution of the average RCS for TE (-o-) and TM (-) polarization in $|\theta| < 45^{\text{o}}$ when the optimized quantity is $I_1^{\text{TE}} + I_1^{\text{TM}} + 0.02R_2$, $\alpha = 0.5$, and the penalty is removed in regions of length $0.15\lambda$ from the tips. Convergence in $\sim 6$ iterations.**

# TE + TM, penalty except at the tips, $\theta_m = 45^o$




**Six first shapes for combined TE and TM optimization with penalty. Radius of circle is two wavelengths**

# TE + TM, penalty except at the tips, $\theta_m = 45^o$

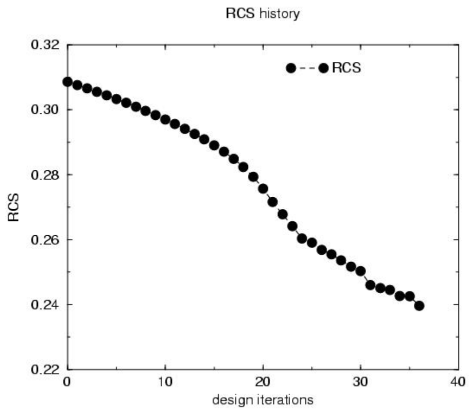


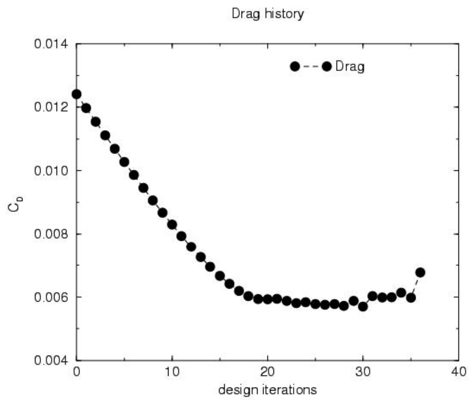

**Shape optimized for TE and TM polarization with a penalty $0.02R_2$ with zero weighting within $0.15\lambda$ of the tips. $I_2^{\mathrm{TE}} = 0.135$, $I_2^{\mathrm{TM}} = 0.245$. Penalty has practically removed the corrugations.**

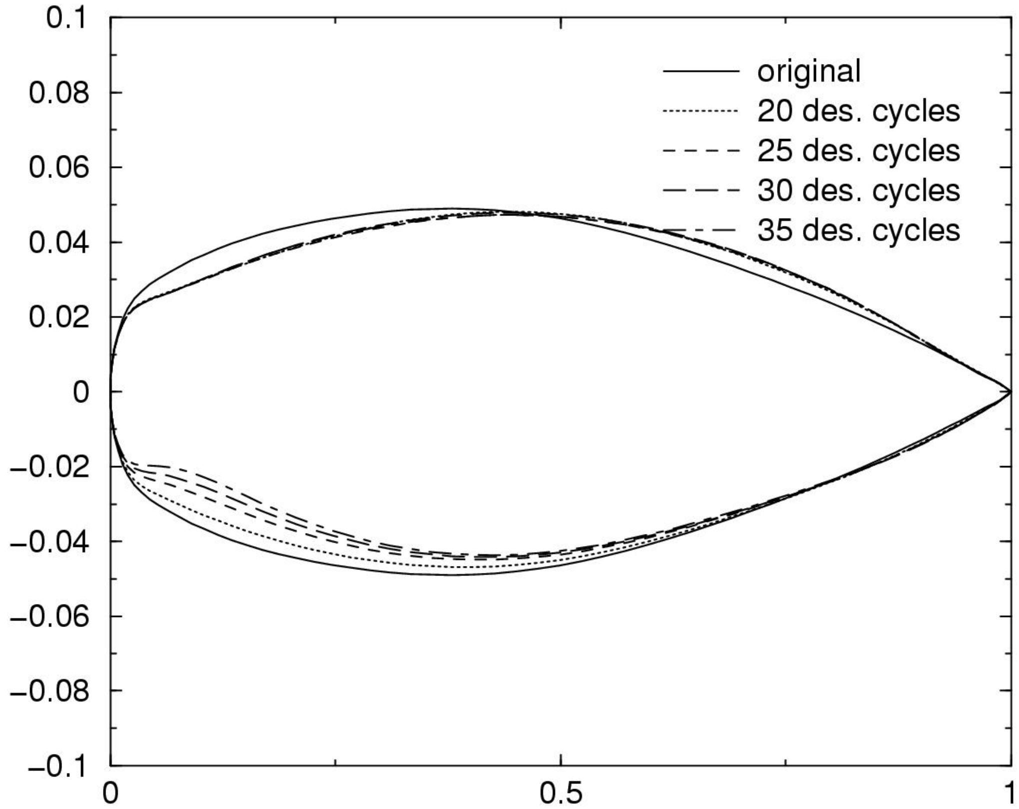
# Combined RCS + CFD optimization

**Steepest descent. Lift fixed as a constraint. $y(x)$ modified with "bump functions".**

**Goal function $0.1 \times$RCS + $0.9 \times$drag.**



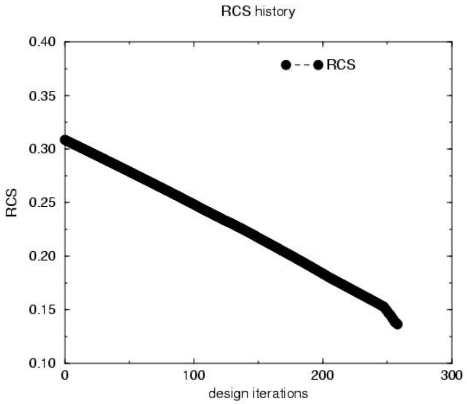



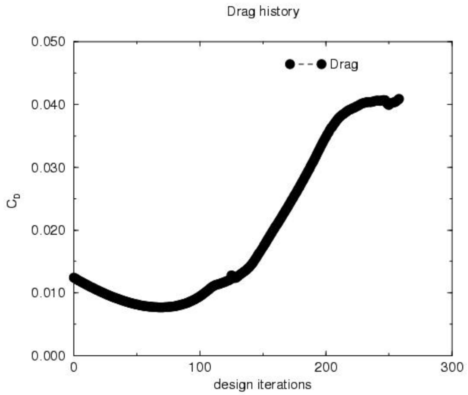



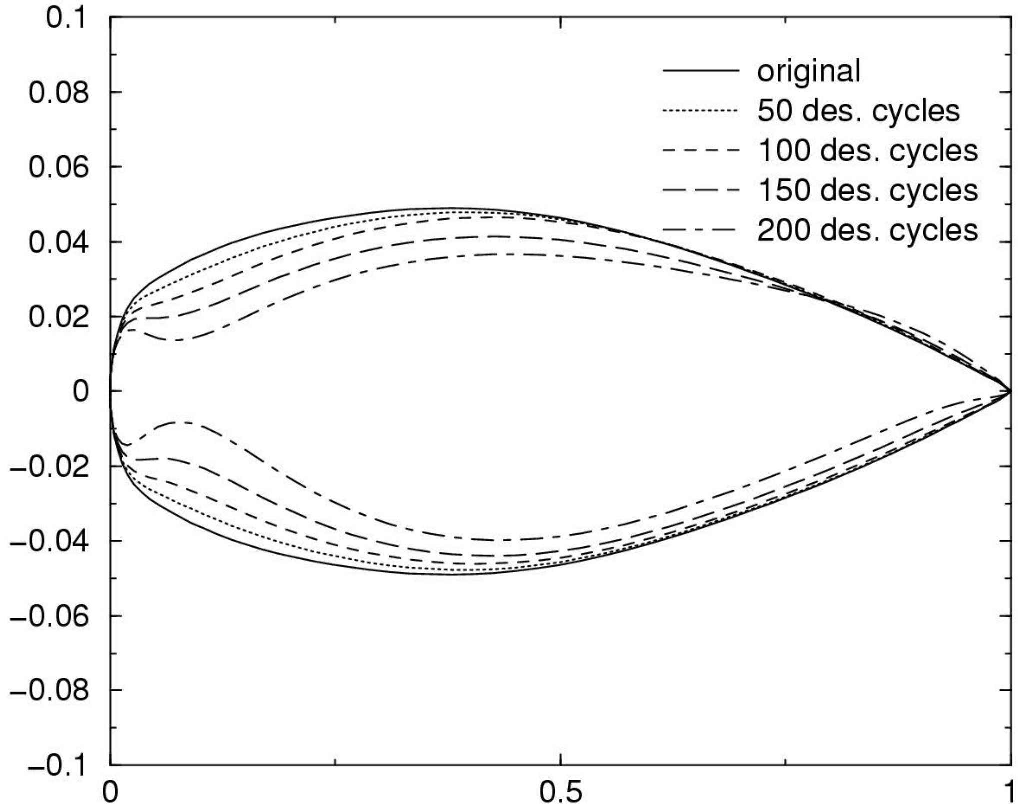
# Combined RCS + CFD optimization

**Steepest descent. Lift fixed as a constraint. $y(x)$ modified with "bump functions".**

**Goal function 0.8×RCS + 0.2×drag.**



# Data assimilation in Bio Fluid Mechanics

joint work with

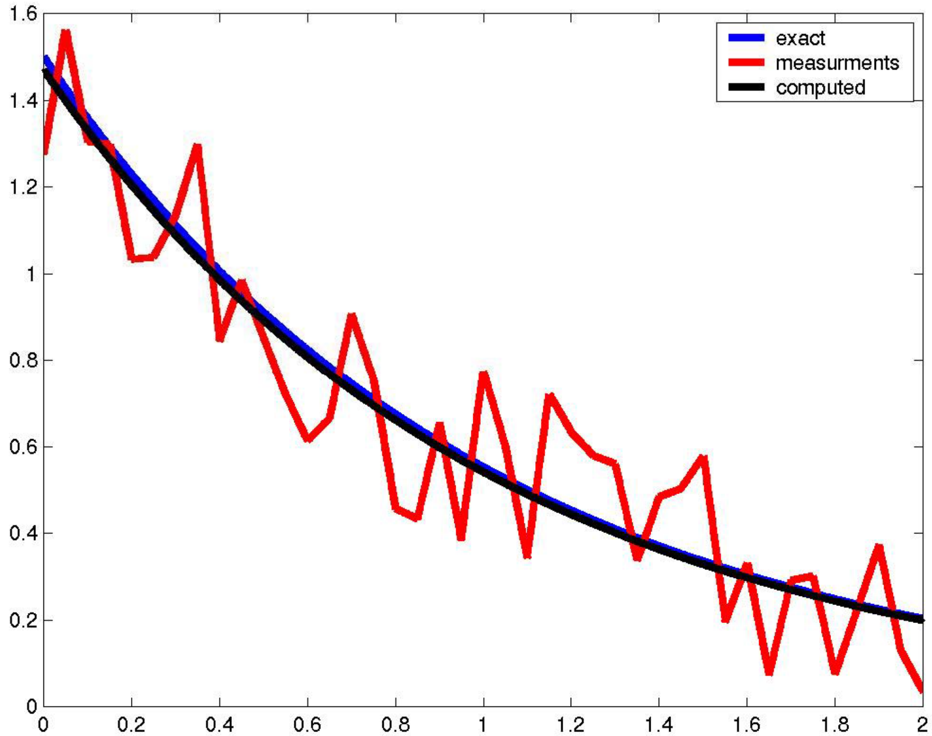
J. Lundvall, V. Kozlov and M. Karlsson
Linköping University

# Data assimilation

Given: Sparse measured data in space and time including noise.

Objective: Compute data on a high resolution grid in space and time assuming that the physics is described by wellknown mathematical equations with an unknown initial condition.

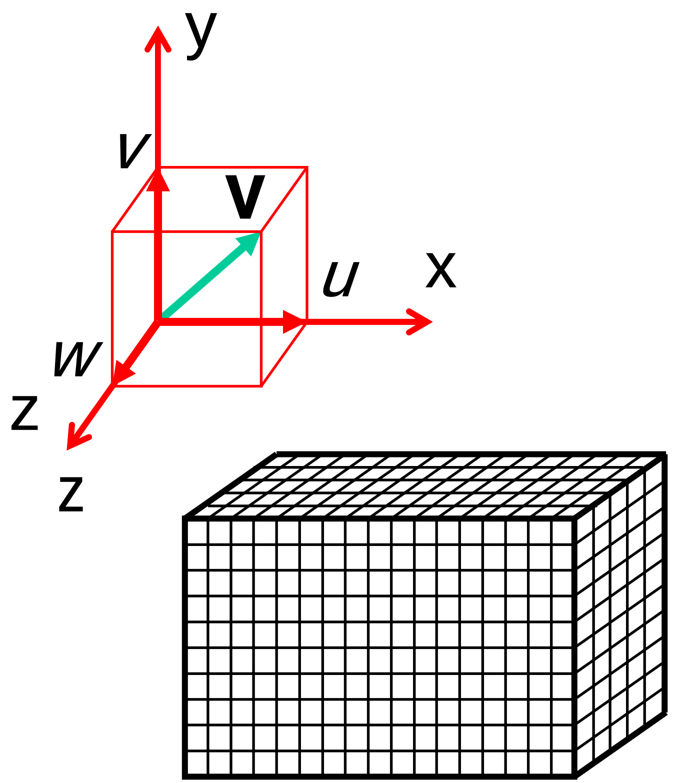
Example:
y'+y=0, y(0)=?



# Example of application fields

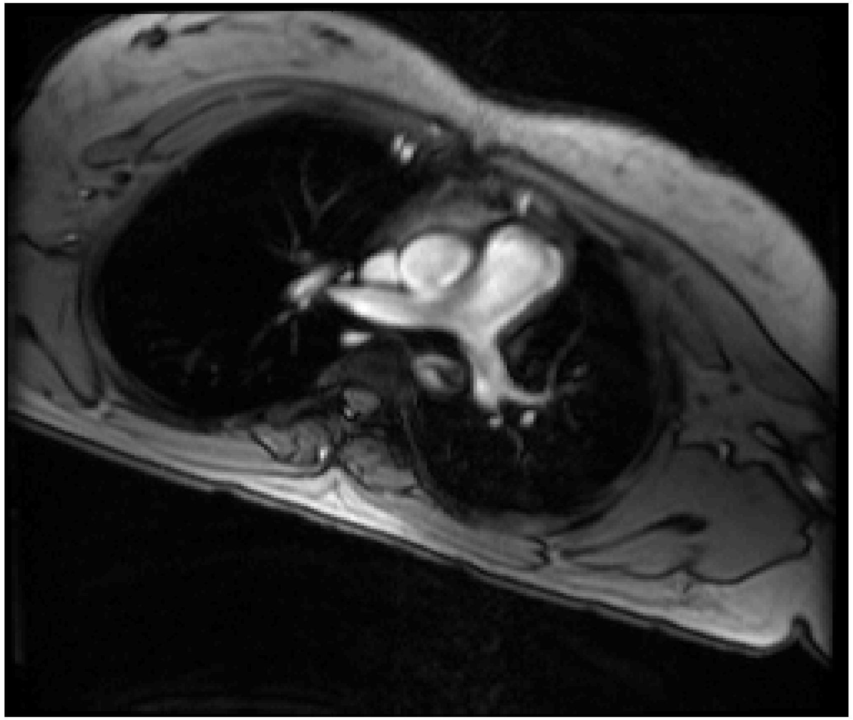
- Meteorology, weather forecast
- Medicine, MR measurements
- Wind tunnel experiments

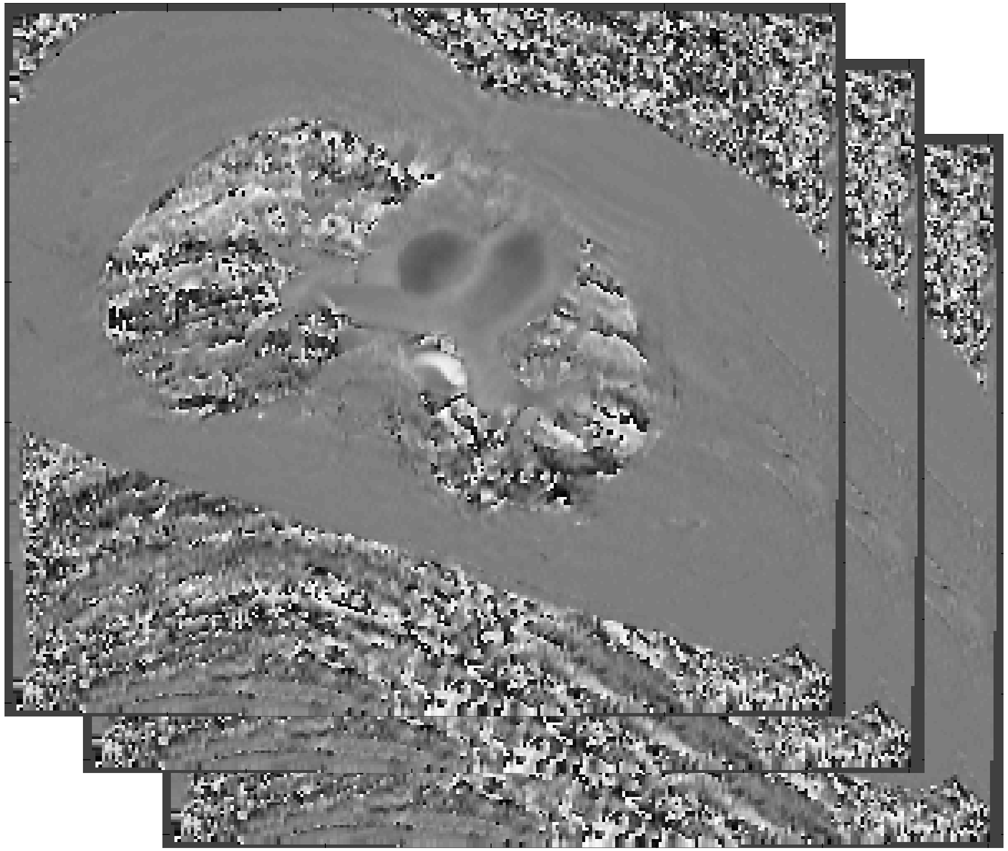
# Velocity Measurement using MR camera




**3D + T = 4D**

# MR-data: Magnitude and Phase Images

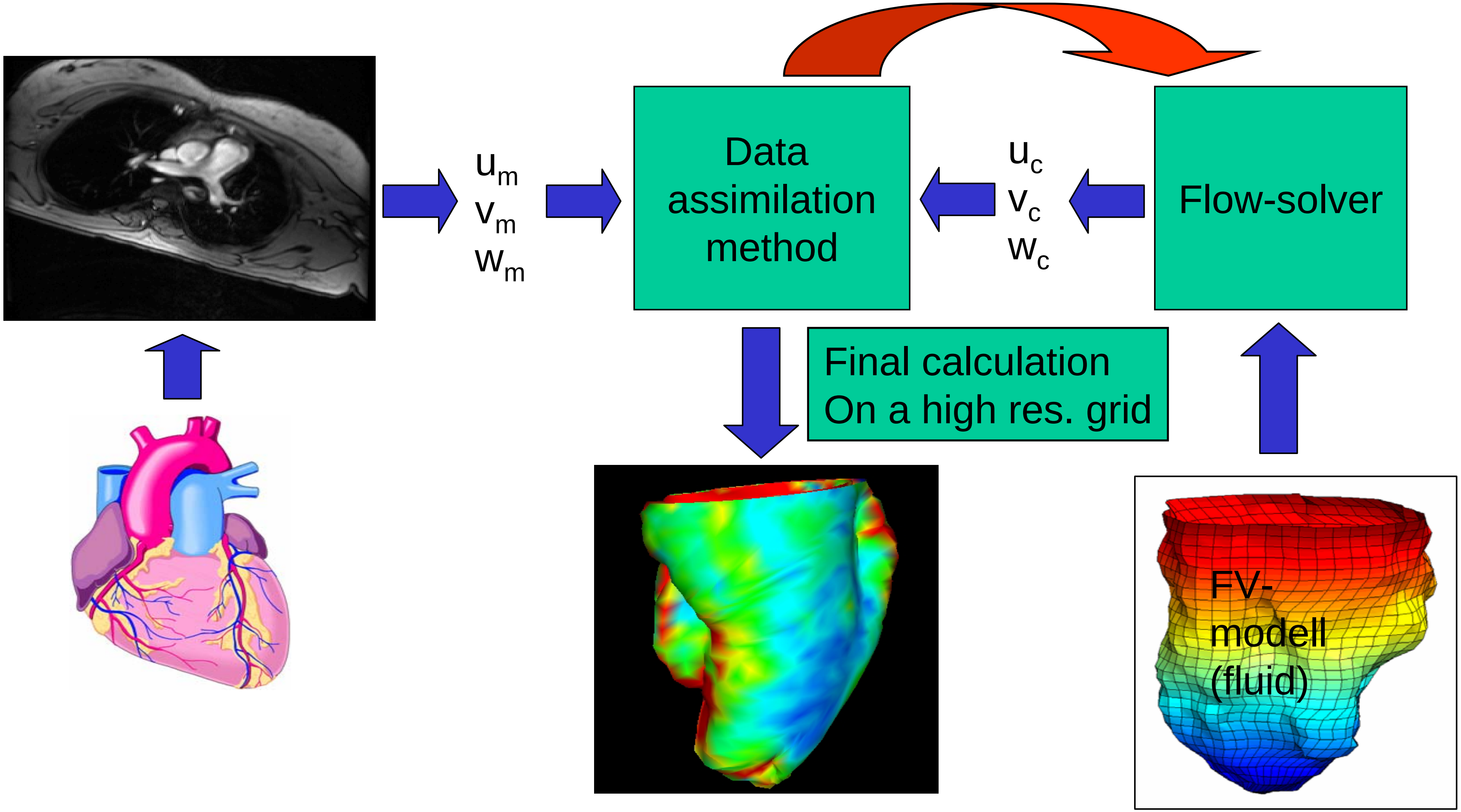




MR Magnitude

Phase (velocity)

# Reconstruction of velocity field from MR-data



# Problem formulation

- measured data u* is given
- assume that the physical model is known and described by a PDE
- the boundary conditions are given
- the initial condition is unknown and used as the control variable

$$Q_T = \Omega \times (0,T), \qquad \min_{u_0} \|u - u^*\|_{L^2(Q_T)},$$

$$\text{s.a. } P(u,\psi) = 0.$$

# Differential equation

$$\begin{aligned}\frac{\partial u}{\partial t}+\frac{\partial}{\partial x}F(u,u_x) &= 0,\\ u(x,t)|_{x\in\partial\Omega} &= 0,\\ u(x,0) &= u_0(x).\end{aligned}$$

Weak formulation : $u \in X$ Is a weak solution if

$$-\int_{Q_T}\left(u\psi_t+F(u,u_x)\psi_x\right)dxdt+\int_\Omega\left(u(x,T)\psi(x,T)-u_0(x)\psi(x,0)\right)dx=0,$$

for all $\psi \in X'$.

P(u,ψ)=0

# Variational formulation

$$L(u) = \frac{1}{2}\int_{Q_T} (u-u^*)^2 dxdt + P(u,\psi)$$

$$\delta L(u;\delta u_0) = \int_{Q_T} (u-u^*)\delta u dxdt + \delta P(u,\psi) =$$

$$= \int_{Q_T} \left( u - u^* - \frac{\partial \psi}{\partial t} - \frac{\partial F}{\partial u}\frac{\partial \psi}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\frac{\partial \psi}{\partial x}\right)\right)\delta u dxdt$$

$$+ \int_{\Omega} (\psi(x,T)\delta u(x,T) - \psi(x,0)\delta u_0(x))\, dx$$

# Adjoint equations

$$
\begin{aligned}
\psi(x,T) &= 0, \\
\frac{\partial \psi}{\partial t} + \frac{\partial F}{\partial u}\frac{\partial \psi}{\partial x} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial u_x}\frac{\partial \psi}{\partial x}\right) &= u - u^*, \\
\psi(x,t)|_{x\in\partial\Omega} &= 0
\end{aligned}
$$

$$\delta L = -\int_\Omega \psi(x,0)\delta u_0 dx,$$

$$\delta u_0 = \alpha\psi(x,0),\ \alpha > 0 \quad \Longrightarrow \quad \delta L < 0.$$

Iteration: $u_0^{k+1} = u_0^k + \alpha\delta u_0^k.$

# Existence of an optimal solution

- Ill posed problem
- No optimal solution exist in general
- Regularization necessary

$$u_0(x) = \sum_{m=1}^{M} c_m \phi_m(x).$$

# Existence of an optimal solution

- $T : L^2(\Omega) \to L^2(Q_T),\ y = Tx.$
- Min. problem is equivalent to solving $y^\delta = Tx$, där $\|y^\delta - y\| \le \delta.$
- T is a compact operator and the inverse problem is ill posed
- The previous iteration is equivalent to the Landweber regularization method for non linear inverse problems

$$x_{k+1}^\delta = x_k^\delta + T'(x_k^\delta)^*(y^\delta - Tx_k^\delta).$$

- The iteration converges to the true solution if δ→ 0

# Application: Viscous Burgers' equation

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{1}{2}u^2 - \mu u_x\right) = 0, \quad \mu > 0.$$

- Adjoint equation

$$\frac{\partial \psi}{\partial t} + u\frac{\partial \psi}{\partial x} + \mu\frac{\partial^2 \psi}{\partial x^2} = u - u^*.$$

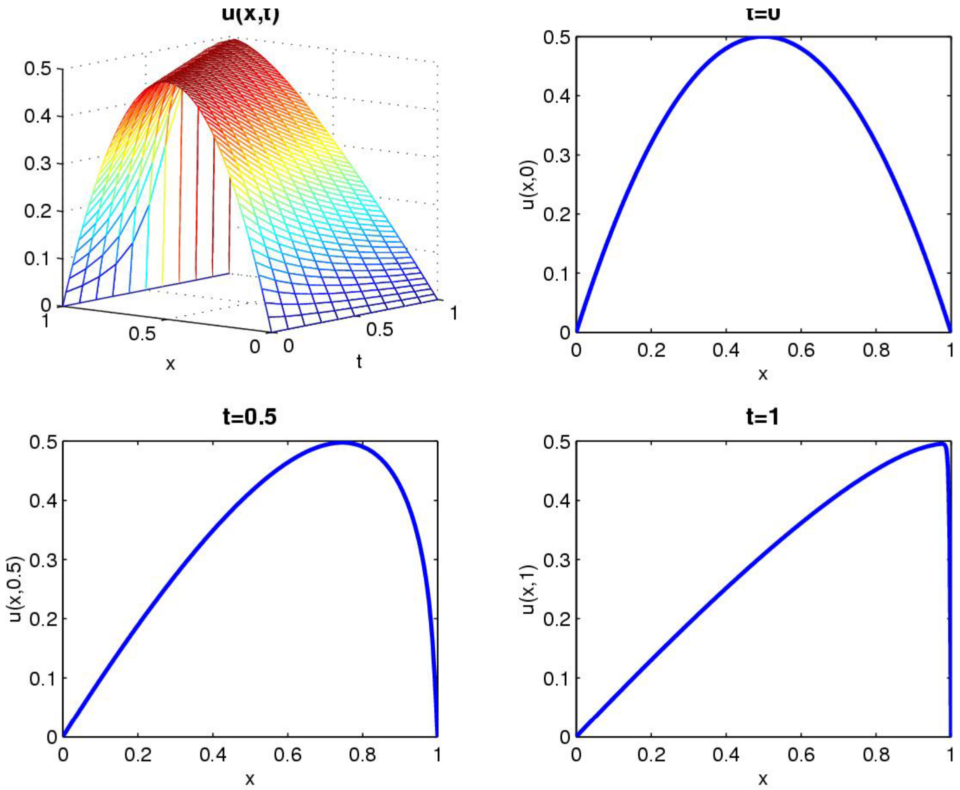
- Data voids: V denotes a void region in the (x,t) plane and m is def. according to

$$m(x,t) = \begin{cases} 0 & \text{om } (x,t) \in V, \\ 1 & \text{om } (x,t) \notin V. \end{cases}$$

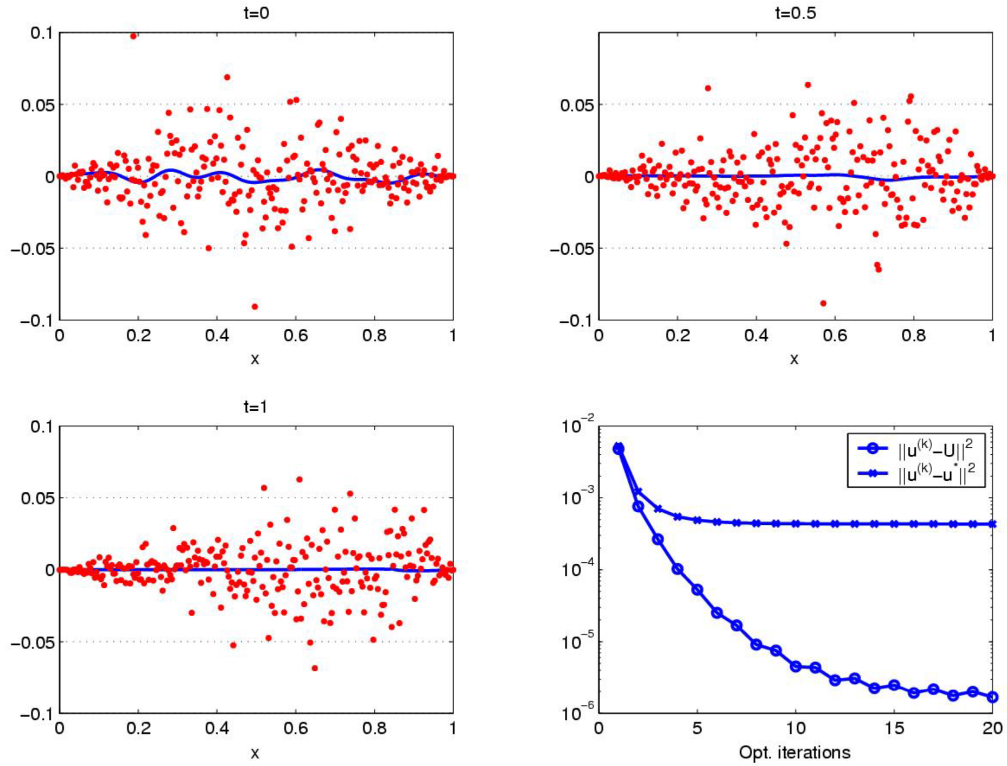
- The adjoint equation

$$\frac{\partial \psi}{\partial t} + u\frac{\partial \psi}{\partial x} + \mu\frac{\partial^2 \psi}{\partial x^2} = m(x,t)(u - u^*).$$

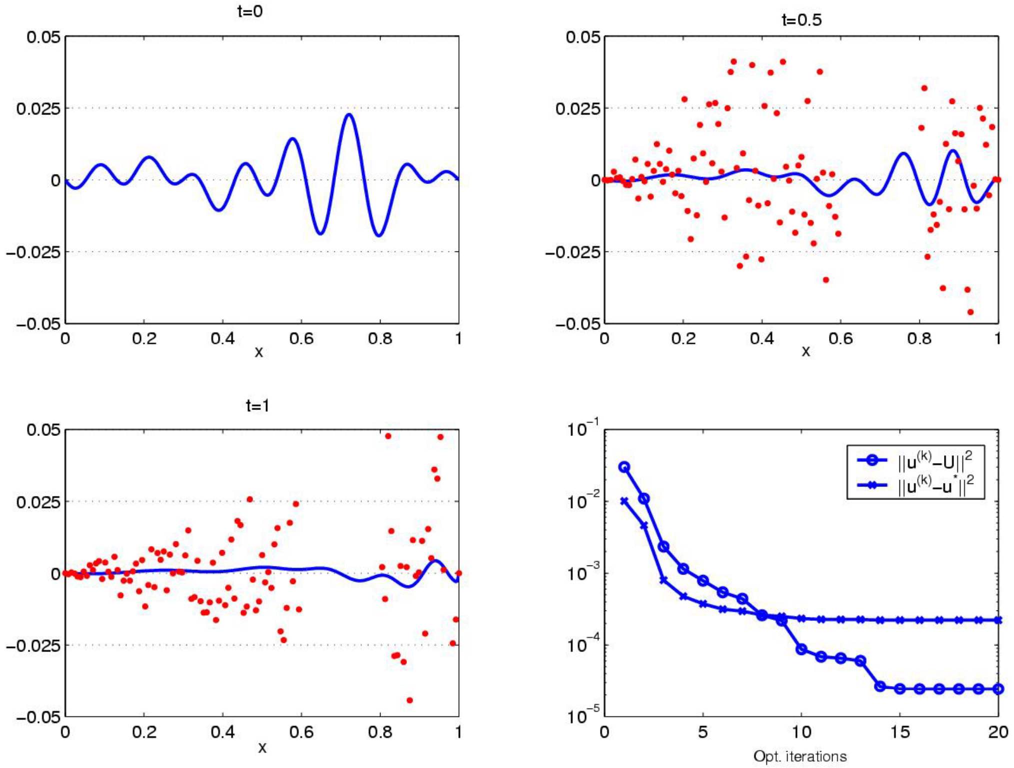
# ”Exact” solution



# Example 1: Noise data



# Example 2: Noise and void data



# Summary and conclusions

- Optimization technique coupled to PDE:s have been presented.
- The optimization gradients are computed from the continuous governing equations and corresponding adjoint equations.
- The optimization technique has been sucessfully applied to three different problems:
  - Aerodynamic shape optimization
  - Electromagnetic RCS optimization
  - Data assimilation in Bio Fluid Dynamics

# Summary and conclusions

- An efficient gradient based optimization technique which can handle large number of design variables.
- Gradients are computed from Euler and Adjoint Euler calc.
- Good agreement with finite difference gradient calculations.
- Applied to realistic 3D aircraft configurations such as the SCT.
- The drag was reduced by 9.5% without thickness modifications and by 16% with thickness modifications.
- Lift and pitching moment was not changed.

## Conclusions

- Gradient method with adjoint solution developed for RCS reduction.
- Both shape and materials optimization in 2D work well.
- Far more efficient than evolutionary methods.
- Optimization problems generally ill-posed, but can be regularized.
- Conflicting demands of RCS reduction and aerodynamics on PEC shape can be resolved by using absorbing materials.
- Gradient method will be tested on moderate-sized 3D problems solved with Multipole code developed at Uppsala University (Nilsson & Lötstedt).
- Method potentially useful in many microwave applications.

# Aerodynamic Shape Optimisation

- Background
- Optimization technique
- The optimization system **cadsos** at SAAB
- Example: Aerodynamic shape optimization of a SCT Aircraft
- Description of the SCT geometry and grid generation
- Aerodynamic shape optimization of the SCT geometry
- Formulation of the optimization problem
- Gradient validation
- Optimization results