Conformal holonomy, symmetric spaces, and skew symmetric torsion

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In the talk I will present recent results obtained in collaboration with J. Alt and A. J. Di Scala about holonomy reductions of normal conformal Cartan connections. Using a recent result by Cap, Gover and Hammerl, we study the case when the conformal holonomy reduces to the isotropy group H of a pseudo-Riemannian irreducible symmetric space G/H, and find that, under certain algebraic conditions, such a reduction not only specifies a metric g in the conformal class but also a metric connection D with totally skew symmetric torsion and holonomy in the stabiliser in H of a null line. Special cases of this situation are the symmetric spaces defined by $G = \mathrm{SL}(3, \mathbb{R})$ and $H = \mathrm{SO}(1, 2)$, in which case we show that the metric g is flat and hence that the conformal class must be flat. Further examples are the symmetric spaces $G = \mathrm{SL}(3, \mathbb{K})$, where $\mathbb{K}$ are the complex numbers or the quaternions and H is either $\mathrm{SU}(1, 2)$ or $\mathrm{Sp}(1, 2)$. It turns out that in both cases the metric g must be Einstein, and moreover, in the complex case that D defines a nearly-para-Kaehler structure. As a consequence we can exclude these H as possible conformal holonomy groups.