Humboldt-Universität zu Berlin
Institut für Mathematik
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Lecture **Analysis on Manifolds**, WS 2010/11

Exercises no. 1

to be submitted by Nov 2nd

1 Let X be a principal G-space (i.e. G acts freely on X), and Y be another G-space. Show that there is a one-to-one correspondence between equivariant maps $f : X \to Y$ and sections of $\pi : X \times Y/G \to X/G$.

2 a) Show that a connected space needs not to be path-connected.

b) Show that X is path-connected iff X is connected and each $x \in X$ has a path-connected neighbourhood.

3 Show that a surjective continuous map $f : X \to Y$ is an identification, if it admits a section $s : Y \to X$.

4 Consider the S^1-action on S^2 given by rotation around the x^3-axis, with orbit space $[-1, 1]$. Show that the orbit map is an identification w.r.t. the natural topologies.

5 Define a 'reasonable' topology on the Cartesian product of an arbitrary family $(X_\alpha)_{\alpha\in\mathcal{A}}$.

6 Is it true that $\partial \overset{\circ}{A} = \emptyset$ for any subset A of a topological space X?