Humboldt-Universität zu Berlin
Institut für Mathematik
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Lecture **Analysis on Manifolds**, WS 2010/11

Exercises no. 7

to be submitted by Dec 14th

Let M be a manifold of dimension m.

1 Let $p_1, p_2 \in M$ be different points. Show that

$$T_{p_1}M \cap T_{p_2}M = 0.$$

2 Construct $f \in C^\infty(\mathbb{R})$ such that the set of critical values is dense (hint: write $\mathbb{Q}$ as a sequence $(r_i)_{i=-\infty}^{\infty}$ and construct f with a critical value in $[i, i+1]$, $i \in \mathbb{Z}$).

3 Prove the transformation law for tangent vectors in T_pM under a change of basis.

4

a) Show that $\tau^1(M)$ is a vector space and a left $C^\infty(M)$-module.

b) Show that the projection $\pi : TM \to M$ is smooth.

5 Consider smooth paths in M through $p \in M$. Call two such paths equivalent iff they define the same derivation at p. Show that the set of equivalence classes can be mapped bijectively to T_pM.

6 Prove the lemma on the local structure of $dj[TM] \subset T\mathbb{R}^n$ given in the lecture of Dec. 8th, where $j : M \to \mathbb{R}^n$ is a smooth embedding.