Humboldt-Universität zu Berlin
Institut für Mathematik
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Lecture **Analysis on Manifolds**, WS 2010/11

Exercises no. 9

to be submitted by Dec 11

1 Proof the Lemma ("Algbraic properties of the Lie Bracket") from the lecture on January 6th.

2 Show that $\mathbb{R}^3$ forms a Lie algebra under the cross-product $(x, y) \mapsto x \times y$.

3

a) Show that any $X \in \tau^1(M)$ is complete if M is compact.

b) Is every $X \in \tau^1(M)$ complete?

c) If $x : U_x \to \mathbb{R}^m$ is a coordinate system for the smooth manifold M^m, then

$$\left[\frac{\partial}{\partial x^i}, \frac{\partial}{\partial x^j}\right](p) = 0 \quad \text{for } p \in U_x.$$

4 Let $X \in \tau^1(M)$. Show that the integral curves of X are immersed submanifolds, but not necessarily submanifolds which form a decomposition of M.

5

a) Describe the flow of any linear vector field X in $\mathbb{R}^2$ if $X = X^\dagger$ and sketch the flow lines.

b) How is the flow of an arbitrary linear vector field related to the flows in a) ?