Humboldt-Universität zu Berlin
Institut für Mathematik
Prof. Dr. Jochen Brüning
Lecture **Analysis on Manifolds**, WS 2010/11

Exercises no. 12

to be submitted by Feb 1st 2011

1 Let (M, g^{TM}) be a Riemannian manifold.

a) Show that

$$L(c) = \int_0^1 \sqrt{g^{TM}[c'(t), c'(t)]}\, dt$$

is well defined for each regular C^1-path $c : [0,1] \to M$.

b) Show that

$$d_M(p,q) := \inf \left\{ L(c) \ : \ c : [0,1] \to M \text{ piecewise } C^1 \text{ and regular} \right\}$$

is a metric which defines the topology of M.

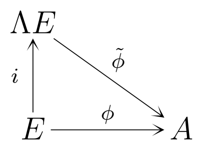
2 Construct the product on $T(E)$ and show that it is associative.

3 Show that the algebras $\mathcal{L}\left(\bigotimes_{i=1}^k E_i\right)$ and $\bigotimes_{i=1}^k \mathcal{L}(E_i)$ are isomorphic (as algebras).

4 If A is a $\mathbb{K}$-algebra and $\phi : E \to A$ linear, for a $\mathbb{K}$-vector space E, such that

$$\phi(v)^2 = 0, \quad v \in E,$$

then the following diagram is commutative for a unique algebra homomorphism $\tilde{\phi}$; i is the natural embedding.



5 Show that

$$\mathcal{L}_X(i(Y)\alpha) = i([X,Y])\alpha + i(Y)\left(\mathcal{L}_X \alpha\right).$$