Humboldt-Universität zu Berlin
Institut für Mathematik
Prof. Dr. Jochen Brüning
Lecture **Analysis on Manifolds**, WS 2010/11

Exercises no. 13

to be submitted by Feb 8th 2011

1 (Kreisketten-Lemma) Let $c : [0,1] \to M$ be a continuous curve. Show that there is a cover $(U_i))_{i=1}^l$ of coordinate domains $U_i = U_{x_i}$, such that

$$\begin{aligned} U_{x_i} \cap c([0,1]) &= c((\delta_i, \epsilon_i)), \quad 2 \le i \le l-1, \\ U_{x_1} \cap c([0,1]) &= c([0,\epsilon_1)), \quad U_{x_l} \cap c([0,1]) = c((\delta_l, 1]), \end{aligned}$$

for some sequences (δ_i), (ϵ_i) with $0 = \delta_1$, $\epsilon_l = 1$, and

$$\delta_{i-1} < \delta_i < \epsilon_{i-1} < \delta_{i+1} < \epsilon_i < 1, \quad 1 < i < l.$$

2 Let M_i be a manifold of dimension m_i, $i = 1, 2$, and $M := M_1 \times M_2$. Show that M is orientable if M_1 and M_2 are orientable.
What can be said if M_1 is not orientable?

3 Let M_i be as in 2), $i = 1, 2$, and let $f \in C^\infty(M_1, M_2)$ be a surjective submersion. Show that $f^{-1}(p)$ is orientable if M_1 and M_2 are.

4
a) Let now M_2 be a hypersurface (:= submanifold of codimension 1) of M_1, with M_1 orientable. Show that M_2 is orientable iff there is $X \in \tau_1(M)$ such that

$$span\,\langle X(p)\rangle \oplus T_pM_2 = T_pM$$

for all $p \in M_2$.

b) Apply a) to the situation $M_2 = f^{-1}(q)$, where $f \in C^\infty(M_1, \mathbb{R})$ with q a regular value.

c) Give an explicit volume form for S^m.

5 Show that a Riemannian metric induces a smooth metric on all tensor bundels (explain also what this means).