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Introduction to Spectral Theory of Differential Operators, SS 2011

Exercises 6, week of May 23, 2011

1 Show that $D := \sqrt{-1}\frac{d}{dt} : C^1_c(\mathbb{R}) \to C^1_c(\mathbb{R})$ is symmetric in $L^2(\mathbb{R})$ and closable but not closed; describe the closure.

2 Let $D \in L_{dense}(H_1, H_2)$, then $D^\star$ is closed.

3 Let $D \in L_{dense}(H_1, H_2)$ and define $U \in L(H_1, H_2)$ by $U((x, y)) := (x, -y)$. Then

$$U(gr\, D^\star) = (gr\, D)^\perp.$$

4 Consider an operator $D \in L_{dense}(L^2(\mathbb{R}^m, \mathbb{C}^{N_1}), L^2(\mathbb{R}^m, \mathbb{C}^{N_2}))$ given by

$$Ds(x) := \sum_{|\alpha| \leq k} D_\alpha \frac{\partial^{|\alpha|} s}{\partial x^\alpha}(x), \quad s \in C^k_c(\mathbb{R}^m, \mathbb{C}^{N_1}),$$

where $D_\alpha \in L(\mathbb{C}^{N_1}, \mathbb{C}^{N_2})$; D is called a differential operator of order k with constant coefficients. Show that D is closable.

5 Let $d : \lambda_c(M) \to \lambda_c(M)$ be the de Rham operator on an oriented Riemannian manifold. Show that d is closable in $\lambda_{(2)}(M)$.