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Introduction to Spectral Theory of Differential Operators, SS 2011

Exercises 9, week of June 13 to June 27, 2011

1 Show that for smooth sections s of the product bundle $E = M \times B$, M smooth manifold, B a Banach space, the description

$$\Delta_X^E s(p) := ds(p)[X], \quad x \in \tau^1(M),\ p \in M,$$

defines a flat connection.

2 Show that for any smooth manifold M and any smooth vector bundle $E \to M$ there is a canonical isomorphism

$$\lambda^k(M,E) \simeq C^\infty(M, \Lambda^k T^\star M \otimes E), \quad k \in \{0,\dots,m\}.$$

3 Define for $\omega \in \lambda^k(M),\ s \in C^\infty(M,E),\ X_i \in \tau^1(M),\ i = 1,\dots,k,$

$$(\omega \wedge s)[X_1,\dots,X_k] := \omega[X_1,\dots,X_k]s \in C^\infty(M,E).$$

Show that $\omega \wedge s \in \lambda^k(M,E)$ and that for $\omega_i \in \lambda^{k_i}(M),\ i = 1,2,$

$$\omega_1 \wedge (\omega_2 \wedge s) = (\omega_1 \wedge \omega_2) \wedge s. \tag{1}$$

Define a wedge product

$$\wedge : \lambda^k(M) \times \lambda^l(M,E) \to \lambda^{k+l}(M,E)$$

and show the analog of (1).

4 Show that the Koszul formula defines the unique torsion-free metric affine covariant derivative on a Riemannian manifold (M, g^{TM}).