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INTRODUCTION TO SPECTRAL THEORY OF DIFFERENTIAL OPERATORS, SS 2011

EXERCISES 9, week of June 13 to June 27, 2011

More exercises

5 Show that for any connection Δ^E on E and all $X, Y \in \tau^1(M)$

$$R^E_{X,Y} = \Delta^E_X \Delta^E_Y - \Delta^E_Y \Delta^E_X - \Delta^E_{[X,Y]}.$$

6 Consider $\eta \in \lambda(M, OM)$, with OM the orientation bundle. In coordinates (U_x, x) with $OM|U_x = OU_x$ trivial we may write, in a suitable frame σ_x,

$$\eta^{\sigma_x} = f^x dx \text{ with } f^x \in C^\infty(U_x).$$

Show that η has a well defined integral based on

$$\int_{U_x} \eta^{\sigma_x} := \int_{x(U_x)} x^{-1,\star}(f^x dx).$$

7 Show that $C_c^\infty(\mathbb{R})$ is dense in the algebra $\mathcal{A}$ with respect to all norms $\|\cdot\|_{n+1},\ n \in \mathbb{N}$.

8 If $F \in C_c^\infty(\mathbb{C})$ and $|F(x+iy)| \leq C_f y^2$ then

$$\int_{\mathbb{C}} \frac{\partial F}{\partial \overline{z}}(z) R_A(z) dx dy = 0,$$

for any self-adjoint operator A.

9 As a consequence of 7), show that the definition

$$f(A) = \frac{1}{\pi} \int_{\mathbb{C}} \frac{\partial \tilde{f}}{\partial \overline{z}}(z) R_A(z) dx dy$$

is independent of the choices of $n \in \mathbb{N}$ and $\tau \in C_c^\infty(\mathbb{R})$ with $\tau(x) = 1$ in a nbhd of 0.