Research Training Group 1128

Analysis, Numerics, and Optimization
of Multiphase Problems

Analysis of thin films

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Abstract

Understanding thin films in practical applications (1) requires some mathematical insight. Modelling of certain physical situations, described by a certain scaling approach, yields a so-called lubrication model (2,3). This poster shows what kind of mathematical structures might be investigated. Here, those will be self-similar rupture solutions.

Setting

We seek a velocity distribution $u(x,t)$, $\mathbf{x} \in \Omega$, and a domain $\Omega(t)$, that deforms with time t. The domain considered here is defined as $\Omega = \{\mathbf{x} \in \mathbb{R}^3 : -h \le z \le h(t,x,y)\}$. The physical setting is

- surface tension and van der Waals-interaction contribute,
- height scale is much smaller than lateral scale.

Governing equations

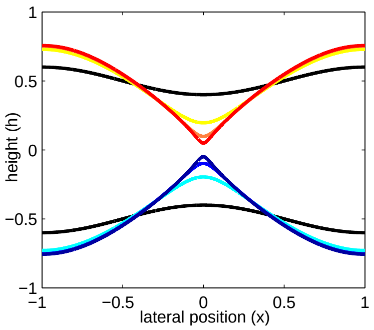
$$\begin{aligned} \rho(\partial_t u + u \cdot \nabla u) &= \mu \Delta u + \nabla(p - \phi(h)) && \text{in } \Omega \\ \nabla \cdot u &= 0 && \text{in } \Omega \\ (n,t) \cdot \pi(n) &= (\sigma, 0) && \text{on } \partial\Omega \\ \partial_t h = (n \cdot u)(n \cdot e_z)^{-1}&|_{\partial\Omega} \end{aligned}$$

Numerics

After scaling the governing equations and working out the leading order approximation in $\epsilon =$ *height scale/lateral scale*, one ends up with the *lubrication model*. For line-rupture geometry, as described above, this boils down to the two equations

$$\begin{aligned} h_t &= -(hu)_x, \\ u_t + uu_x &= \frac{4}{h}(hu_x)_x + (3Sh_{xxx} - \phi(h))_x. \end{aligned}$$

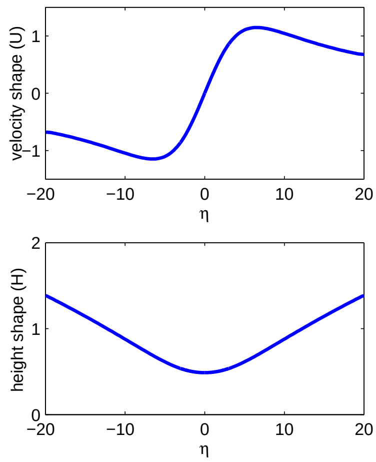
As a numeric solution of this equation shows, an initially sinusoidal perturbation leads to a film rupture after a finite time.



A simulation of the lubrication model reveals a self-similar form

$$\begin{aligned} h(x,t) &= \tau^{\alpha} H(\eta), \\ u(x,t) &= \tau^{\gamma} U(\eta), \end{aligned}$$

with $\eta = (x - x_*)\tau^{-\beta}$, here with $\alpha = 1/3$, $\beta = 1/2$ and $\gamma = -1/2$. Solving an ODE for the self-similar shape functions H and U, gives



The knowledge obtained by this analysis might be used in the case where one would like to have, prevent or just understand a rupture. A later project is to investigate properties of lubrication model with slip-boundary conditions, *i.e.* $(u - bu_z)_{z=0} = 0$.

Personal profile

From 1997–2004 I studied physics at the Humboldt-Universität with specialization on the *theory of elementary particles*. My diploma thesis (d) was done in the group of Prof. M. Müller-Preussker in the field of lattice gauge theories. The work supported the validity of a semiclassical model for the deconfinement-confinement phase transition. This research was closely related to the DFG research group 465 *lattice–hadron–phenomenology*.

Supported by DFG graduate college *The Standard Model of Particle Physics - structure, precision tests and extensions*, I continued this work until March 2005. Since then, I have a position in the DFG training group 1128, doing research on the analysis of thin films.

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