4. Conclusion

The aforementioned Wiren's model can be used for the developed probability model of fatigue crack propagation which is based on experimental results. The described improvements of the reliability model of fatigue crack growth propagation allow for the assessment of fatigue crack length as a function of ΔK, and the derivation of the increasing reliability $\mathsf{R}(\Delta K)$ at lower loading conditions ΔF_{0k}. The reliability tends towards the final value of reliability outside experimental observations.

It is proved that improvements are valid under the zone of lower loading conditions of a specimen with crack, because the microstructural and mechanical properties are preserved. But inside this zone of loading conditions, the reliability $\mathsf{R}(\Delta K_0)$ that the crack length will be shorter than in case of experimental testing is substantially increased.

References

1 Legat, J.; Gubeljak, N.; Primec, Č.: Probabilistic model of fatigue crack growth in HSLA steel weldments. GAMM Jahrestagung '92, Leipzig 1992.
2 Legat, J.; Gubeljak, N.; Primec, Č.: The use of statistical methods in fatigue behaviour assessment of HSLA steel weldments, Vol. 2. Budapest 1990, pp. 507–512.
3 Paris, P. C.; Erdoğan, F.: Trans. ASME, J. Basic Eng. **85**, 4, 528ff.
4 Tang, J.; Spencer, Jr., B. F.: Eng. Frac. Mech. **34** (1989) 2, 419–433.
5 Ghonem, H.; Dore, S.: Eng. Frac. Mech. **27** (1987) 1, 1–25.

Address: Doz. Dr.-Ing. Janko Legat, Mr.-Ing. Nenad Gubeljak, Mr.-Ing. Čedomir Primec, Faculty of Technical Sciences, POB 224, Smetanova 17, SLO-62000 Maribor, Slovenia

5.2 STOCHASTISCHE MODELLE II

ZAMM · Z. angew. Math. Mech. **74** (1994) 6, T 587–T 589 Akademie Verlag

Römisch, W.; Schultz, R.

Quantitative Stability of Two-Stage Stochastic Programs

MSC (1991): 90C15, 90C31

We study quantitative stability properties of solutions to certain two-stage stochastic programs which are given by

$$\mathrm{P}(\mu): \quad \min\{g(x) + Q_\mu(Ax) : x \in C\},$$

$$\text{where} \quad Q_\mu(\chi) := \int_{\mathbb{R}^s} Q(z-\chi)\,\mu(\mathrm{d}z) \quad \text{and} \quad Q(t) := \min\{q^\top y : Wy = t,\, y \geqq 0\}.$$

We assume that $g:\mathbb{R}^m \to \mathbb{R}$ is a convex function, $C \subset \mathbb{R}^m$ is a non-empty closed convex set, $q \in \mathbb{R}^{\bar m}$ and A, W are matrices of proper dimensions, and μ is a (Borel) probability measure on $\mathbb{R}^s$. Throughout, we have the following general assumptions:

(A1) $W(\mathbb{R}^{\bar m}_+) = \mathbb{R}^s$ (complete recourse),

(A2) $\{u \in \mathbb{R}^s : W^\top u \leqq q\} \neq \emptyset$ (dual feasibility),

(A3) $\mu \in \mathcal{M}_1(\mathbb{R}^s) := \{\nu : \nu \text{ probability measure}, \int_{\mathbb{R}^s} \|z\|\,\nu(\mathrm{d}z) < \infty\}$.

(A1)–(A3) ensure Q_μ to be a real-valued convex function on $\mathbb{R}^s$ ([1]). Assigning to each $\nu \in \mathcal{M}_1(\mathbb{R}^s)$ the set $\psi(\nu)$ of solutions to $\mathrm{P}(\nu)$ we study quantitative continuity properties of the multifunction ψ at some original measure μ. For motivation, background and further references on stability in stochastic programming see [4]. Since the uniqueness of solutions in $\mathrm{P}(\mu)$ is rather exceptional (see e.g. the discussion in [5]), standard techniques of parametric optimization fail (e.g. those using traditional sufficient second order conditions). Our main result is the following

Theorem 1: *Suppose* (A1)–(A3) *and that $\psi(\mu)$ is nonempty and bounded. Let $U \subset \mathbb{R}^m$ be an open convex bounded set containing $\psi(\mu)$.*

a) *Let the following growth condition be fulfilled:*

(GC) *There exists a constant* $\alpha > 0$ *such that*

$$g(x) + Q_\mu(Ax) \geqq \inf_{x \in C} \{g(x) + Q_\mu(Ax)\} + \alpha d(x, \psi(\mu))^2 \quad \textit{holds for all} \quad x \in C \cap U .$$

Then there exists a constant $L > 0$ *such that*

$$\sup_{x \in \psi(\nu)} d(x, \psi(\mu)) \leqq L \sum_{j=1}^{l} \sup_{t \in A(U)} |F_{\mu \circ (-B_j)}(-B_j^{-1} t) - F_{\nu \circ (-B_j)}(-B_j^{-1} t)|$$

whenever $\nu \in \mathscr{M}_1(\mathbb{R}^s)$ *has the property that the right-hand side is sufficiently small.*

b) *Let g be convex quadratic and C be convex polyhedral. Assume that* Q_μ *is strongly convex on* $A(U)$, *i.e., there exists a* $\varkappa > 0$ *such that for all* $\chi, \tilde{\chi} \in A(U)$, $\lambda \in [0, 1]$,

$$Q_\mu(\lambda\chi + (1 - \lambda)\tilde{\chi}) \leqq \lambda Q_\mu(\chi) + (1 - \lambda) Q_\mu(\tilde{\chi}) - \varkappa\lambda(1 - \lambda) \|\chi - \tilde{\chi}\|^2 .$$

Then there exists a constant $L > 0$ *such that*

$$d_{\mathrm{H}}(\psi(\mu), \psi(\nu)) \leqq L \sum_{j=1}^{l} \sup_{t \in A(U)} |F_{\mu \circ (-B_j)}(-B_j^{-1} t) - F_{\nu \circ (-B_j)}(-B_j^{-1} t)|$$

whenever $\nu \in \mathscr{M}_1(\mathbb{R}^s)$ *has the property that the right-hand side is sufficiently small.*

(Here d_{H} *denotes the Hausdorff distance on subsets of* $\mathbb{R}^m$, B_j, $j = 1, \ldots, l$, *denote the optimal basis submatrices of W, and* $F_{\nu \circ (-B_j)}$ *denotes the distribution of the linearly transformed measures* $\nu \circ (-B_j)$.*)*

The Proof of a) is a consequence of the results in [7] together with Proposition 2.3 and Corollary 2.11 in [5]; b) is proved in [5].

The assumptions of b) imply (GC) (Proposition 2.5 in [5]), but (GC) does not imply the assertion of b) (Example 2.6 in [5]). Condition (GC) and both estimates are lost for general convex $C \subset \mathbb{R}^m$ and g even if Q_μ is strongly convex (Example 2.7 in [5]). Q_μ is strongly convex on some open convex set $V \subset \mathbb{R}^s$ if (i) (A1), (A3) are fulfilled, (ii) there exists a $\bar{u} \in \mathbb{R}^s$ such that $W^{\mathrm{T}}\bar{u} < q$ componentwise, and (iii) μ has a density Θ_μ and $\Theta_\mu(t) \geqq r > 0$ for all t in a neighbourhood of V (Theorem 2.2 in [6]).

Finally we show how Theorem 1 can be used to derive asymptotic properties of optimal solutions when estimating the underlying distribution by empirical ones. We obtain a large-deviation estimate for solutions without imposing the unique solvability of $\mathrm{P}(\mu)$. Let $\xi_1, \xi_2, \ldots, \xi_n, \ldots$ be independent $\mathbb{R}^s$-valued random on some probability space $(\Omega, \mathscr{A}, \mathsf{P})$ having common distribution μ, and let us consider the empirical measures

$$\mu_n(\omega) := \frac{1}{n} \sum_{i=1}^{n} \delta_{\xi_i}(\omega) , \qquad \omega \in \Omega ; \qquad n \in \mathbb{N} .$$

Corollary 2: *Under the assumptions of Theorem* 1 b) *there exists a constant* $\varepsilon_0 > 0$ *such that for all* $\varepsilon \in (0, \varepsilon_0]$ *holds*

$$\limsup_{n \to \infty} \frac{1}{n} \log \mathsf{P}(\{\omega : d_{\mathrm{H}}(\psi(\mu), \psi(\mu_n(\omega))) \geqq \varepsilon\}) \leqq -2 \left(\frac{\varepsilon}{Ll}\right)^2 ,$$

where L and l denote the Lipschitz modulus and the number of basis matrices, respectively, arising in Theorem 1.

Proof: Well-known measurability arguments imply that the Hausdorff distance of $d_{\mathrm{H}}(\psi(\mu), \psi(\mu_n(\cdot)))$ is an $\mathscr{A}$-measurable random variable with values in $\mathbb{R} \cup \{+\infty\}$. We introduce the notation

$$\Phi_n(\omega) := \max_{j=1,\ldots,l} \eta_{n,j}(\omega), \eta_{n,j}(\omega) := \sup_{t \in \mathbb{R}^s} |F_{\mu \circ (-B_j)}(t) - F_{\mu_n(\omega) \circ (-B_j)}(t)| , \qquad \omega \in \Omega ; \qquad n \in \mathbb{N} .$$

Next we select $\varepsilon_0 > 0$ such that $Ll\Phi_n(\omega) < \varepsilon_0$ implies $d_{\mathrm{H}}(\psi(\mu), \psi(\mu_n(\omega))) \leqq Ll\Phi_n(\omega)$ (Theorem 1). Then we have for each $\varepsilon \in (0, \varepsilon_0]$ and all $n \in \mathbb{N}$,

$$\mathsf{P}(\{\omega : d_{\mathrm{H}}(\psi(\mu), \psi(\mu_n(\omega))) \geqq \varepsilon\}) \leqq \sum_{j=1}^{l} \mathsf{P}\left(\left\{\omega : \eta_{n,j}(\omega) \geqq \frac{\varepsilon}{Ll}\right\}\right) .$$

The multivariate Dvoretzky-Kiefer-Wolfowitz inequality ([3]) then implies that, for each $\delta > 0$ and $j \in \{1, \ldots, l\}$, there exist constants $C_j > 0$ such that

$$\mathsf{P}(\{\omega : \eta_{n,j}(\omega) \geqq \varepsilon/Ll\}) \leqq C_j \exp(-(2 - \delta) n(\varepsilon/Ll)^2) , \qquad n \in \mathbb{N} .$$

Hence, we obtain for each $n \in \mathbb{N}$,

$$\log \mathsf{P}(\{\omega : d_H(\psi(\mu), \psi_n(\omega)) \geqq \varepsilon\}) \leqq \log\left(\sum_{j=1}^{l} C_j\right) - (2 - \delta)\, n \left(\frac{\varepsilon}{Ll}\right)^2 .$$

Since $\delta > 0$ was arbitrary, the proof is complete. □

By making use of Theorem 1a) the analogue of Corollary 2 for the one-sided Hausdorff distance is valid, too. Large deviation estimates were also obtained in [2] and [8] for more general stochastic programs. However, the results of [2] require the unique solvability of the original program and Theorem 2 of [8] only applies to measures μ with bounded support.

References

1 KALL, P.: Stochastic linear programming. Springer-Verlag, Berlin 1976.
2 KANIOVSKY, Y. M.; KING, A. J.; WETS, R. J.-B.: Probabilistic bounds (via large deviations) for the solutions of stochastic programming problems. IBM Research Division, Research Report, RC 18752, 1993.
3 KIEFER, J.: On large deviations of the empiric D.F. of vector chance variables and a law of the iterated logarithm. Pacific J. Math. **11** (1961), 649–660.
4 RÖMISCH, W.; SCHULTZ, R.: Stability analysis for stochastic programs. Ann. of Operations Res. **30** (1991), 241–266.
5 RÖMISCH, W.; SCHULTZ, R.: Lipschitz stability for stochastic programs with complete recourse. Schwerpunktprogramm der Deutschen Forschungsgemeinschaft „Anwendungsbezogene Optimierung und Steuerung", Report No. 408, 1992.
6 SCHULTZ, R.: Strong convexity in stochastic programs with complete recourse. J. Computat. Appl. Math. **56** (1994) (to appear).
7 SHAPIRO, A.: Quantitative stability in stochastic programming. Math. Program. (to appear).
8 VOGEL, S.: Stability results for stochastic programming problems. Optimization **19** (1988), 269–288.

Address: Prof. Dr. sc. WERNER RÖMISCH, Dr. RÜDIGER SCHULTZ, Humboldt-Universität Berlin, Fachbereich Mathematik, Unter den Linden 6, D-10099 Berlin, Germany

5.3 OPTIMALE STEUERUNG

ZAMM · Z. angew. Math. Mech. **74** (1994) 6, T 589–T 591 Akademie Verlag

CALLIES, R.

Antriebsoptimierung bei Raumsonden

MSC (1991): 93C15, 49J15, 70Q05, 70N05

An dem realistischen Modell einer Raumsonde zur Venus werden Möglichkeiten zur kombinierten Optimierung von Flugbahn und Raumfahrzeug aufgezeigt. Der Schwerpunkt der Geräteoptimierung liegt auf dem Antriebssystem.

Das Modell (1)

In einem heliozentrischen Kugelkoordinatensystem wird die Bewegung des Massenschwerpunktes einer Raumsonde mathematisch durch das folgende hoch-nichtlineare Differentialgleichungssystem dimensionsloser Größen der Form $\dot{x} = f(x, u, t)$ (x Zustandsvektor, u Steuerungsvektor; t unabhängige Variable, hier die Zeit) beschrieben:

$$\dot{r} = v_r, \qquad \dot{\varphi} = \frac{v_\varphi}{2\pi r \sin\vartheta}, \qquad \dot{\vartheta} = \frac{v_\vartheta}{r}, \qquad \dot{m} = -\beta_1 + \beta_2 ,$$

$$\dot{v}_r = \frac{c_1\beta_1 + c_2\beta_2}{m} \sin\Psi \sin\Xi + \frac{v_\varphi^2 + v_\vartheta^2}{r} - \frac{1}{r^2} + \frac{\xi}{mr^2}$$
$$- \sum_{j=1}^{n} \frac{m_j}{s_j^3} \left[r - r_j \cos\vartheta \cos\vartheta_j - r_j \sin\vartheta \sin\vartheta_j \cos(2\pi(\varphi - \varphi_j))\right],$$

$$\dot{v}_\varphi = \frac{c_1\beta_1 + c_2\beta_2}{m} \cos\Psi \cos\Xi - \frac{v_\varphi v_\vartheta}{r} - \frac{v_\varphi v_\vartheta}{r} \cdot \cot\vartheta - \sum_{j=1}^{n} \frac{m_j}{s_j^3} \left[r_j \sin\vartheta_j \sin(2\pi(\varphi - \varphi_j))\right],$$