Surgery along Chekanov's knots

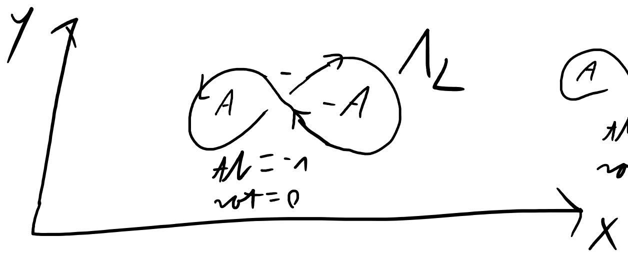
OneNote Link: https://1drv.ms/u/s!ApRQR77A3CIHijiwhmG14XyY51Mq

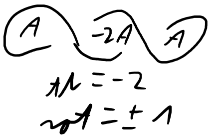
1. Lagrangian projection:

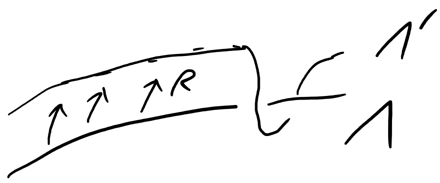
$$\Lambda \subset (\mathbb{R}^3, \xi_{st} = \ker(x\,dy + dz)) \subset (S^3, \xi_{st}) \text{ a Leg knot}$$

The Lagrangian projection:

$$(x, y, z) \longmapsto (x, y)$$



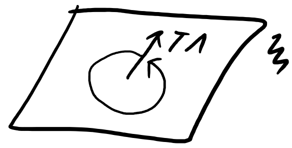




2. Classical invariants:

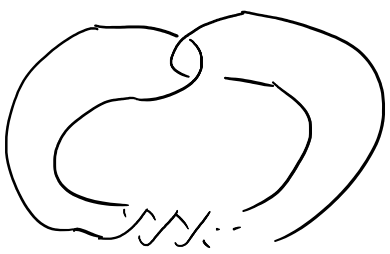
$$tb(\Lambda) := \text{writhe}(\Lambda_L)$$

$$rot(\Lambda) := rot(\Lambda_L)$$

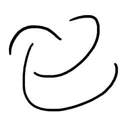


3. Chekanov's knots:

Twist knots:

$T_n :=$ 

$T_0 = O$

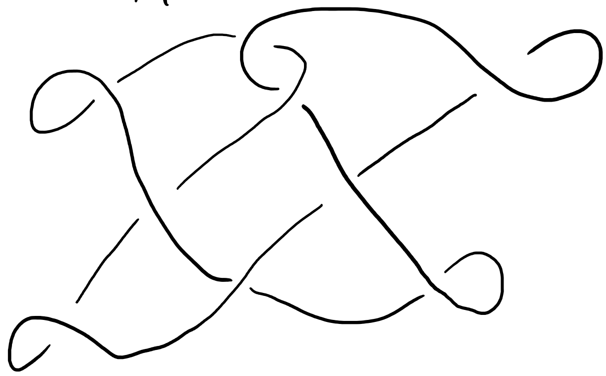
$T_1 =$ 

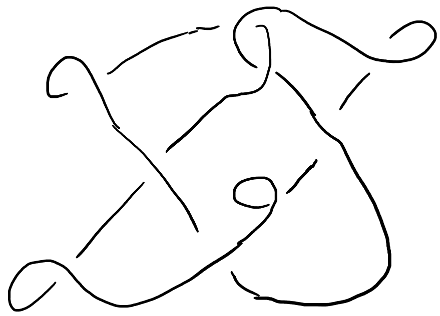
$T_2 =$ Leg 8

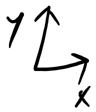
T_3

Legendr. realizations of T_3:

Λ^a Λ^b







$$tb(\Lambda^a) = 1 = tb(\Lambda^b)$$

$$rot(\Lambda^a) = 0 = rot(\Lambda^b)$$

<u>THM 1</u> (CHEKANOV)

(1) $\exists$ comb. description of $LHA(\Lambda)$

(2) $LHA(\Lambda^a) \not\cong LHA(\Lambda^b)$

(3) $\Lambda^a \not\cong \Lambda^b$

<u>THM 2</u> (BEE, Ng) $\qquad (S^3, \xi_{st}) = (\partial D^4, \omega_{st})$

$\Lambda(-1) = (D^4 \cup$ Weinstein 2-handle attached along Λ $, \omega)$

$\partial\Lambda(-1) = \partial($ " $, \xi)$

(1) $\Lambda^a(-1) \overset{sympl}{\not\cong} \Lambda^b(-1)$

(2) $\partial\Lambda^a(-1) \overset{cont}{\not\cong} \partial\Lambda^b(-1)$

<u>Rmk</u>: * Proof of T. 2. uses parts of T. 1.

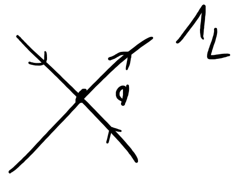
Idea of T.1 (1):

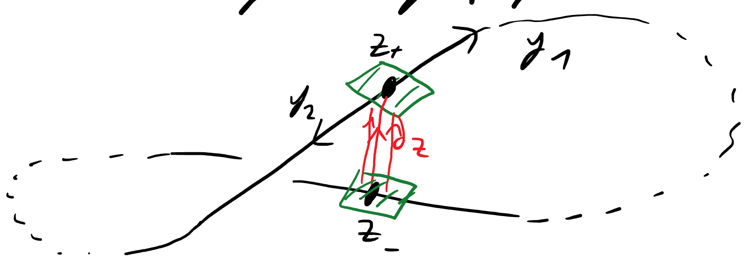
* GENERATORS of LHA(Λ): Reeb chords

$$\updownarrow 1:1$$

crossings in Lag proj of Λ

$R_{\alpha_{st}} = \partial_z$





* GRADINGS = rel. CONLEY-ZEHNDER INDEX

$$\mathrm{rot}(\gamma_{i,L}) = \frac{N_i}{2} + \frac{1}{4}$$

$$\Rightarrow N_1 - N_2 = \pm \mathrm{rot}(\Lambda)$$

$$\text{grading of } (a) = [N_1] = [N_2] \in \mathbb{Z}/_{\mathrm{rot}(\Lambda)}\mathbb{Z}$$

Ex.

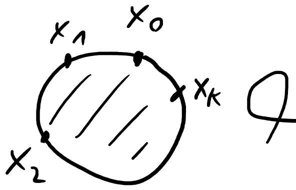
$|a| = 1 \in \mathbb{Z}$

$\partial a = 1 + 1 = 0 \in \mathbb{Z}_2$

$\mathbb{K} = \mathbb{Z}_2$

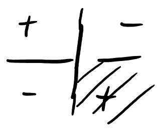
* BOUNDARY OP:

$W_k^+(a) := \Big\{ f:$

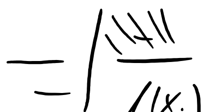


$\longrightarrow \mathbb{R}^2$ s.t.

$f(\partial D^2) \subset \Lambda_L$ $\quad f(x_i) = $ crossings, $f(x_0) = a$



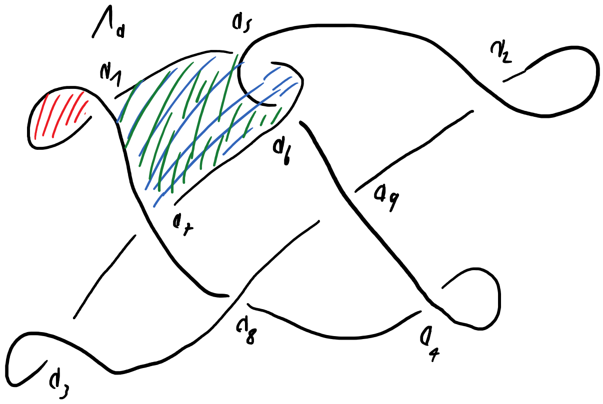
$a = f(x_0)$



$f(x_i)$

$\Big\} /\text{rep.}$

$$\partial_k(a_j) = \sum_{f \in W^+_{k+1}(a_j)} f(x_1)\cdots f(x_k)$$



Λ_b

$|b_i| = 1 \quad i = 1,\dots,4$

$|b_i| = 0 \quad i = 5,\dots,9$

$|a_i| = 1 \quad ; i = 1,\dots,4$

$|a_5| = 2$

$|a_6| = -2$

$|a_i| = 0 \quad ; i = 7,8,9$

$d a_1 = 1 + a_7 + a_7 a_6 a_5$

$d a_2 = 1 + a_9 + a_5 a_6 a_9$

$d a_3 = 1 + a_8 a_7$

$d a_4 = 1 + a_8 a_9$

$d a_i = 0 \quad i = 5,\dots,9$

Proof of T.2.(1):

Idea:

$$\begin{tikzcd} CH(\Lambda(-1)) \arrow[rr, "deg=0"] & & CH(D^4) \arrow[dl, "deg = \pm 1 \ ?"] \\ & LH^{cyc}(\Lambda) \arrow[ul, "deg=0"] & \end{tikzcd}$$

$$0 = CH_{-3}(D^4) \to LH^{cyc}_{deg=-2}(\Lambda) \xrightarrow{\cong} CH_{deg=-2}(\Lambda(-1)) \longrightarrow CH_{deg=-2}(D^4) = 0$$

$\Lambda = \Lambda^b \qquad 0$

$\Lambda = \Lambda^a \qquad$ to show $a_6 \neq 0$

$a_6 \neq 0 \in LH^{cyc}_{deg=-2}(\Lambda^a)$

$\ulcorner \quad A = \mathbb{Z}_2 \langle a_6 \rangle \quad \urcorner$

Consider $\phi : LHA(\Lambda_a) \longrightarrow A$ def by

$$a_6 \longmapsto a_6$$
$$a_7, a_9 \longmapsto 1$$
$$a_8 \longmapsto -1$$
$$a_i \longmapsto 0 \quad \text{for } i < 6$$

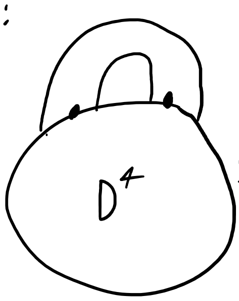
induces a map $LH^{cyc}(\Lambda_a) \longrightarrow A^{cyc}$

which is non-trivial on homology. $\blacksquare$

<u>T.2.(2):</u> via augmentations: ϕ is an augmentation

* collection of all augmentations is an invariant of $\partial\Lambda(-1)$
* $\Lambda^a(-1)$ has an augmentation of degree $= -2$
* $\Lambda^b(-1)$ has <u>NOT</u>. $\blacksquare$

<u>Remark:</u>



$S^3 = \partial D^4$

$S^3 \,|\, VK \qquad V\, S^1 \times D^2$

$$\partial\Lambda^a(+1) \overset{cont}{\cong} \partial\Lambda^b(+1)$$

<u>ETNYRE</u>