Thm: (Bourgeois & Oancea)

(W^{2n}, ω) cpt. sympl. mfd. with ct. type bdary $M = \partial W$

Under some transversality assumptions there ex. a long exact sequence of pos. sympl. homology and linearized contact homology of M

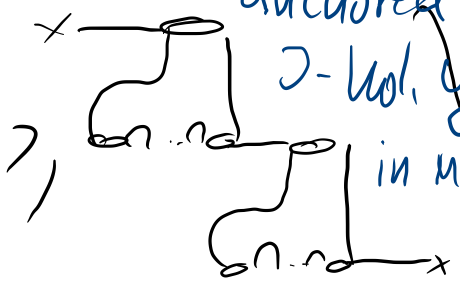
$$\begin{array}{ccc} CH_*(M) & \xrightarrow{D} & CH_{*-2}(M) \\ & \nwarrow{\scriptstyle [+(n-3)]} \quad \swarrow & \\ & SH^+_{*-1(n-3)}(W) & \end{array}$$

Further D is determined entirely by the count of rat. curves in the symplectization of M and rigid hol. planes in the completion $\hat{W}$.

Pf: 1) Filtrations $\Rightarrow$ spectral sequence $\Rightarrow$ long exact sequence

2) Iso of filtered complexes

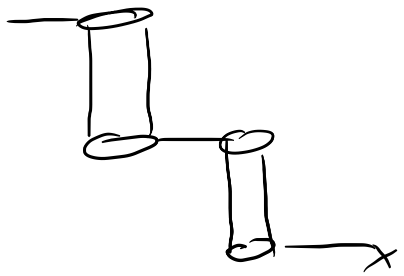
$$BC_*(\lambda) := \Big(\bigoplus_{\gamma \in P(\lambda)} \langle \hat{\gamma} \rangle \oplus \langle \check{\gamma} \rangle, \Big)$$



Cascades of anchored J-hol. cyl. in $M \times \mathbb{R}$

„S^1-param. lin CH complex"

For $H \in \mathcal{H}$ with $H|_{collar} = \alpha e^r + \beta$
$\alpha \notin \operatorname{spec}(\lambda)$

$$C_*^+(H) = \Big(\bigoplus_{\substack{\gamma \in \mathcal{P}^{\le \alpha}(\lambda) \\ \cong \mathcal{P}(H)/S^1}} \langle \hat\gamma \rangle \oplus \langle \check\gamma \rangle ,$$  $$\Big)$$

Floer cylin. with cascades

"Floer complex" over Λ_ω = completion of $\mathbb{Q}[H_2(W)]$ with

$$|\hat\gamma e^A| = \mu_{CZ}(\gamma) + \mu_{Morse}(\tilde\gamma) - 2c_1(A)$$

here $\tilde\gamma$ eighter $\check\gamma$ or $\hat\gamma$.

$$|\tilde\gamma e^A| = \underbrace{\mu_{CZ}(\gamma) - 2c_1(A)}_{|\tilde\gamma e^A|_b} + \underbrace{\mu_{Morse}(\tilde\gamma)}_{|\hat\gamma e^A|_f}$$

obtain a filtration via $|\cdot|_b$ i.e.

$$\mathcal{F}_\ell BC_*(\lambda) := \bigoplus_{|\tilde\gamma e^A|_b \le \ell} \mathbb{Q}\langle e^A \hat\gamma \rangle \oplus \mathbb{Q}\langle e^A \check\gamma \rangle$$

incr. Filtration? $\langle \partial \tilde\gamma_-, e^A \tilde\gamma_+ \rangle \neq 0$

wts: $|\tilde\gamma_-|_b \geq |e^A \tilde\gamma_+|_b$

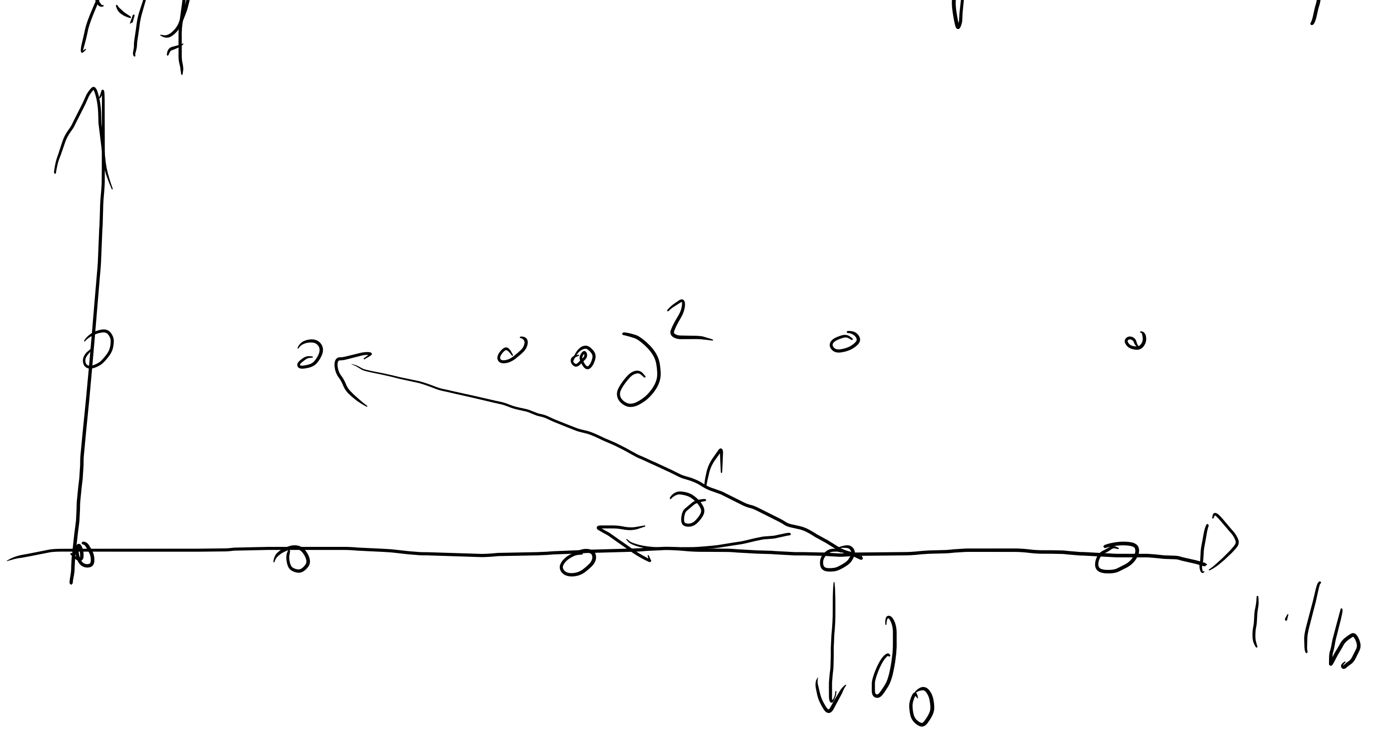
know $\dim \mathcal{M}^A_{anc}(\tilde\gamma_-, \tilde\gamma_+) = 0$

$\Leftrightarrow$ $|\tilde\gamma_-| - |e^A \hat\gamma_+| - 1 = 0$

$|\tilde\gamma_-|_b + \mu_{Morse}(\tilde\gamma_-) - |e^A \tilde\gamma_+| - Morse(\tilde\gamma_+) = 1$

$\Leftrightarrow$ $|\tilde\gamma_-|_b - |e^A \tilde\gamma_+| = 1 - \mu_{Morse}(\tilde\gamma_-) + \mu_{Morse}(\tilde\gamma_+) \geq 0$ $\square$

split $\partial = \partial^0 + \partial^1 + \partial^2$ according to the diff. of the base index for the associated spectral seq.



similar have filtration for $C_*(H)$

gen. theorem if (C_*, ∂, F) is a filtered complex s.t. $\forall n\ \exists t, s$ with

$$0 \subset F_s C_n \subset F_{s+1} C_n \subset \dots \subset F_t C_n = C_n$$

(i.e. Filtration is bounded)

there ex. a spectral seq. (E^r, d^r) s.t.

$$E^1 = H(gr\, C)$$

$$E^\infty = gr\, HC$$

more prec.

$$E^1_{pq} = H_{p+q}(F_pC / F_{p-1}C)$$

$$E^\infty_{pq} = F_p H_{p+q} / F_{p-1} H_{p+q}$$

(here $s = n-1$ $t = n$)

have $0 = F_j H_n \qquad j < n-1$

$$F_{n-1} H_n \subset F_n H_n = H_n$$

$$\Rightarrow \quad 0 \to E^3_{n-1,1} \to H_n \to E^3_{n,0} \to 0$$

and since $E^\infty = E^3$

$$0 \to E^3_{n,0} \to E^2_{n,0} \xrightarrow{\partial^2} E^2_{n-2,1} \to E^3_{n-2,1} \to 0$$

put together into a LES

$$\begin{tikzcd}
E^2_{k-1,1} \arrow[rr, dashed] \arrow[dr] & & H_k \arrow[rr, dashed] \arrow[dr] & & E^2_{n,0} \arrow[r, "\partial^2"] & E^2_{n-2,0} \arrow[rr, dashed] \arrow[dr] & & {} \\
& E^3_{n-1,1} \arrow[ur] & & E^3_{n,0} \arrow[ur] & & & E^3_{n-2,1} \arrow[ur] &
\end{tikzcd}$$

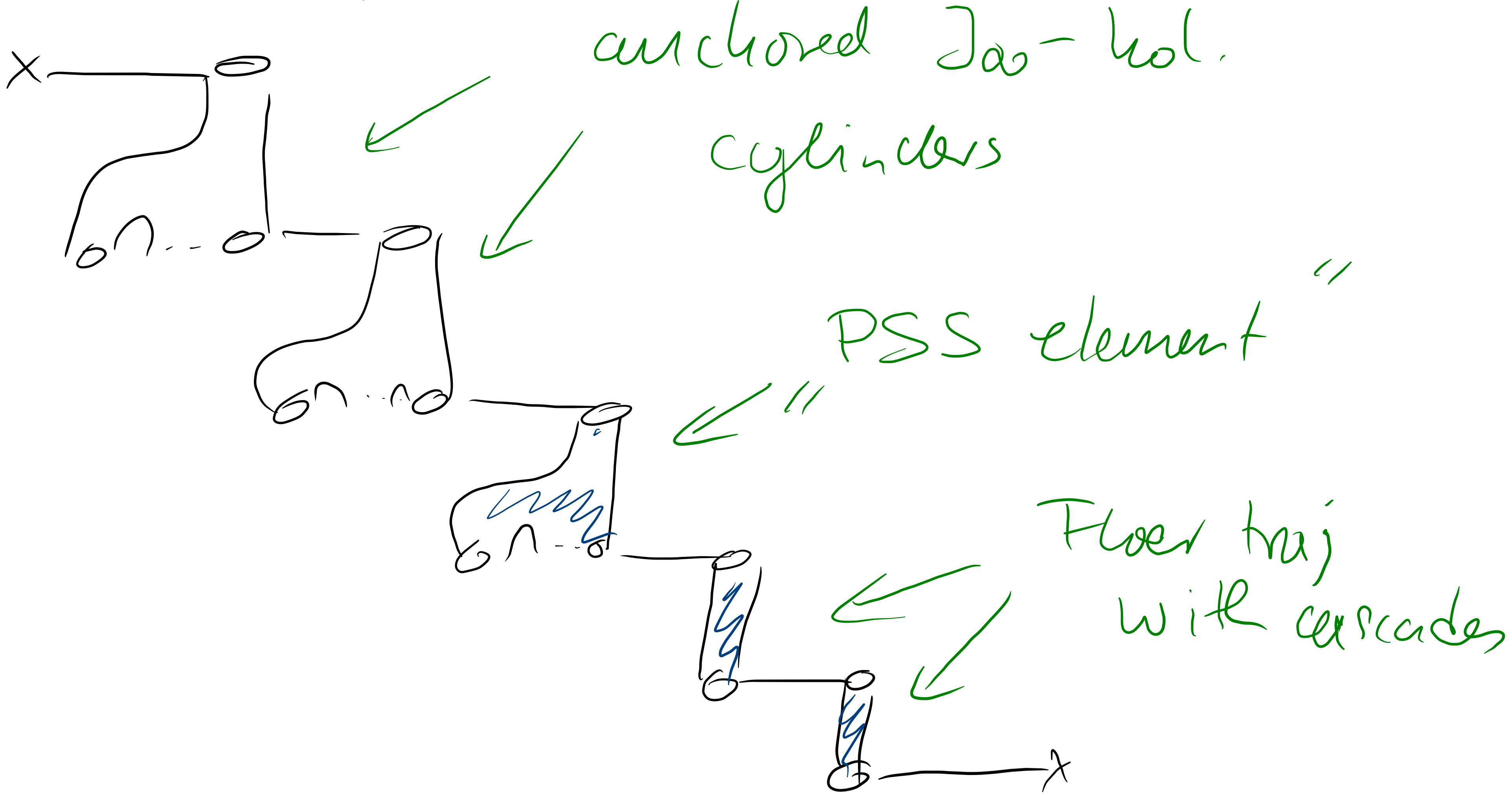
get two LES

$$\begin{tikzcd} \to H_*^{\leq\alpha}(BC) \arrow[r] \arrow[d] & E^2(\lambda) \arrow[r] \arrow[d] & E^2(\lambda) \arrow[r] \arrow[d] & H_*^{\leq\alpha}(BC) \arrow[d] \\ HF_*(H) \arrow[r] & E^2(H) \arrow[r] & E^2(H) \arrow[r] & HF_*(H) \end{tikzcd}$$

1) construct vertical iso's
2) direct limit argument
3) identify $E^2(\lambda)$ with lin. contact homology.

$$\phi : BC_*^{\leq \alpha}(\lambda) \longrightarrow C_*(H)$$

via count of



„PSS solution"

$u : \mathbb{R} \times S^1 \setminus \{z_1, \dots, z_n\} \to M \times \mathbb{R}$

$$\partial_s u + J_\infty \left(\partial_t u - \rho X_{H_\infty} \right) = 0$$

where $\rho : \mathbb{R} \to \mathbb{R}$ s.t.

$$\rho(s) = \begin{cases} 1 & s \gg 0 \\ 0 & s \ll 0 \end{cases}$$

+ asymptotic cond. as before