Course description and syllabus

General information

Instructor: Prof. Chris Wendl
HU Institute for Mathematics (Rudower Chaussee 25), Room 1.301
wendl@math.hu-berlin.de
Office hour: Wednesdays 15:00–16:00

Course webpage: http://www.mathematik.hu-berlin.de/~wendl/Winter2017/Topologie2/

Lectures: Wednesdays 13:00–15:00 in 1.013 (Rudower Chaussee 25)
Fridays 9:00–11:00 in 1.013 (Rudower Chaussee 25)

Problem classes: Wednesdays 11:00–13:00 in 1.013 (Rudower Chaussee 25)

Language: The course will be taught in English.

Prerequisites: The contents of the HU's course *Topologie I* as taught in Summer Semester 2017, including: essentials of point-set topology, fundamental group and covering spaces, cell complexes, basics of singular homology.

Students who have not previously seen the main definitions of singular homology are advised to read through the first portion of Section 2.1 (approximately pages 97 to 126) in Hatcher's book before the start of the semester. We will quickly review this material in the first week.

Depending on students' backgrounds, we may also assume some knowledge of smooth manifolds and differential topology (including the implicit function theorem and Sard's theorem, which were covered in Topologie I last semester), but we can also review this during the semester if necessary.

Course description

This is a course in algebraic topology for students with background knowledge of the material in *Topologie I* as described above under **Prerequisites**. We will develop the singular homology and cohomology functors in depth, with emphasis on the homology of CW-complexes and manifolds, and also their role within the wider context of axiomatic homology theories and their relationship with higher homotopy groups. The tentative program includes the following topics:

1. Introduction to categories and functors
2. Review of singular homology (homotopy invariance, excision, long exact sequence of the pair)
3. Mayer-Vietoris sequence and applications
4. Singular cohomology
5. The Eilenberg-Steenrod axioms for homology and cohomology theories
6. Brief sketch of alternative homology/cohomology theories (Čech and Alexander-Spanier)
7. Axiomatic computation of homology/cohomology for CW-complexes

8. Cup product on cohomology
9. Künneth formula
10. Universal coefficient theorem
11. Topological manifolds, fundamental classes and Poincaré duality
12. Homological intersection theory, transversality and counting intersections of smooth submanifolds
13. Higher homotopy groups
14. Serre fibrations and the homotopy exact sequence
15. Hurewicz homomorphism $\pi_k(X) \to H_k(X)$
16. Whitehead's theorem on weak homotopy equivalences

If there is extra time near the end of the semester, we may also delve into some of the following, depending on students' interests:

17. Lefschetz fixed point theorem
18. Differential forms and de Rham's theorem
19. Bundles and classifying spaces
20. Obstruction theory
21. Bordism groups
22. Characteristic classes
23. Exotic spheres

Werbung: *In Summer Semester 2018 there will also be a student seminar called "Topics in Topology," which is conceived as a sequel to this course. The precise focus of the seminar remains to be decided; input on this from students is welcome. Possible topics include each of the extra topics listed above, plus topological K-theory and spectral sequences.*

Literature

Almost everything we will discuss in this course is contained in at least one of the following two books:

- Allen Hatcher, *Algebraic Topology*, Cambridge University Press 2002
(also freely downloadable from the author's homepage:
`https://www.math.cornell.edu/~hatcher/AT/ATpage.html`)
- Glen Bredon, *Topology and Geometry*, Springer GTM 1993
(online access available via the HU library)

One of the few drawbacks of Hatcher's book is that it hardly mentions smooth manifolds or differential topology at all; by contrast, these topics are slightly over-emphasized in Bredon's book. Here are some other standard algebraic topology books that overlap heavily with each of these:

- James W. Vick, *Homology Theory*, Springer GTM 1994
(online access available via the HU library)
- R. Stöcker und H. Zieschang, *Algebraische Topologie - Eine Einführung*, Teubner 1994
(available in the HU library, Freihandbestand)

And here are a few standard sources for topics in *differential* topology:

- John Milnor, *Topology from the Differentiable Viewpoint*, Princeton University Press 1997
- Morris W. Hirsch, *Differential Topology*, Springer GTM 1976
  (available in the HU library, Freihandbestand)
- Raoul Bott and Loring W. Tu, *Differential Forms in Algebraic Topology*, Springer GTM 2010
  (available in the HU library, Freihandbestand)

Milnor's book is a classic that is relatively easy to read (and everyone should), while Hirsch's book is an important reference for all the slightly tedious but necessary proofs of basic results on transversality. The book by Bott and Tu is really an algebraic topology book but with an explicit focus on smooth manifolds, so e.g. singular cohomology (one of the main topics we will talk about in the course) is largely replaced by de Rham cohomology, which is defined in terms of differential forms and thus only makes sense in the smooth category. Finally, the following book is a classic which I cannot recommend as a textbook for learning the material, but its importance as a historical document earns it a place on this list:

- Samuel Eilenberg and Normal Steenrod, *Foundations of Algebraic Topology*, Princton U. Press 1952
  (available in the HU library, Freihandbestand)

Exam and problem sets

Grades in the course will be determined by a short **oral exam** soon after the end of the semester (with a resit option shortly before the beginning of the following semester). In the exam, you will need to be able to write down the main definitions in the course, discuss their meaning and significance (with reference to examples where appropriate), and describe the most important applications of the major theorems and the main ideas behind their proofs.

There will be one graded assignment midway through the semester, a so-called **take-home midterm**, which you will have two weeks to work on. Achieving a score of 75% or better on the take-home midterm can boost your final exam grade by one notch, i.e.

- $\geq 75\%$ on midterm = (2,0 $\rightarrow$ 1,7 or 1,7 $\rightarrow$ 1,3 etc.)

There will also be ungraded **problem sets** handed out every Wednesday and discussed in the problem class on the following Wednesday.